Transition to superfluid turbulence in quasi two-dimensional confinement.

Emil Varga

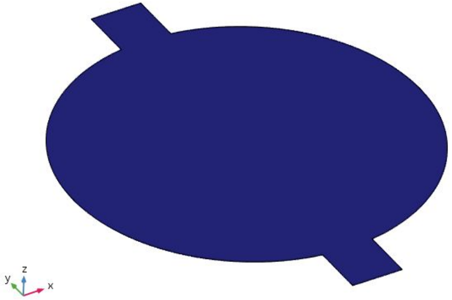
Charles University, Prague, Czech Republic (University of Alberta, Canada)

emil.varga@matfyz.cuni.cz

Bridging classical and quantum turbulence, Cargèse 2023

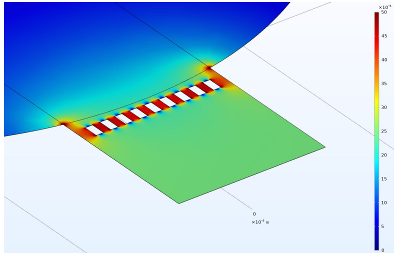
Outline

1. Superfluid helium and turbulence in 3D and 2D

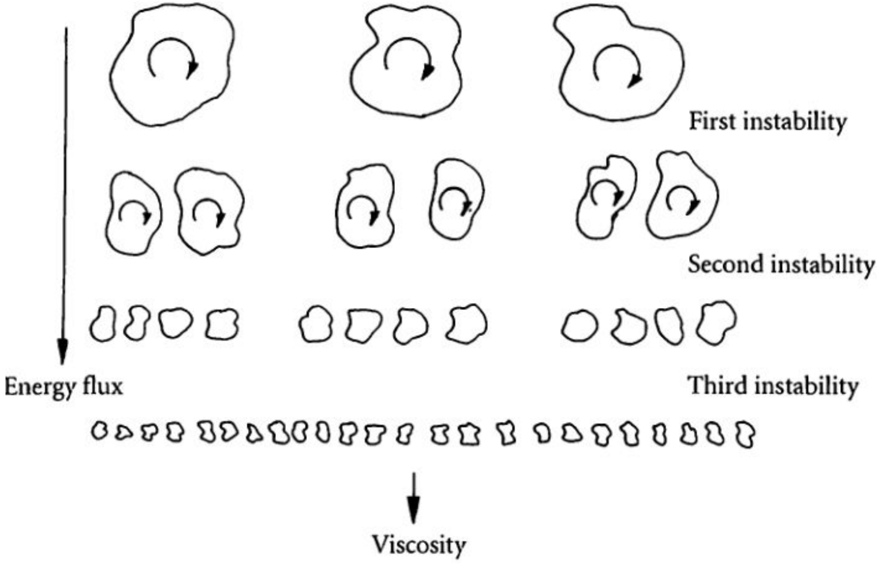


2. (Quasi) two-dimensional quantum turbulence

3. A critical point in transition to turbulence?

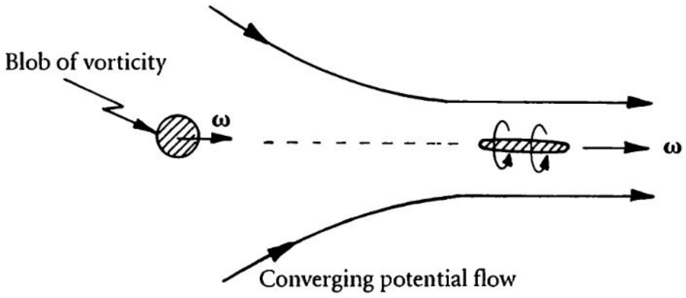


In 3D, turbulent kinetic energy tends to be transported from large scales to small, where it is dissipated.



$$\boldsymbol{\omega} = \nabla \times \mathbf{v}$$

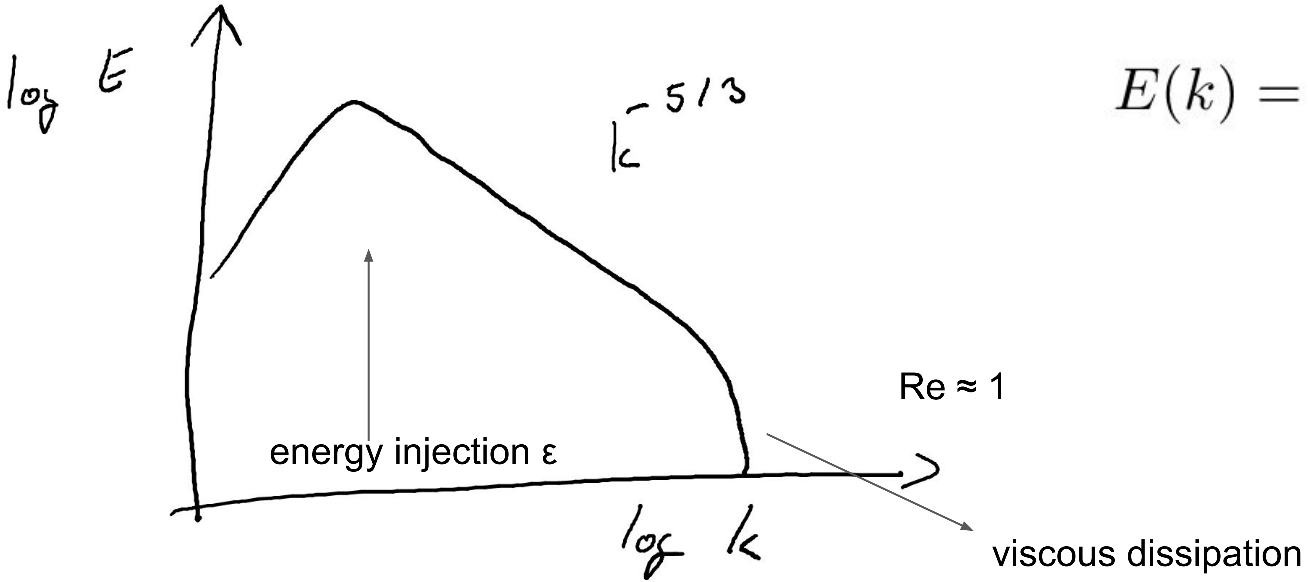
$$\frac{\mathrm{D}\boldsymbol{\omega}}{\mathrm{D}t} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} + \nu \nabla^2 \boldsymbol{\omega}$$



P. A. Davidson, *Turbulence*, Oxford University Press (2004)

In 3D, turbulent kinetic energy tends to be transported from large scales to small, where it is dissipated.

$$\frac{1}{2}\langle u^2 \rangle = \int_0^\infty E(k)\mathrm{d}k$$



Kolmogorov

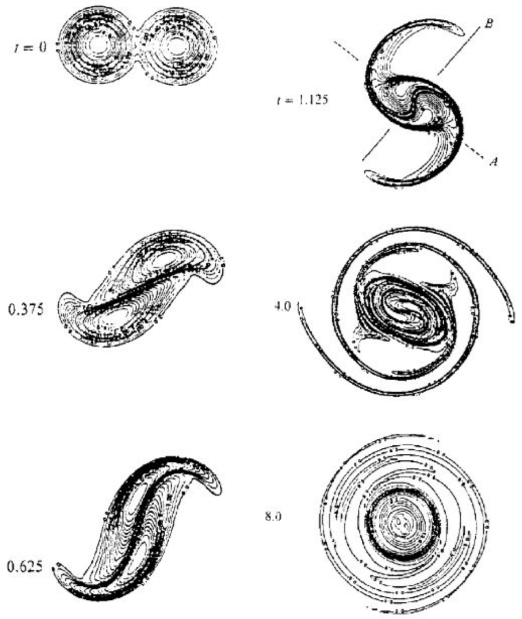
$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

Turbulence in 2D behaves differently from turbulence in 3D – vortex stretching is not allowed, vortices tend to merge.

$$\mathbf{v} = (v_x, v_y, 0)$$

$$\boldsymbol{\omega} = (0, 0, \omega_z)$$

$$\frac{\mathrm{D}\boldsymbol{\omega}}{\mathrm{D}t} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} + \nu \nabla^2 \boldsymbol{\omega}$$



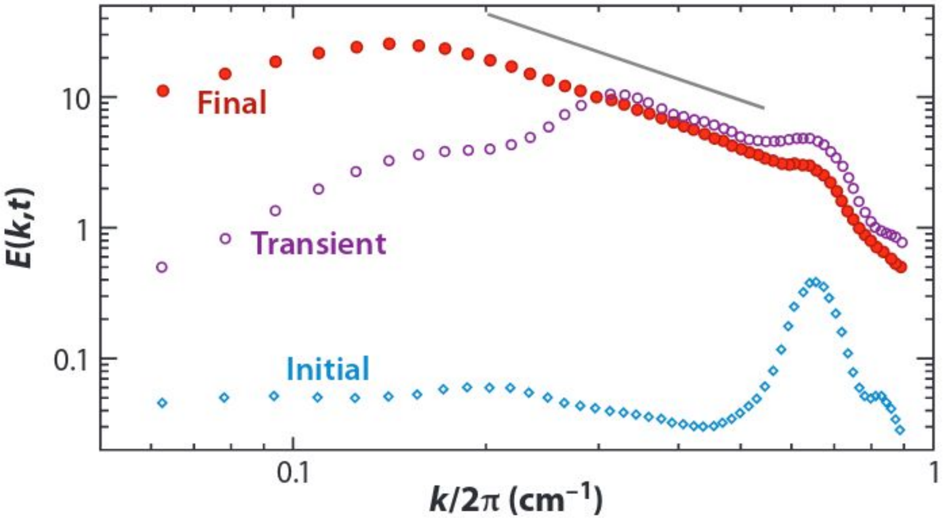
M. V. Melander *et al.*, J. Fluid Mech. **195**, 303-340 (1988)

Turbulence in 2D behaves differently from turbulence in 3D – energy accumulates at the largest scales.

$$\mathbf{v} = (v_x, v_y, 0)$$

$$\boldsymbol{\omega} = (0, 0, \omega_z)$$

$$\frac{\mathrm{D}\boldsymbol{\omega}}{\mathrm{D}t} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} + \nu \nabla^2 \boldsymbol{\omega}$$

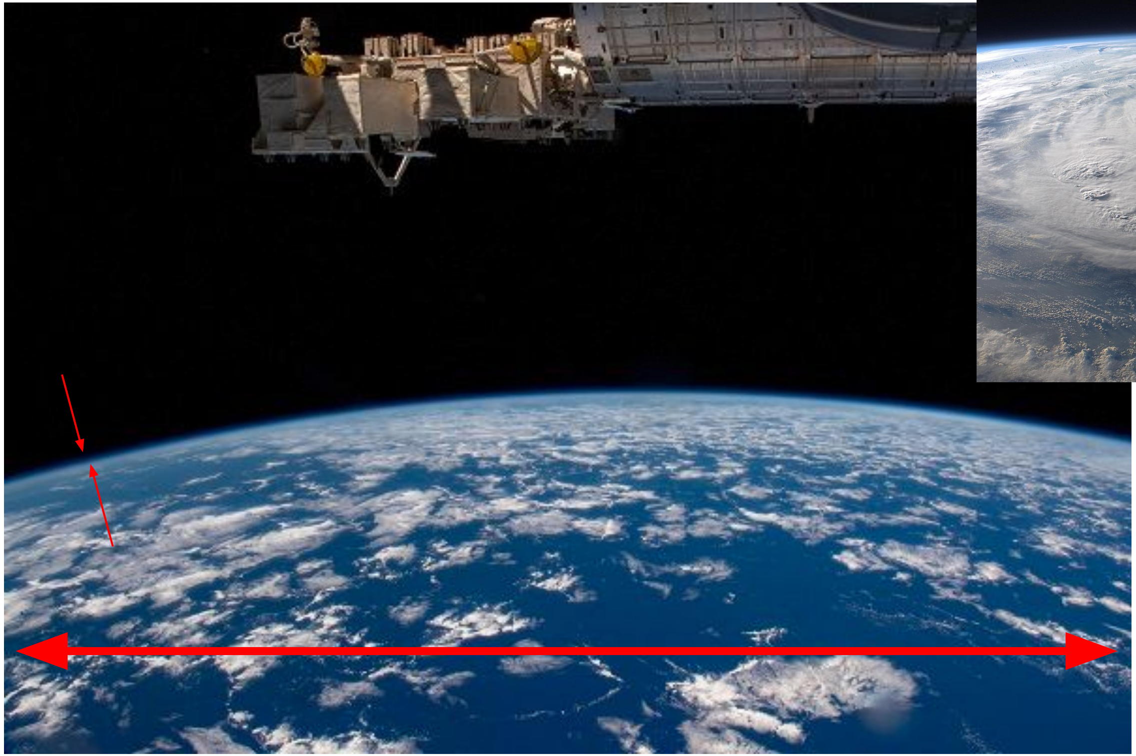


G. Boffetta, R. E. Ecke, *Two-dimensional turbulence*, Annu. Rev. Fluid Mech. **44**, 427 (2012)

Our atmosphere is an example of quasi-2D system

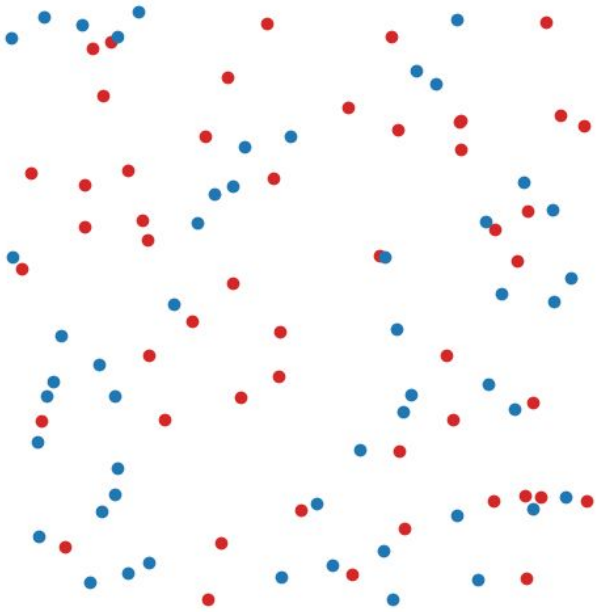
~10 km

10^4 km



Credit: **NASA**

In 2D, a set of discrete vortices can be considered as a gas of interacting particles (Onsager vortex gas)

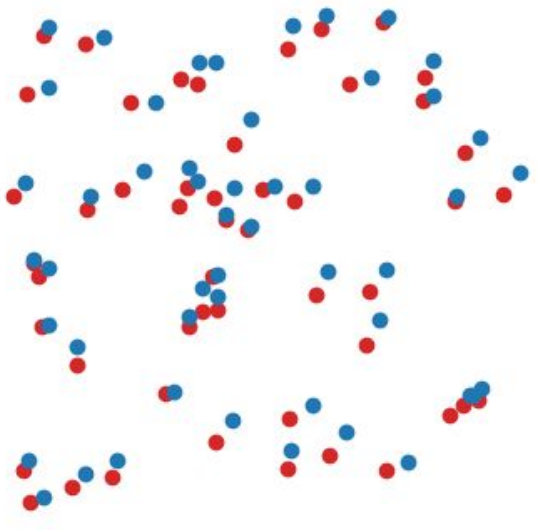


$$\dot{\mathbf{r}}_i = \frac{1}{4\pi} \sum_j \frac{\boldsymbol{\kappa}_j \times (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^2}$$

$$H = -\rho_s \sum_{ij} \kappa_i \kappa_j \log r_{ij}$$

L. Onsager, Statistical Hydrodynamics, Nuovo Cimento 6, 279 (1949)

Increasing the energy per vortex leads to reduction of the distance between like-signed vortices



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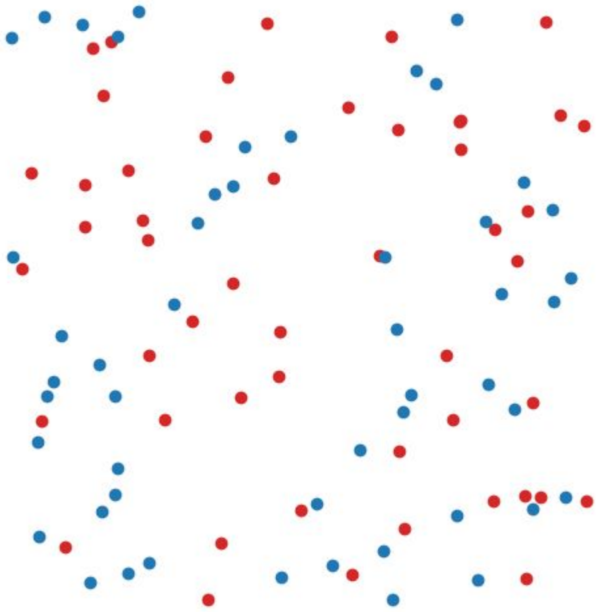

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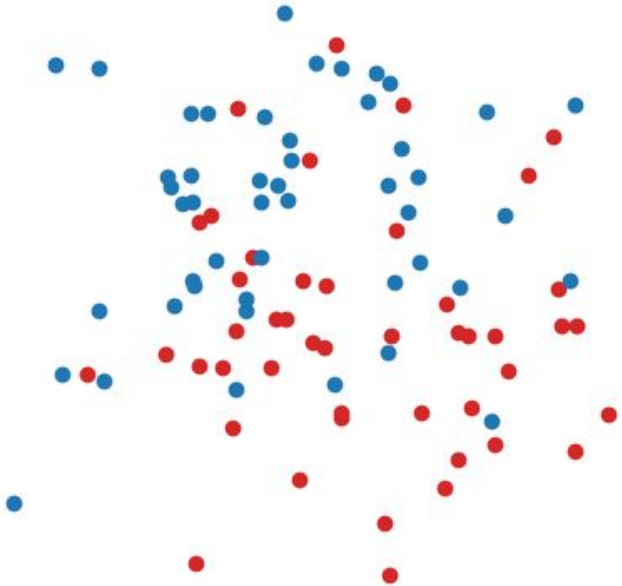

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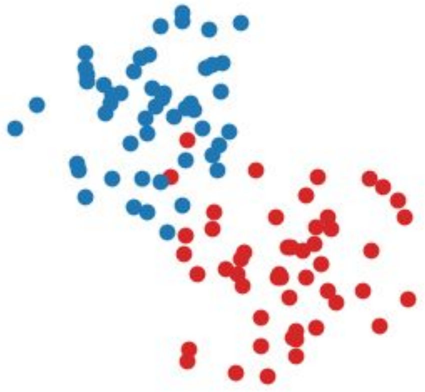

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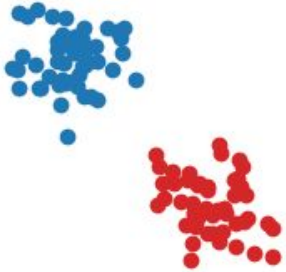
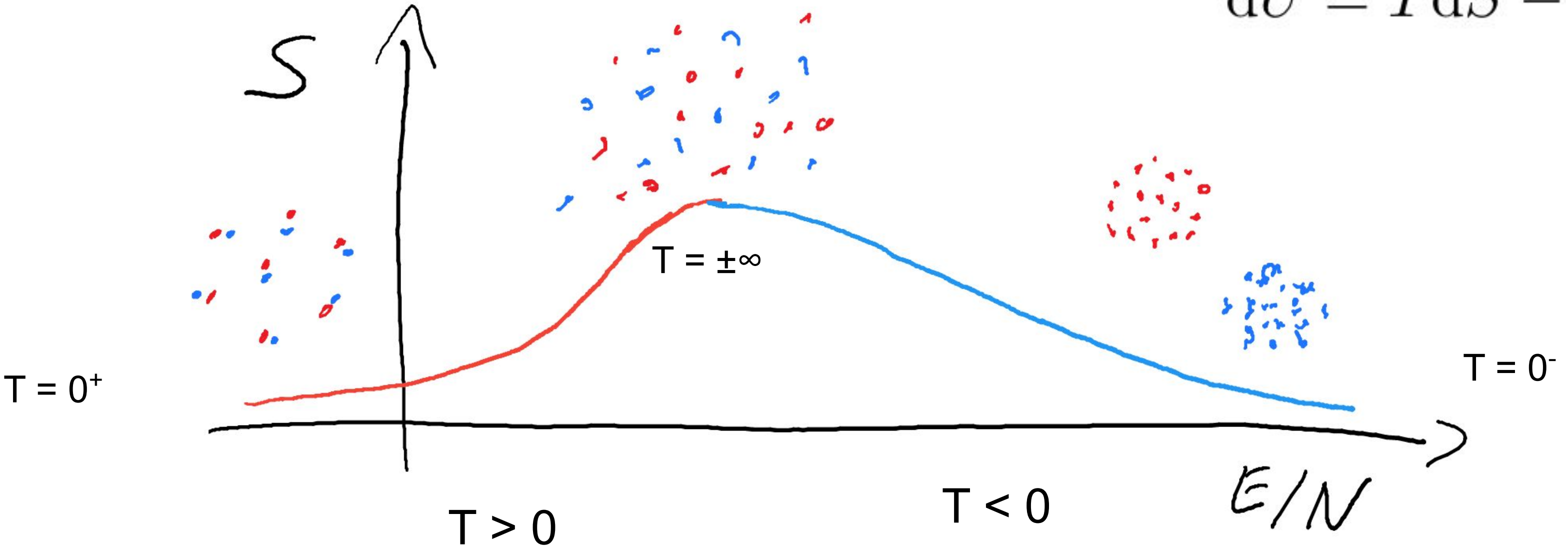
Increasing the energy per vortex leads to reduction of the distance between like-signed vortices



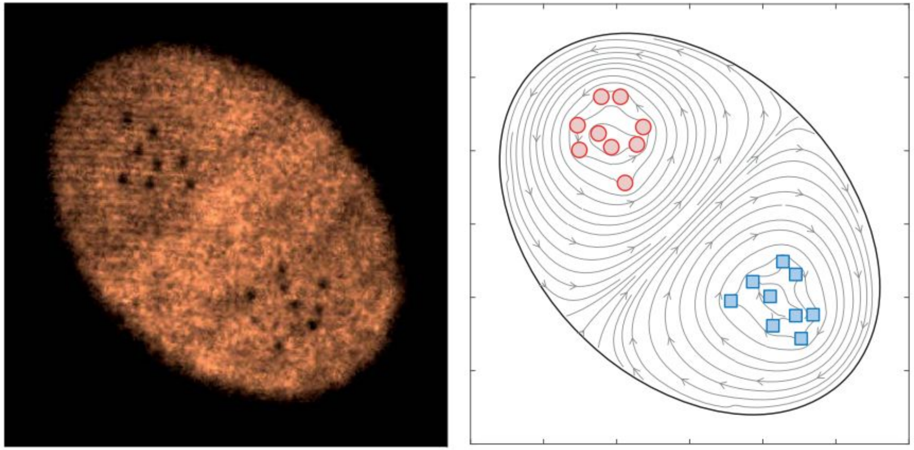
$$H = -\rho_s \sum_{ij} \kappa_i \kappa_j \log r_{ij}$$

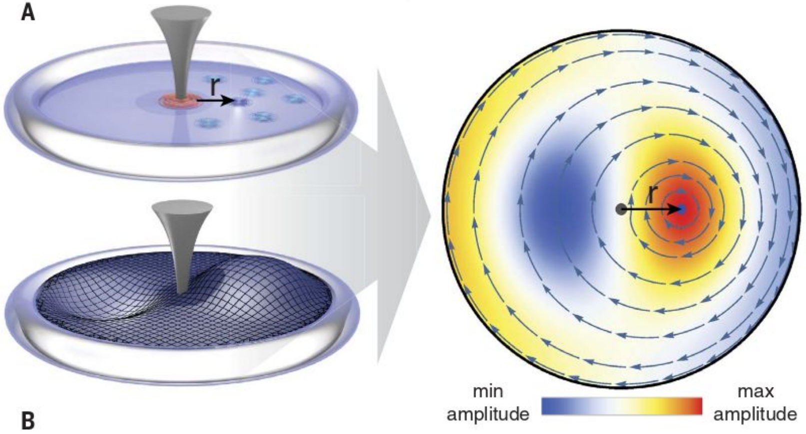
Within the gas analogy, the entropy of the system is decreasing as the energy increases – *T < 0*

$$\mathrm{d}U = T\mathrm{d}S - p\mathrm{d}V$$



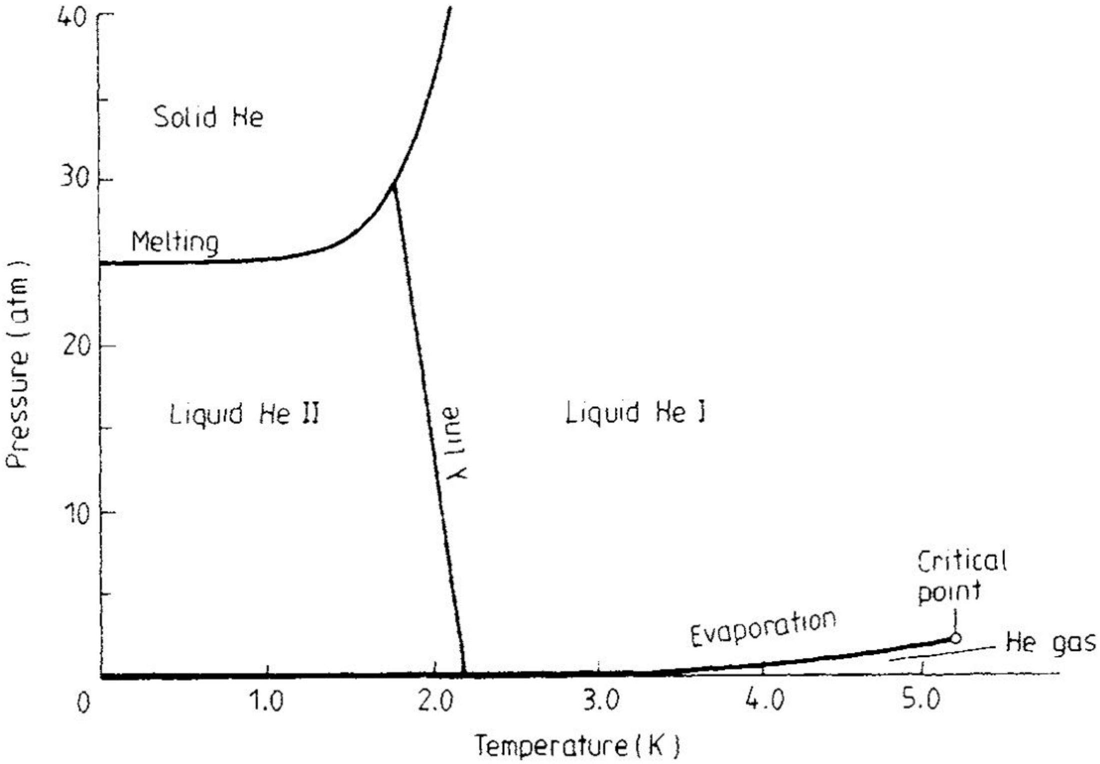
Onager vortex gas at negative temperature was observed in Bose-Einstein condensates and ^{4}He films



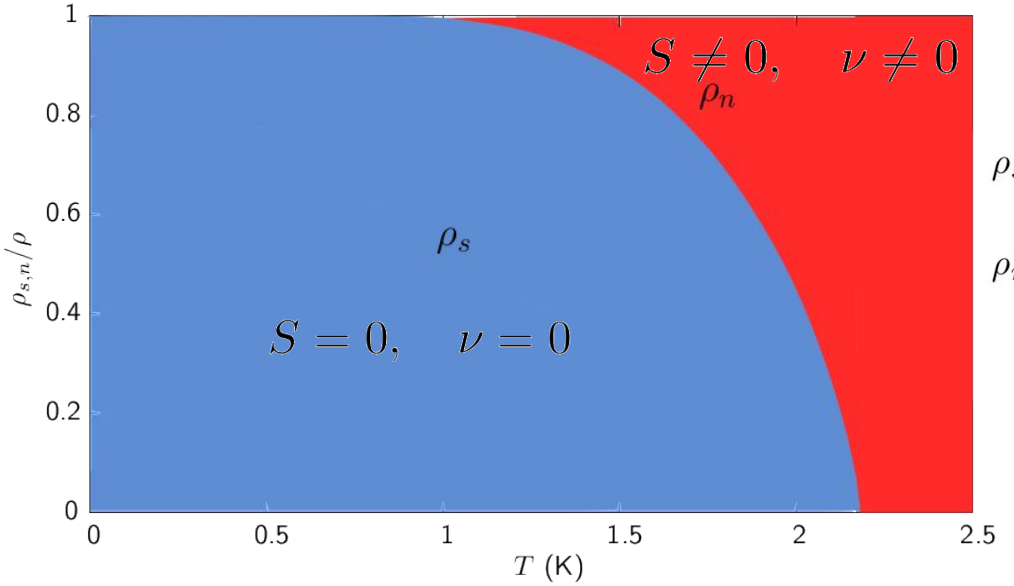


Gauthier et al., Science 364, 1264–1267 (2019)
Johnstone et al., Science 364, 1267–1271 (2019)
Y. P. Sachkou et al., Science 366, 1480–1485 (2019)

^4He never solidifies at low pressure, but enters a superfluid phase around 2 K



Superfluid ^{4}He can undergo two motions at once – viscous normal flow and inviscid superflow

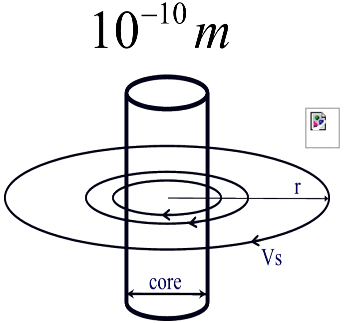


Two equations of motion

$$\rho_s \left(\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s \right) = -\frac{\rho_s}{\rho} \nabla p + \rho_s \sigma \nabla T$$

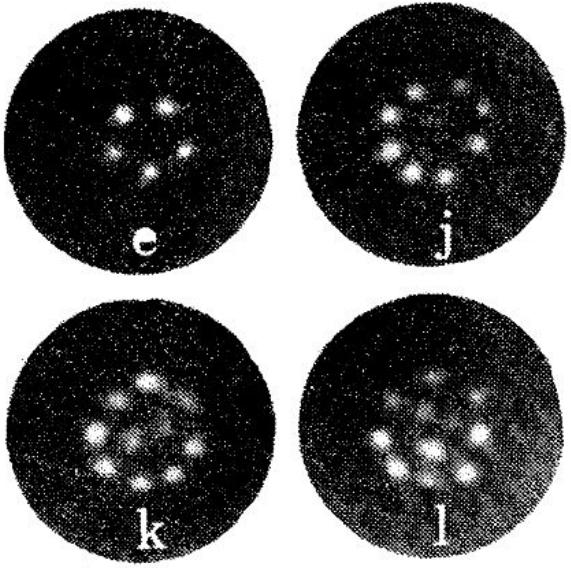
$$\rho_n \left(\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n \right) = -\frac{\rho_n}{\rho} \nabla p - \rho_s \sigma \nabla T + \eta \nabla^2 \mathbf{v}_n$$

Quantization of circulation restricts vorticity into topological defects



$$\oint_C \mathbf{v} \cdot d\mathbf{l} = n\kappa$$

$$\kappa = \frac{h}{m_4} \approx 9.96 \times 10^{-4}\ \mathrm{cm}^2/\mathrm{s}$$

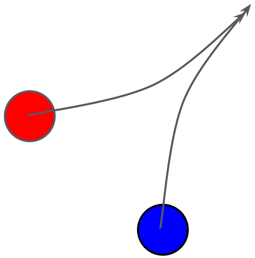


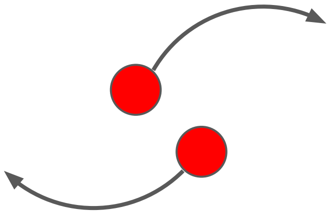
Visualization of vortices generated by rotation

E. J. Yarmchuk, M. J. V. Gordon, and R. E. Packard Phys. Rev. Lett. **43**, 214 (1979)

Dissipation causes like-signed vortices to repel and opposite vortices to attract

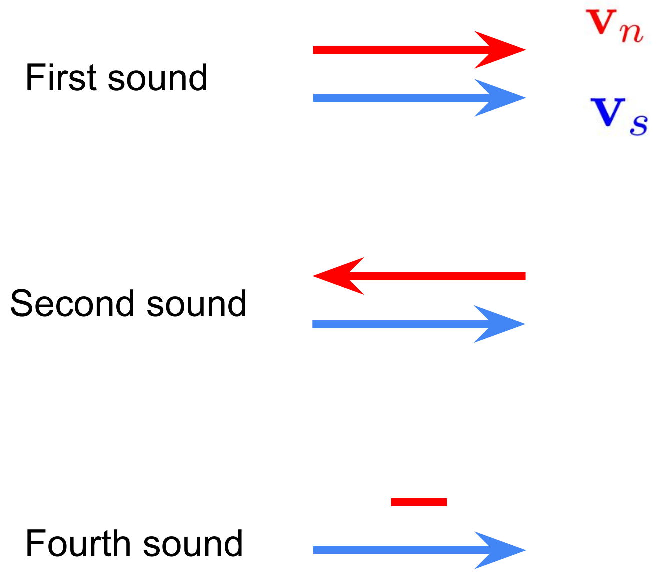
$$\frac{\partial \boldsymbol{s}}{\partial t} = \boldsymbol{v}_s + \beta \boldsymbol{s}' \times \boldsymbol{s}'' + \alpha \boldsymbol{s}' \times (\boldsymbol{v}_n - \boldsymbol{v}_s) + \alpha' \boldsymbol{s}' \times (\boldsymbol{s}' \times (\boldsymbol{v}_n - \boldsymbol{v}_s))$$

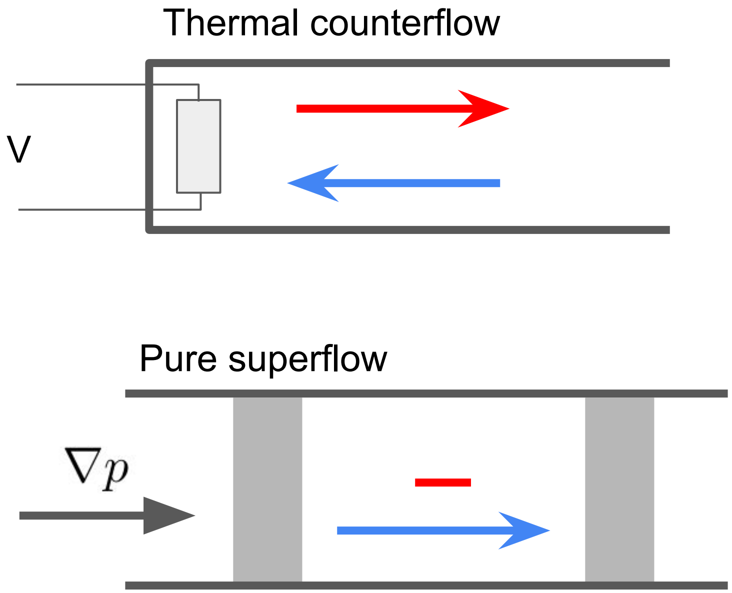




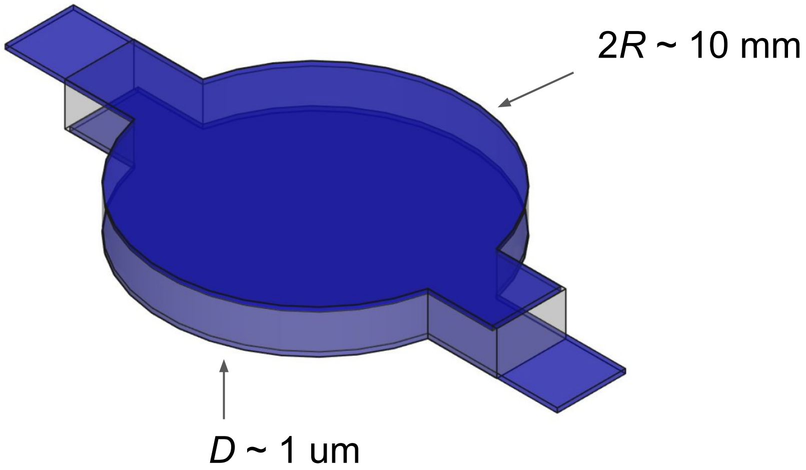
$$H = -\rho_s \sum_{ij} \kappa_i \kappa_j \log r_{ij}$$

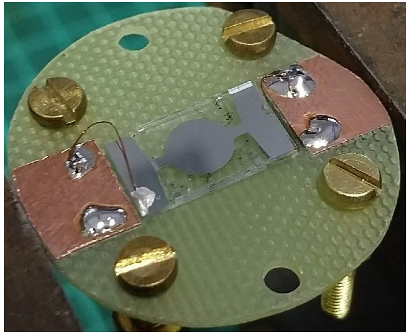
The two-fluid nature of superfluid helium offers more richness in the type of generated flows

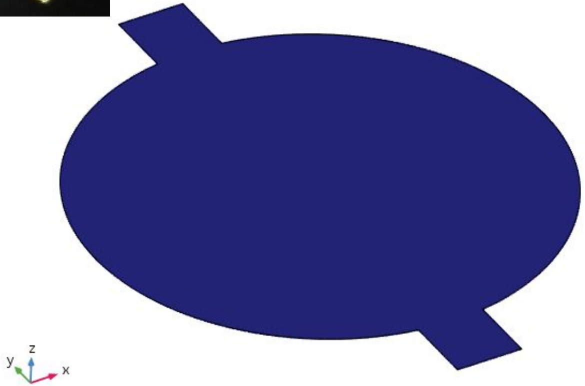




We study strongly confined superfluid ^{4}He using nanofluidic Helmholtz resonators







$$\delta C(t) \propto y(t)$$

$$\Omega_m = \sqrt{\frac{k_{\mathrm{eff}}}{m_{\mathrm{eff}}}} \propto \sqrt{\frac{\rho_s}{\rho^2}}$$

F. Souris *et al.*, Phys. Rev. Applied (2017)
X. Rojas *et al.*, Phys. Rev. B (2015)

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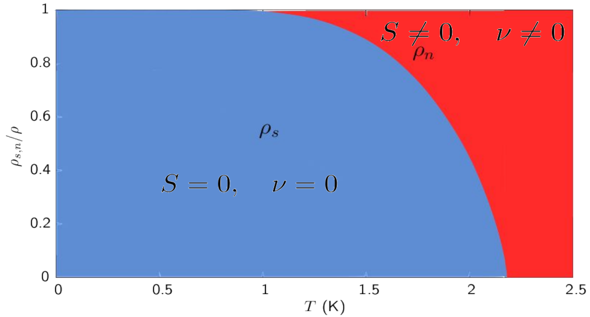
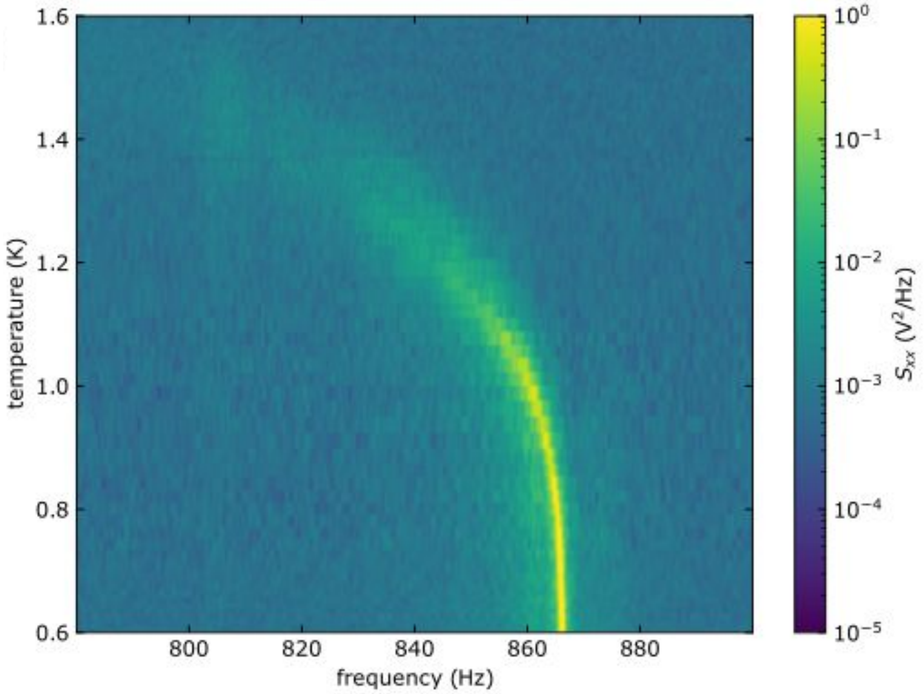
The frequency of the mode follows the superfluid fraction

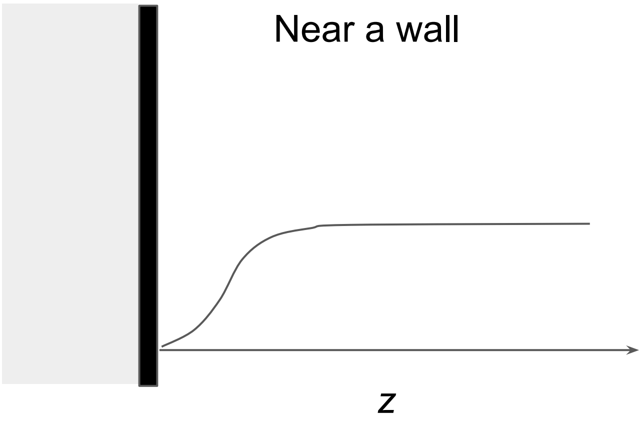


$$\Omega_m = \sqrt{\frac{k_{\text{eff}}}{m_{\text{eff}}}} \propto \sqrt{\frac{\rho_s}{\rho^2}}$$

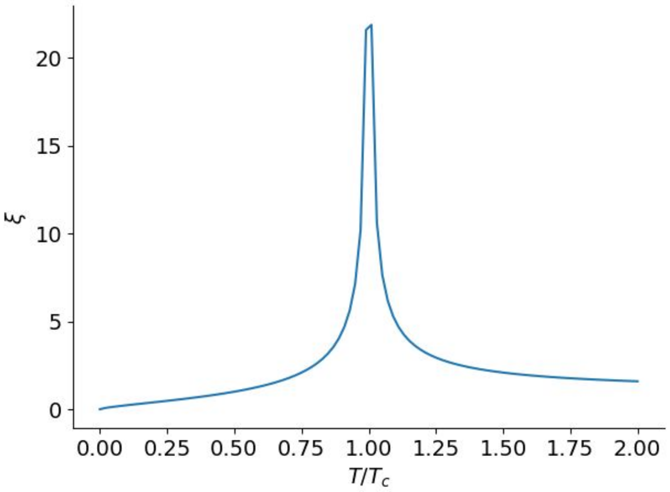


Near the wall the macroscopic wave function must go to zero.

$$\rho_s = |\Psi|^2$$

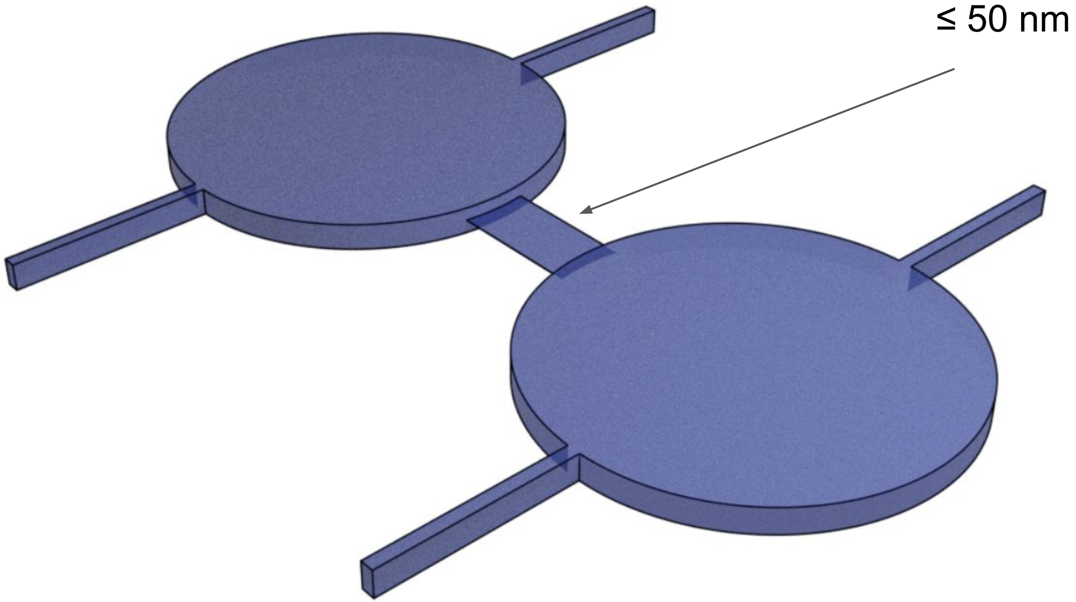


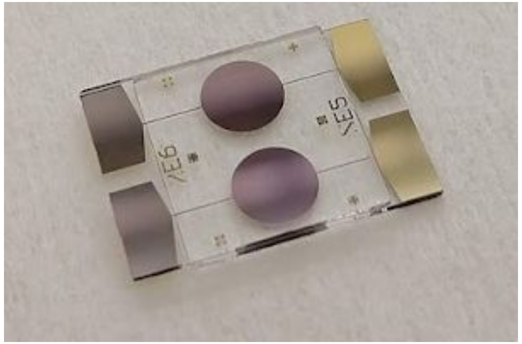
$$\Psi \propto (1 - e^{-z/\xi}) \qquad \xi_0 \approx 1\ \text{Å}$$



$$\xi = \xi_0 t^{-\nu} \qquad t = \left|1 - \frac{T}{T_c}\right|$$

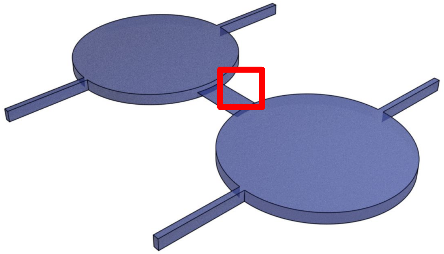
Fully enclosed channels with sub-50 nm height are possible.



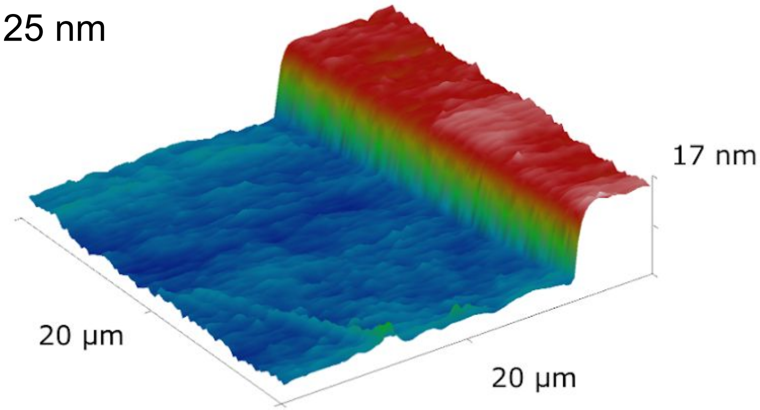


EV, C. Undershute, and J. P. Davis, Phys. Rev. Lett. **129**, 145301 (2022)

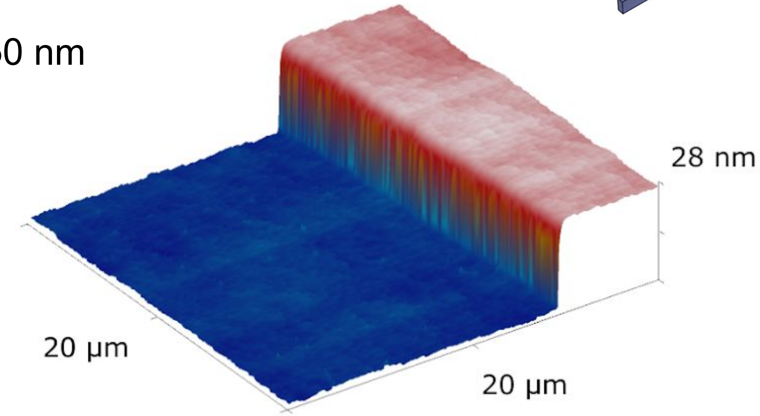
The confined channel is well defined through wet etching of quartz



25 nm



50 nm



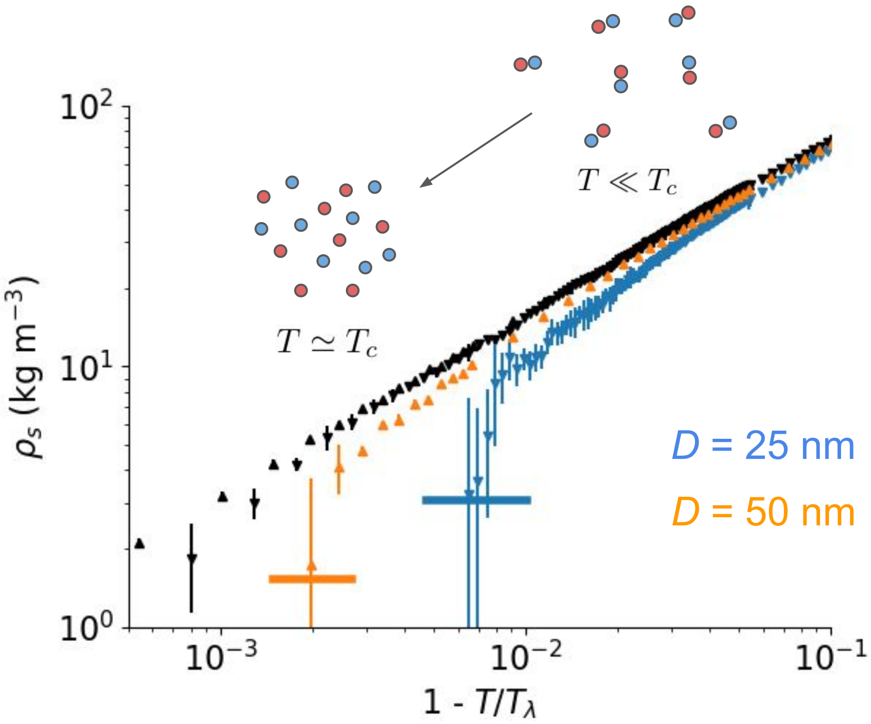
In thin channels superfluidity is suppressed and eventually lost via thermal activation of vortices (Berezinskii-Kosterlitz-Thouless transition)

$$\lim_{T \to T_c^-} \frac{D\rho_s(T)}{T} = \sigma_{\mathrm{KT}}$$

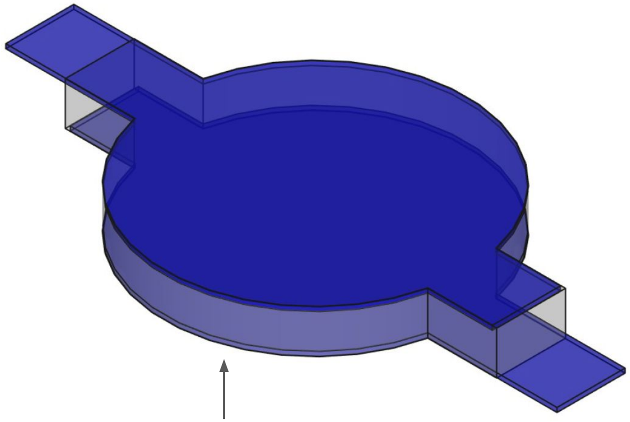
$$\sigma_{\mathrm{KT}} = \frac{2m^2 k_B}{\hbar^2 \pi}$$

$$\sigma_{\mathrm{KT}} = 3.52 \times 10^{-8}\ \mathrm{kg\ m^{-2}K^{-1}}$$

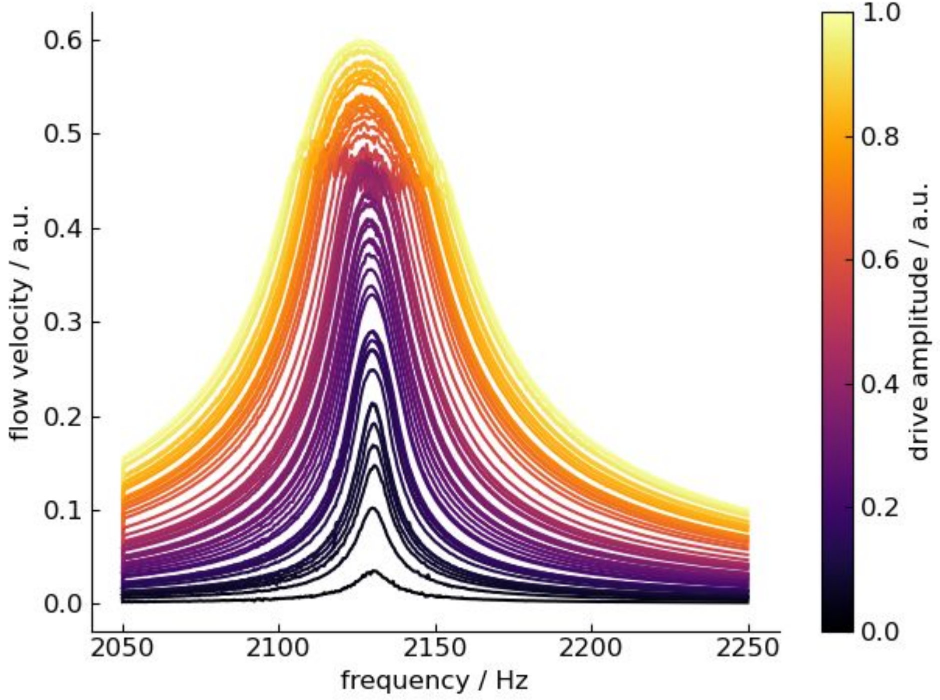
D. R. Nelson, J. M. Kosterlitz, Phys. Rev. Lett. **39,** 1201 (1977)



When driven strongly enough, the Helmholtz resonance becomes nonlinear.

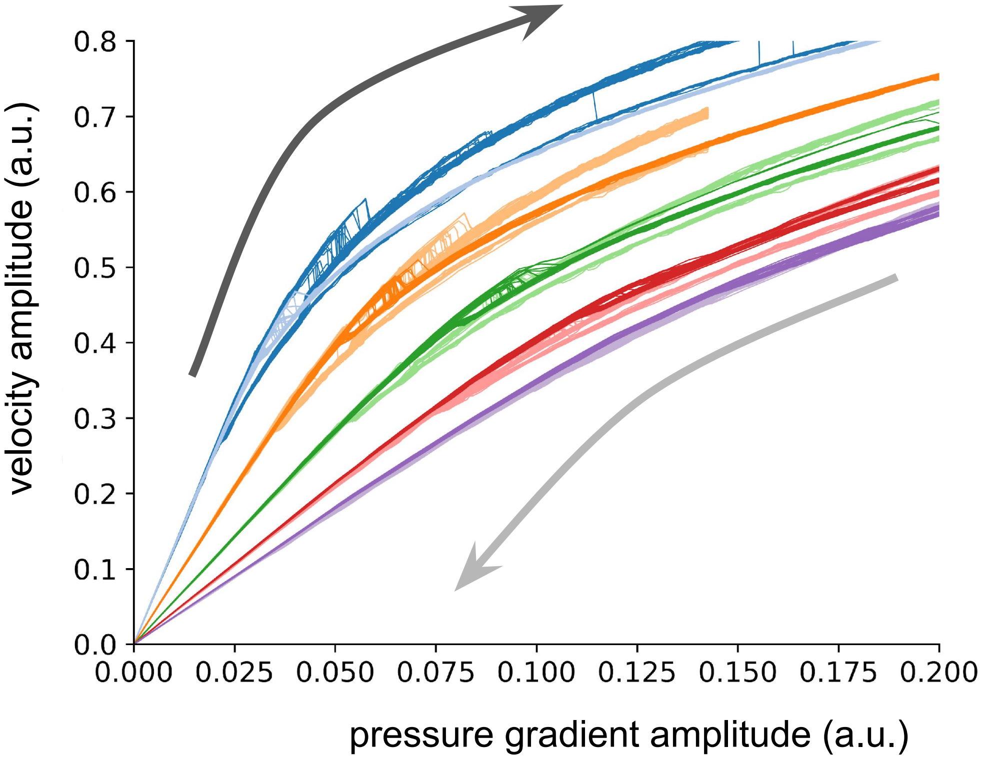


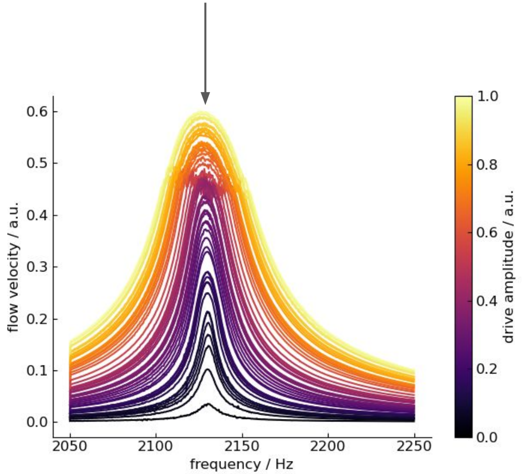
D ~ 0.5 – 2 um



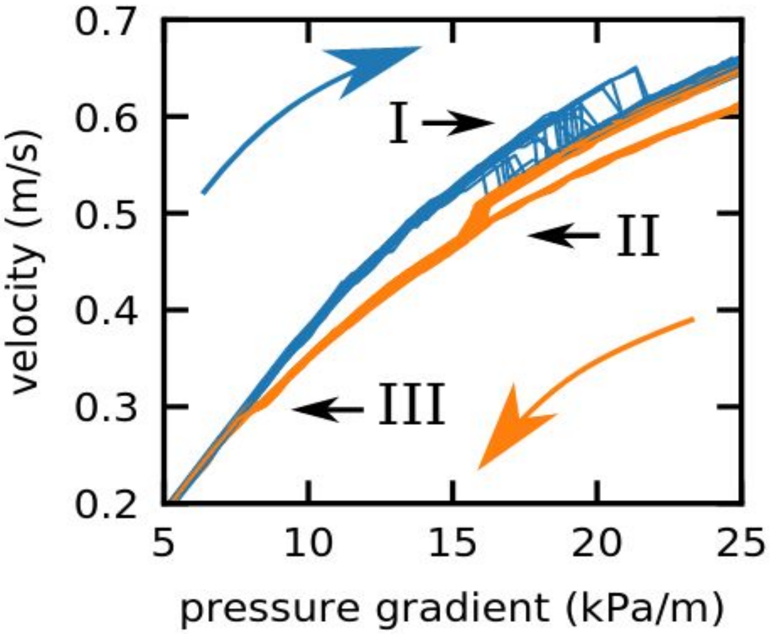
EV, V. Vadakkumbatt, A. J. Shook, P. H. Kim, and J. P. Davis, Phys. Rev. Lett. **128**, 025301 (2020)

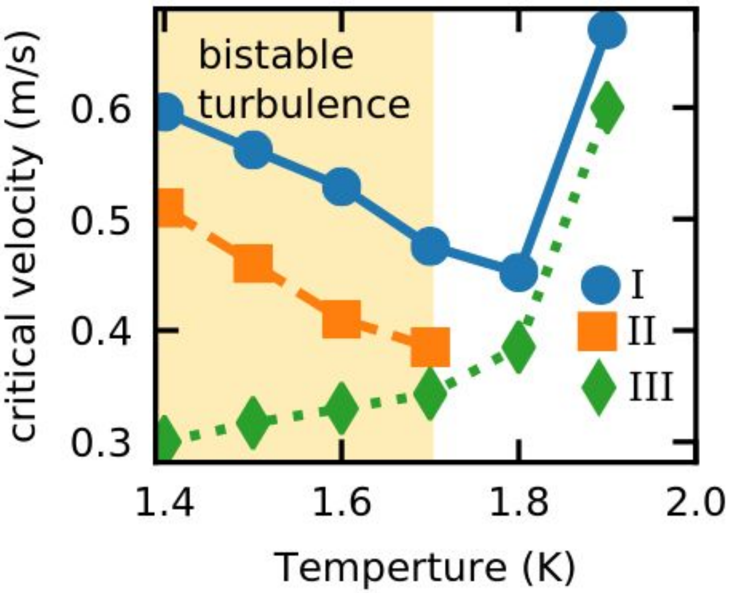
Continuous ramping of applied force reveals hysteretic transition to nonlinear dissipation



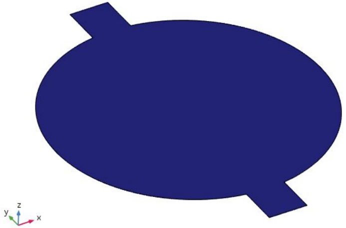


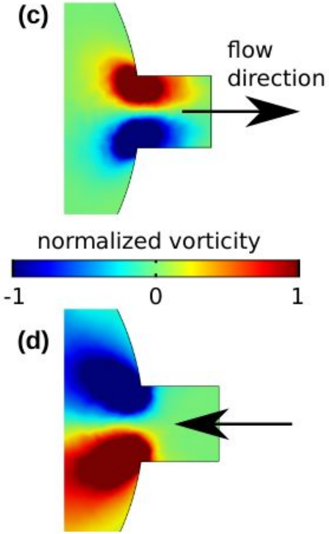
A new “backward” critical velocity appears at sufficiently low temperatures





A likely mechanism behind the dynamics of transition(s) is the competition between two large-scale polarisation states





Quasi-2D vortex dynamics

$$\frac{\mathrm{d}n_+}{\mathrm{d}t} = an_+ + bn_- - dn_+n_- + g_+$$

$$\frac{\mathrm{d}n_-}{\mathrm{d}t} = an_- + bn_+ - dn_+n_- + g_-$$

1) **Splitting and advection out of the domain.**
2) **Splitting (hence quasi-2D).**
3) **Annihilation by collision.**
4) **Creation of new vortices.**

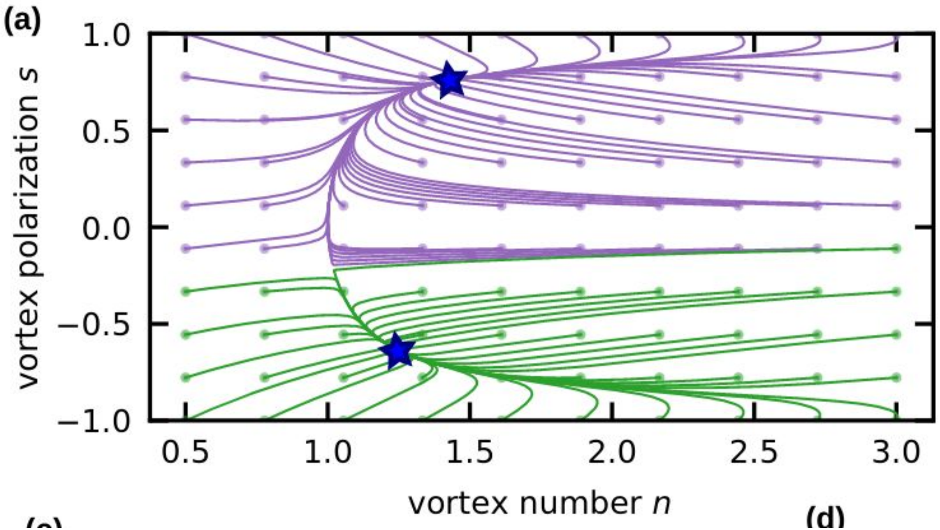
The experiment cannot differentiate between signs of vortices, we need dynamics of the total density n.

$$n = n_+ + n_- \qquad s = \frac{n_+ - n_-}{n}$$

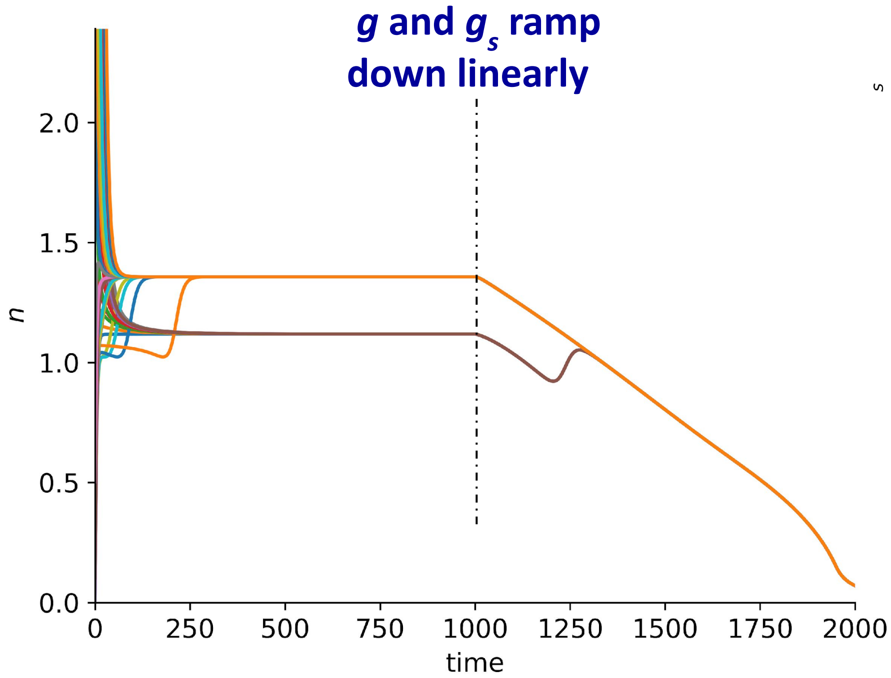
$$\frac{\partial n}{\partial t} = (a+b)n - \frac{1}{2}dn^2(1-s^2) + g,$$

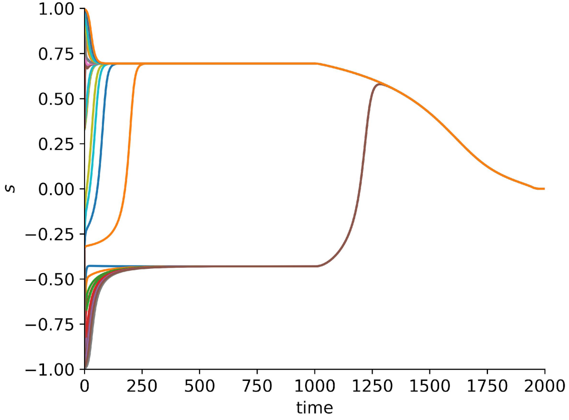
$$\frac{\partial s}{\partial t} = -2bs + \frac{1}{2}dns(1-s^2) + \frac{g_s}{n},$$

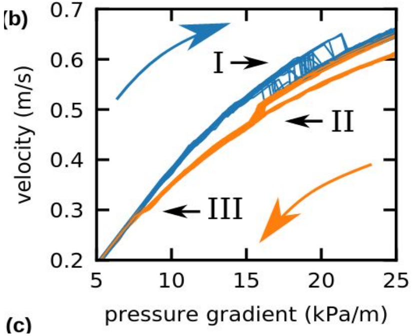
$s > 0$ **$s < 0$**



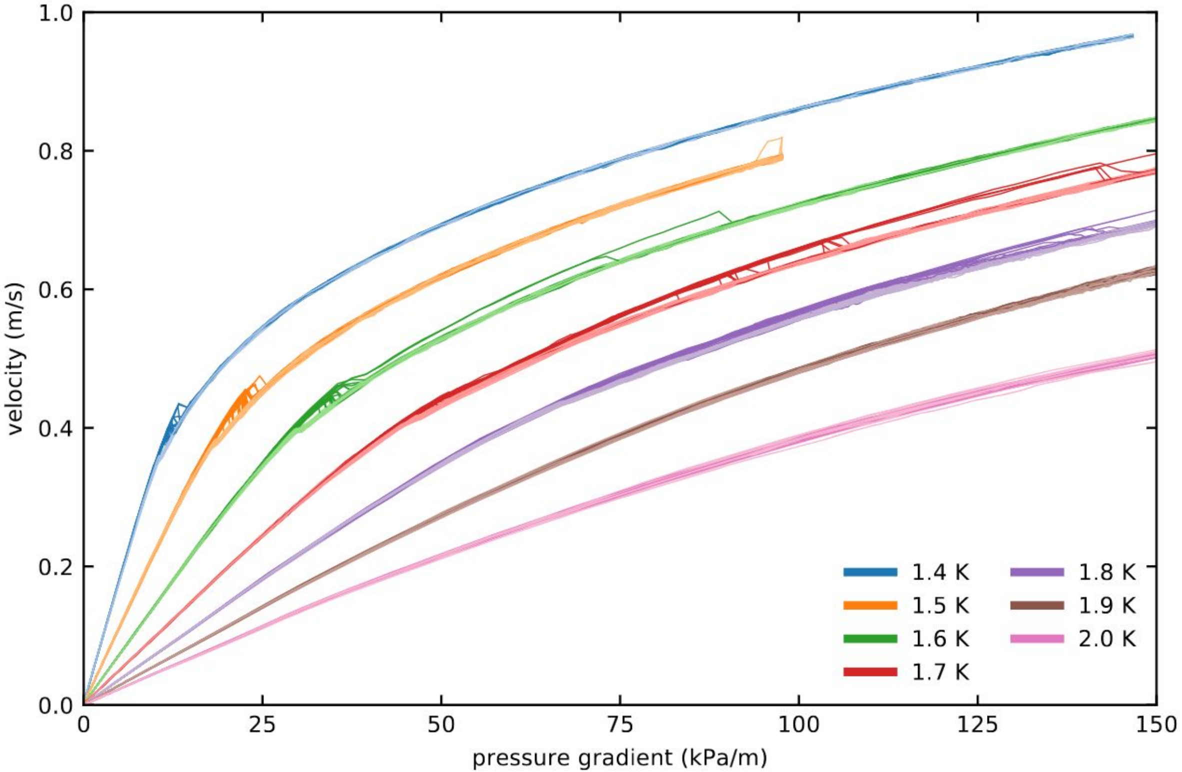
The dynamical system does reproduce the backward transition







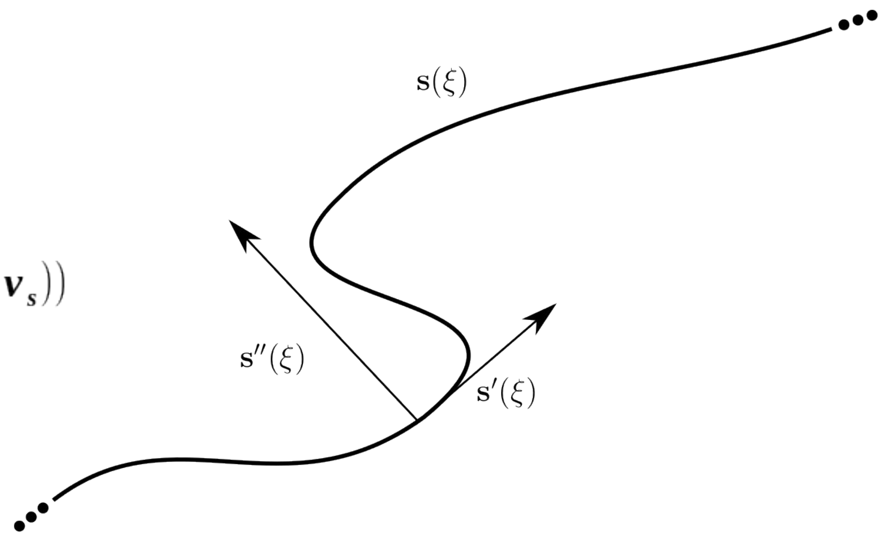
Smaller confinement (800 nm) makes the intermediate state only metastable



We can check whether the vortices can be considered 2D with vortex filament model

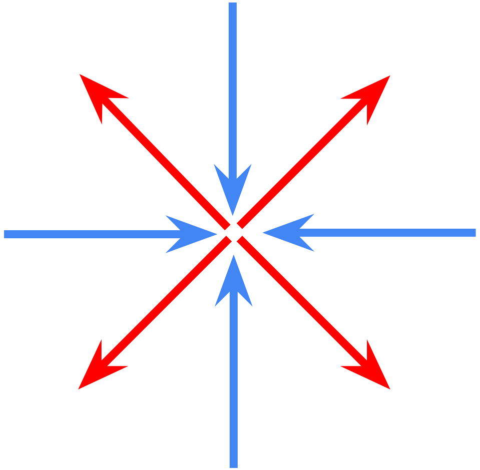
$$\boldsymbol{v}_s(\boldsymbol{r}) = \frac{\kappa}{4\pi} \oint_{\mathcal{L}} \frac{\boldsymbol{s}'(\xi) \times [\boldsymbol{r} - \boldsymbol{s}(\xi)]}{|\boldsymbol{r} - \boldsymbol{s}(\xi)|^3} \, \mathrm{d}\,\xi,$$

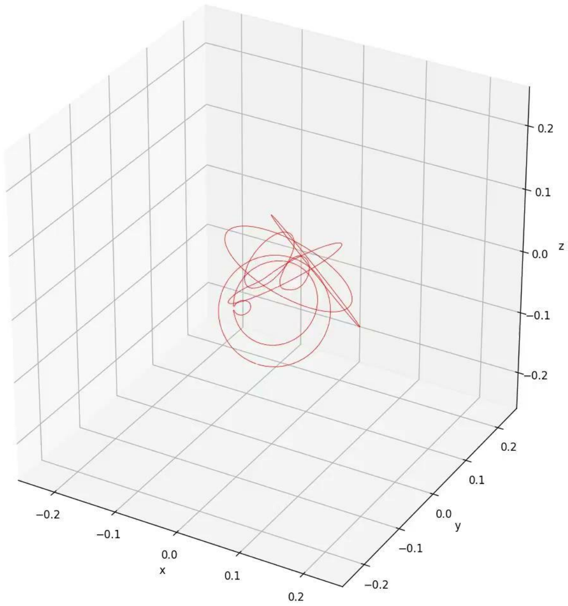
$$\frac{\partial \boldsymbol{s}}{\partial t} = \boldsymbol{v}_s + \beta \boldsymbol{s}' \times \boldsymbol{s}'' + \alpha \boldsymbol{s}' \times (\boldsymbol{v}_n - \boldsymbol{v}_s) + \alpha' \boldsymbol{s}' \times (\boldsymbol{s}' \times (\boldsymbol{v}_n - \boldsymbol{v}_s))$$



With vortex filaments you can make pretty videos

Simulation of a flow
near a point heat source

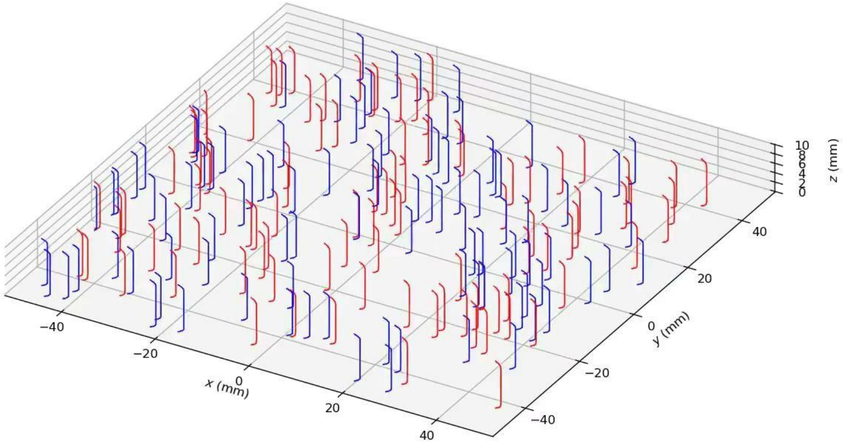




EV, J. Low Temp. Phys. **196**, 28 (2019)

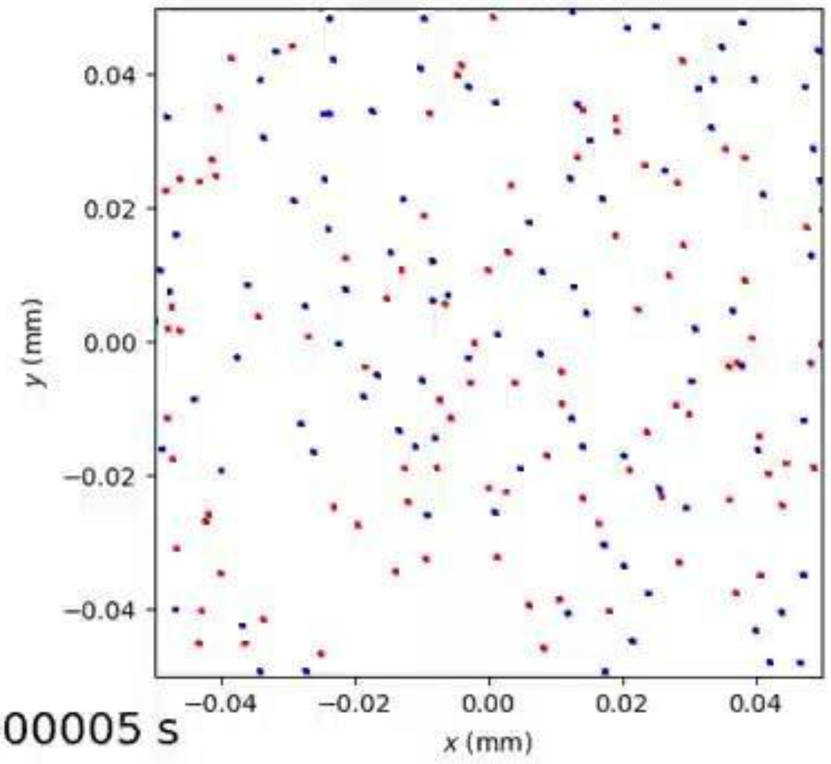
Can the vortices be considered 2D?

10 um



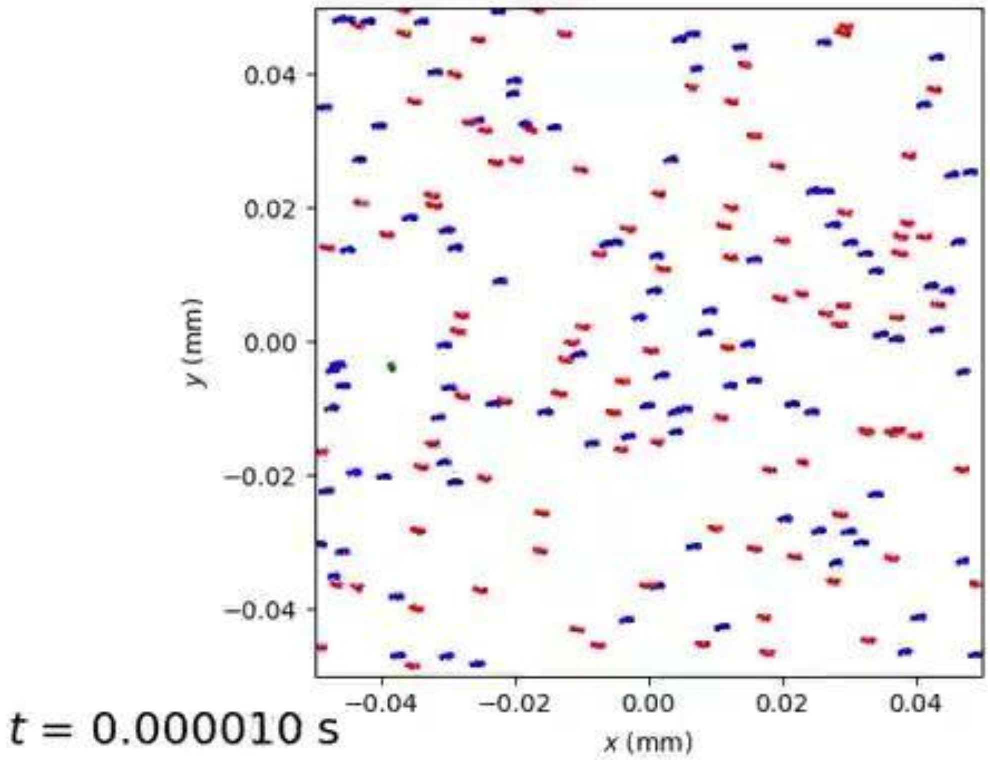
$t = 0.000001$ s

At low velocities vortices cannot move because of pinning



$t = 0.000005$ s

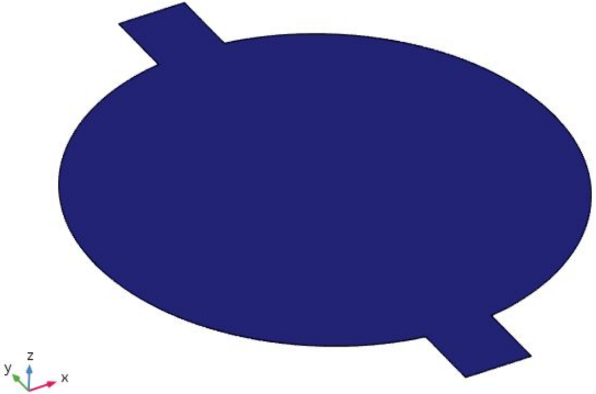
At higher velocities vortices deform enough to slide

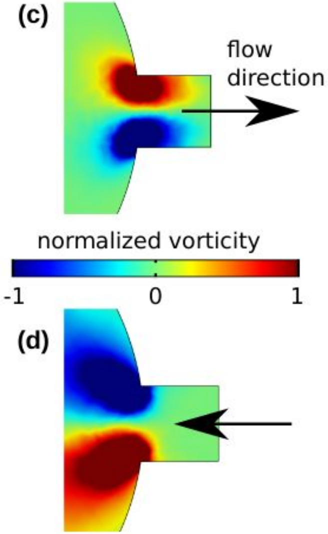


The simulation and post-processing code is free and available to play with

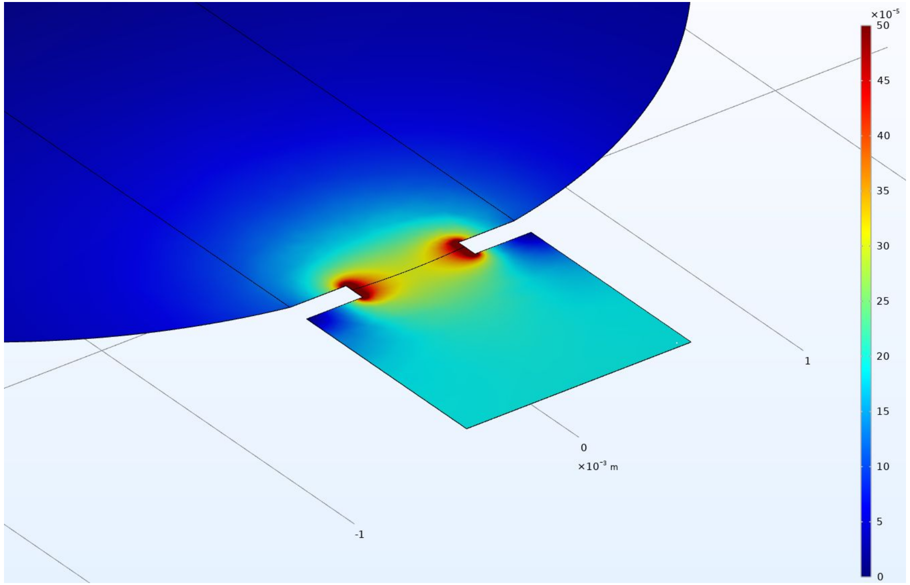
- GPLv3 licensed
- C and Python
- pretty config files (libconf)
- Barnes-Hut (‘tree’) approximation up-to quadruploar terms
- https://bitbucket.org/emil_varga/openvort

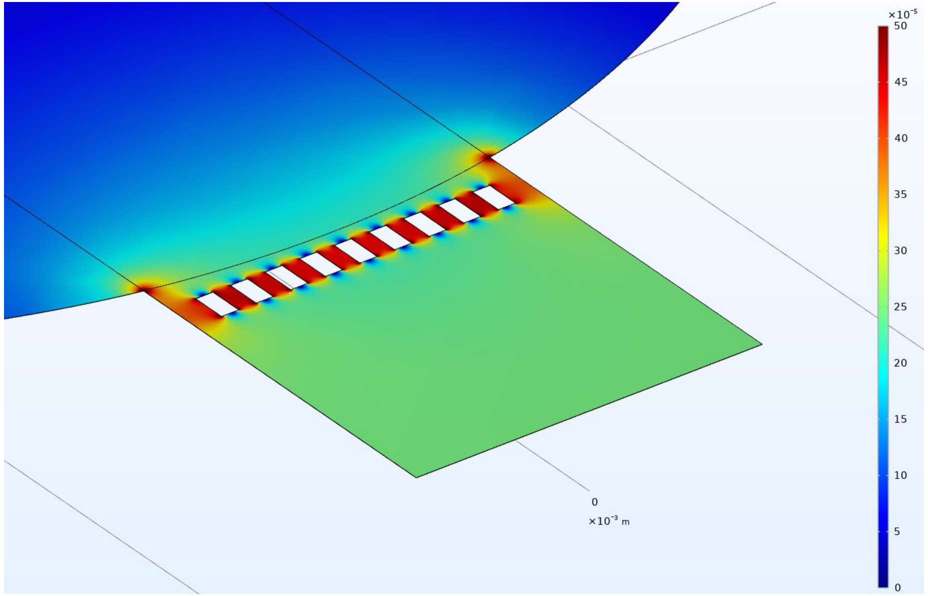
The turbulence was driven at the largest scale.





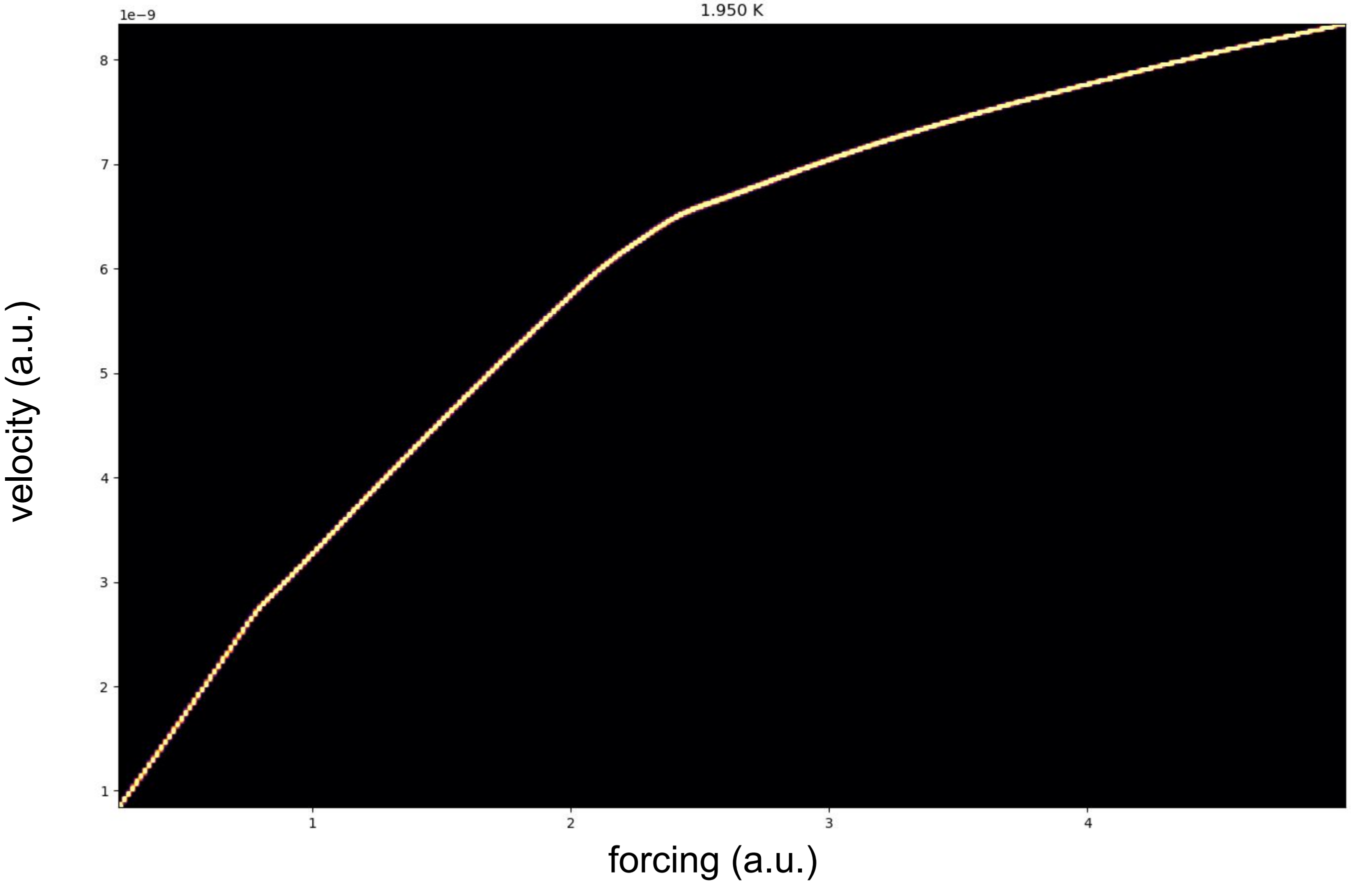
In order to see any influence of the inverse cascade, ratio of energy injection scale to system size must increase

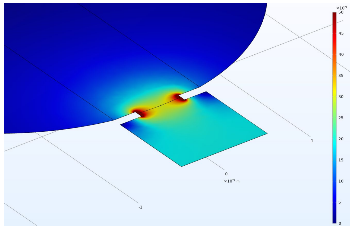


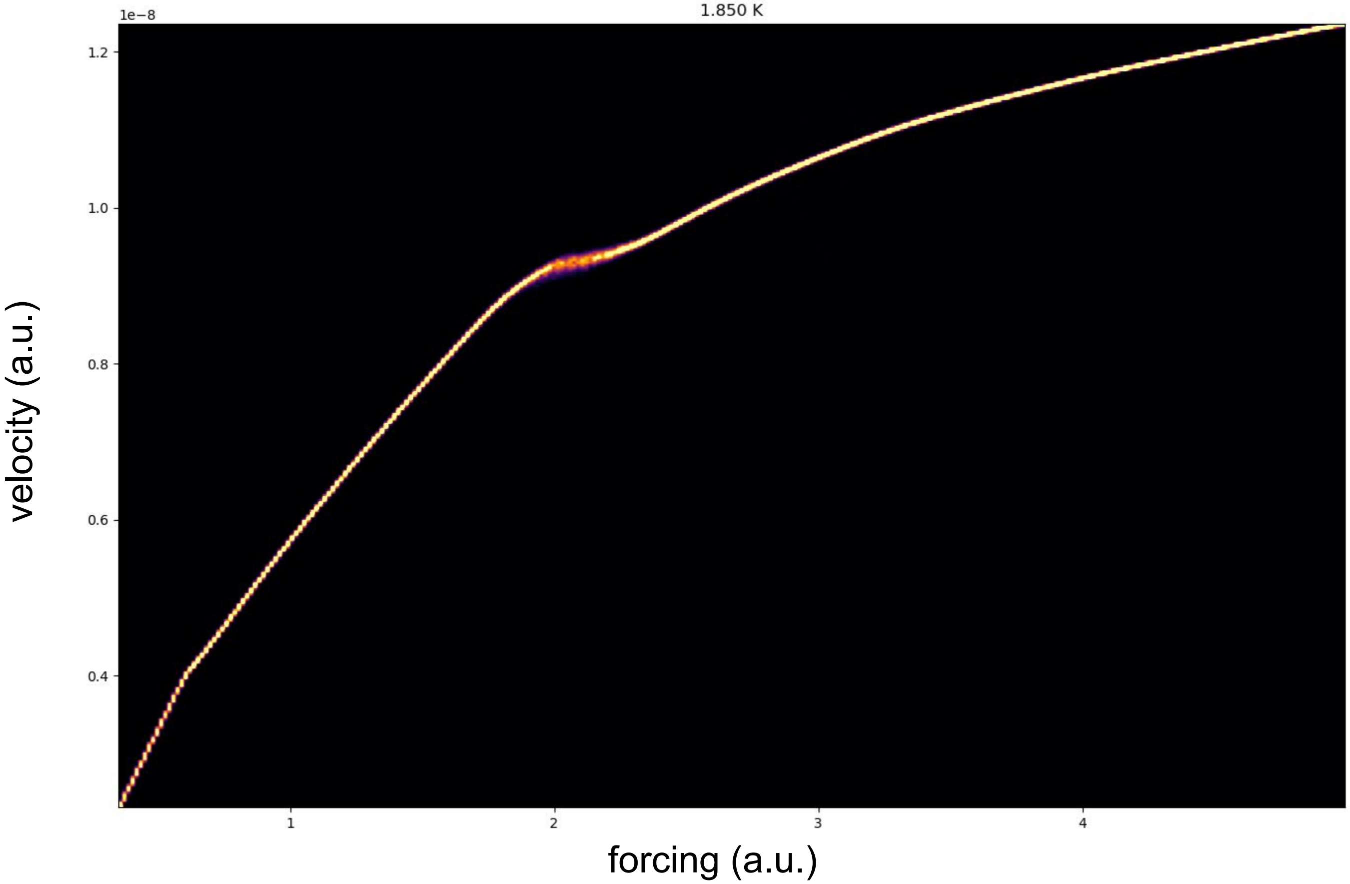


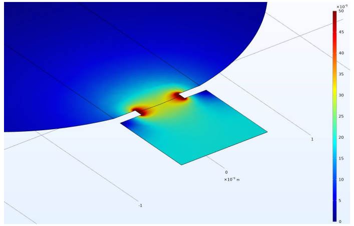
All due to students (Ph.D.) Filip Novotný and (Master’s) Marek Talíř

At high T, at least two continuous transitions occur

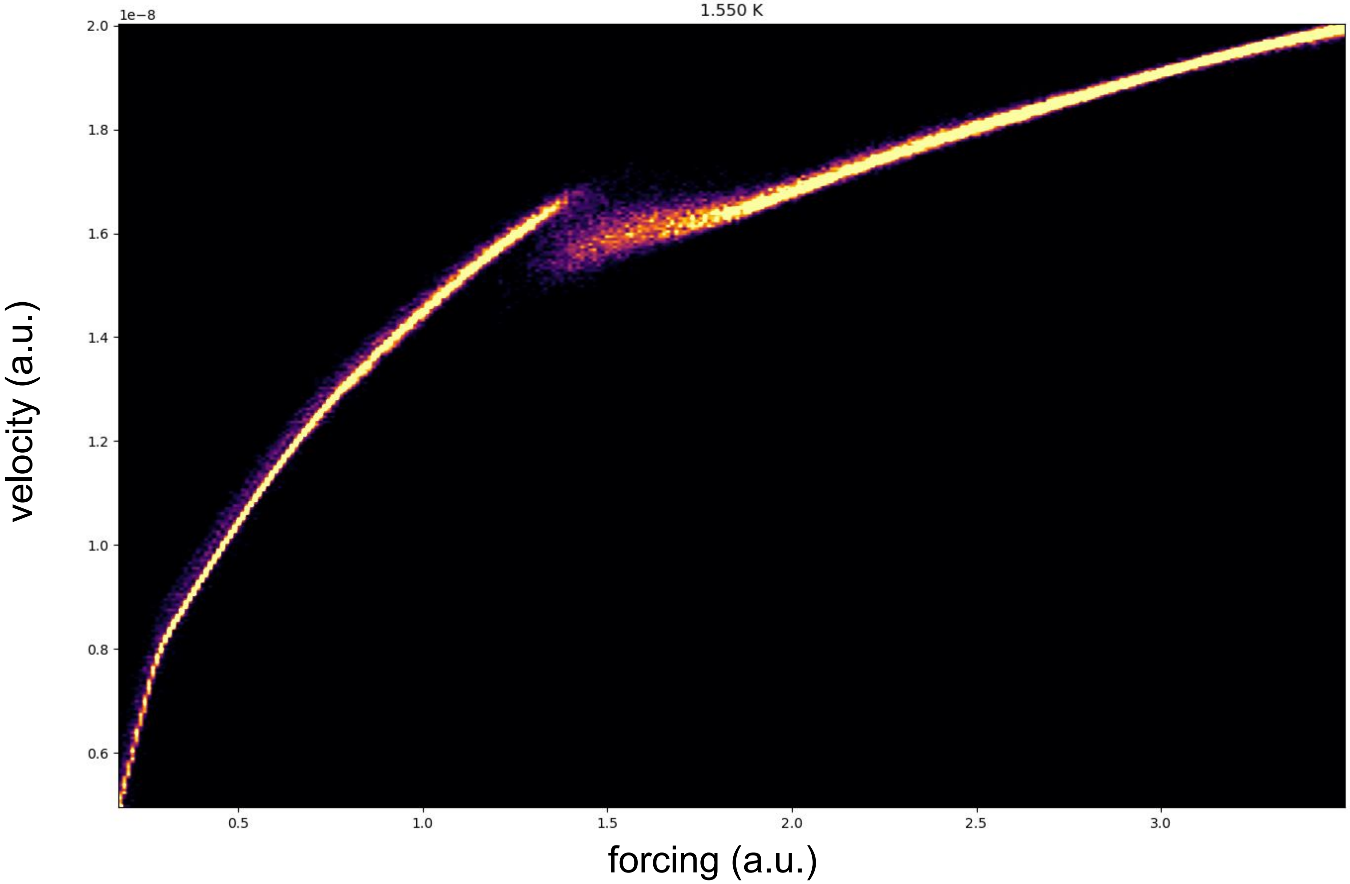


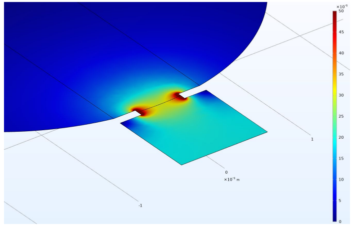



1e−8
1.850 K
1.2
1.0
0.8
0.6
0.4
velocity (a.u.)
1
2
3
4
forcing (a.u.)

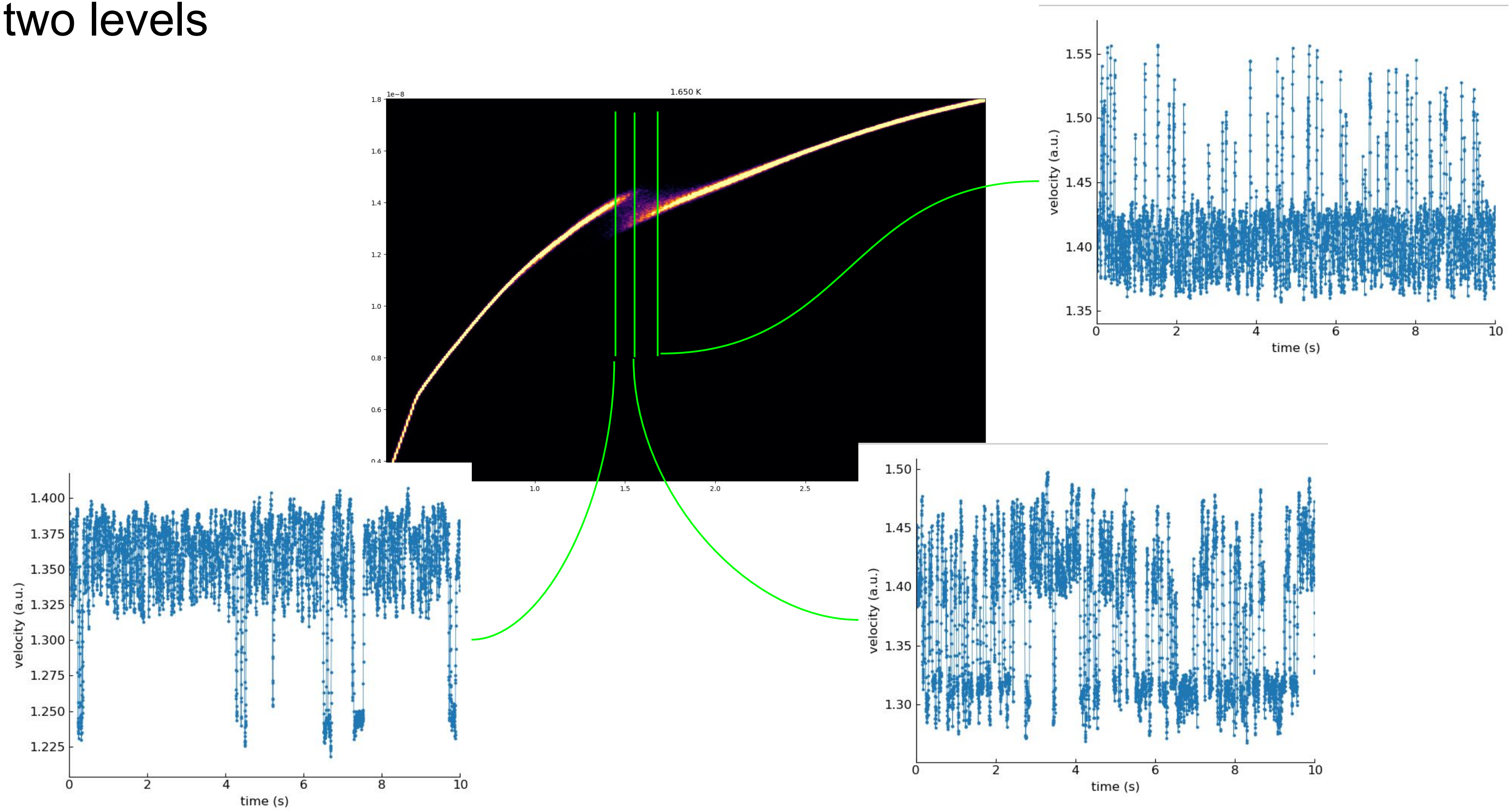


At low T, the second one becomes discontinuous.



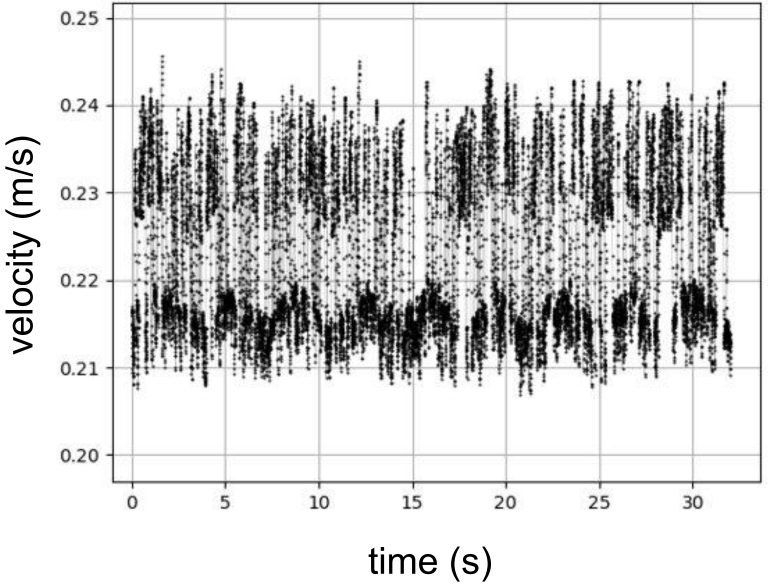


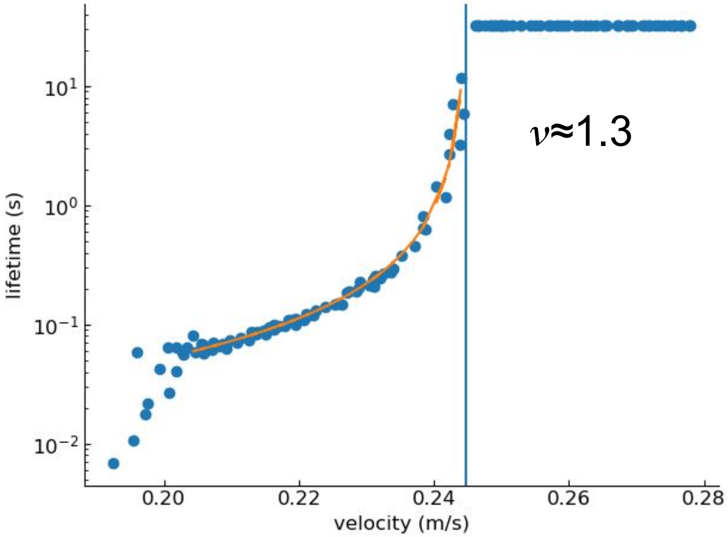
Within the discontinuity, the flow velocity randomly switches between two levels



Mean lifetime of the “more turbulent” state grows near the critical velocity as a power law

$$\tau \propto |v - v_c|^{-\nu}$$





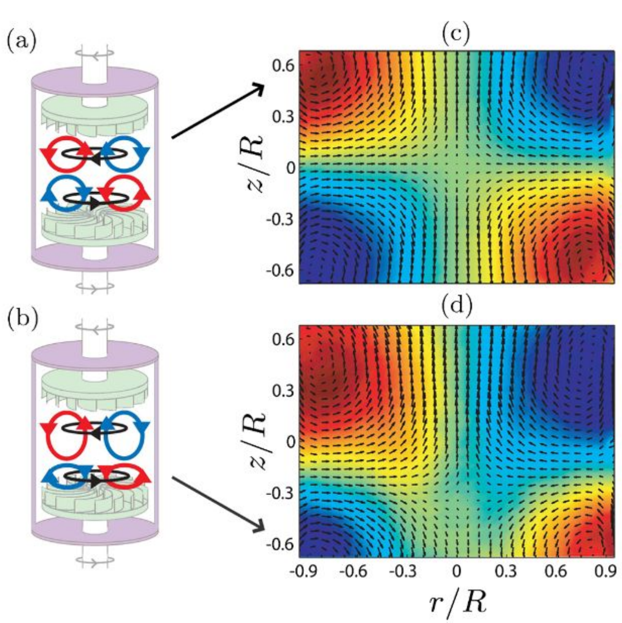
$\nu \approx 1.3$

A phase transition?

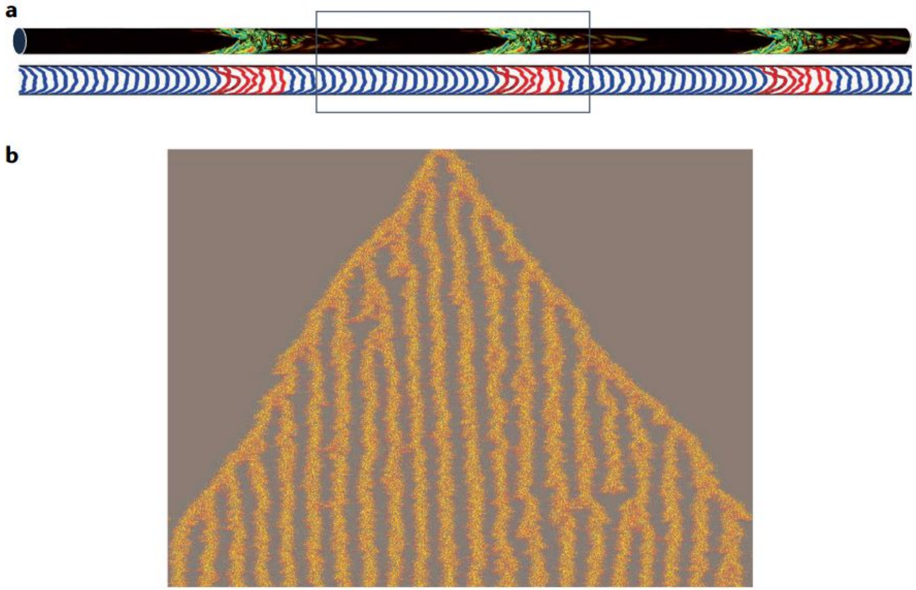
PRL **105,** 214501 (2010) PHYSICAL REVIEW LETTERS week ending 19 NOVEMBER 2010

Experimental Evidence of a Phase Transition in a Closed Turbulent Flow

P.-P. Cortet, A. Chiffaudel, F. Daviaud, and B. Dubrulle
CEA, IRAMIS, SPEC, CNRS URA 2464, Groupe Instabilités et Turbulence, 91191 Gif-sur-Yvette, France



Early (low *Re*) turbulence transition in classical pipe flow is believed to be described by directed percolation

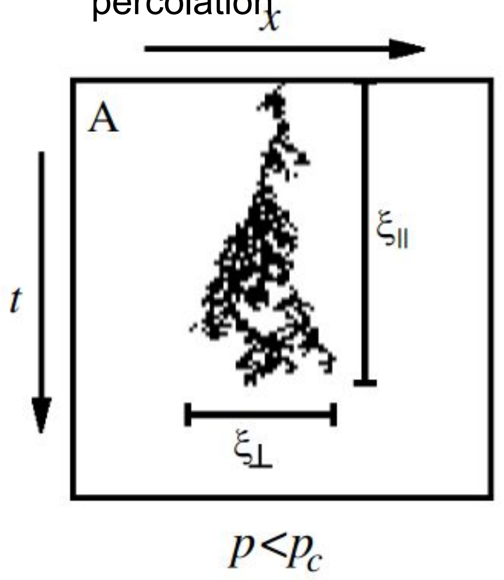


1+1 DP in “one dimensional” pipe flow

B. Hof, *Nat. Rev. Phys.* **5**, 62-72 (2023)
Shi, L. *et al.*, *Phys. Rev. Lett.* **110**, 204502 (2013)

The exponent is compatible with 2D directed percolation

(1+1)D directed percolation



Haye Hinrichsen (2000) Non-equilibrium critical phenomena
and phase transitions into absorbing states,
Advances in Physics, 49:7, 815-958

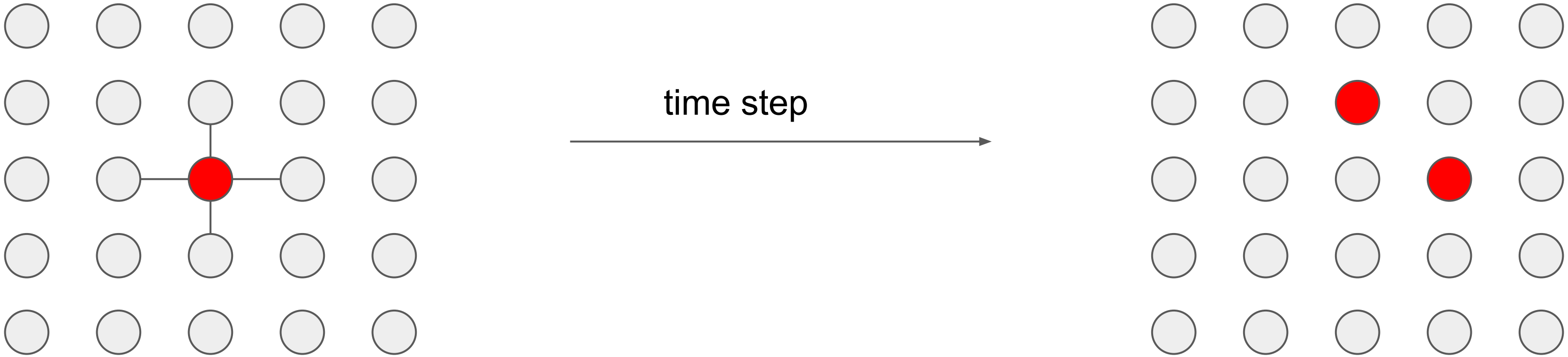
$$\xi_{||} \propto |p - p_c|^{-\nu_{||}}$$

Géza Ódor: Universality classes in nonequilibrium lattice systems

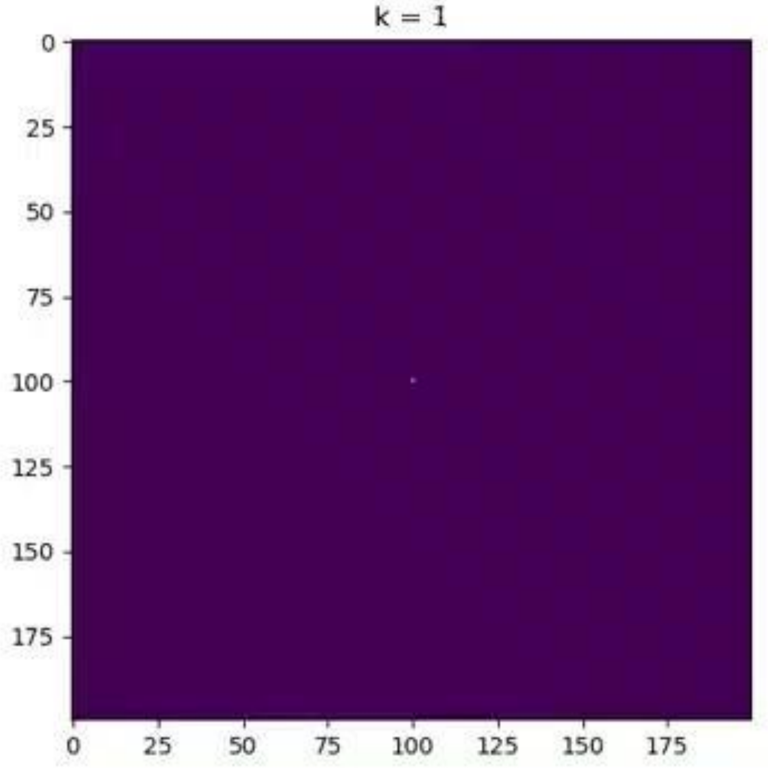
TABLE XII. Estimates for the critical exponents of directed percolation. One-dimensional data are from Jensen (1999a); two-dimensional data are from Voigt and Ziff (1997); three-dimensional data are from Jensen (1992); four-dimensional-ϵ data are from Bronzan and Dash (1974) and Janssen (1981).

Critical exponent	$d=1$	$d=2$	$d=3$	$d=4-\epsilon$
$\beta=\beta'$	0.276486(8)	0.584(4)	0.81(1)	$1-\epsilon/6-0.01128\epsilon^2$
$\nu_\perp$	1.096854(4)	0.734(4)	0.581(5)	$1/2+\epsilon/16+0.02110\epsilon^2$
$\nu_\parallel$	1.733847(6)	1.295(6)	1.105(5)	$1+\epsilon/12+0.02238\epsilon^2$
$Z=2/z$	1.580745(10)	1.76(3)	1.90(1)	$2-\epsilon/12-0.02921\epsilon^2$
$\delta=\alpha$	0.159464(6)	0.451	0.73	$1-\epsilon/4-0.01283\epsilon^2$
η	0.313686(8)	0.230	0.12	$\epsilon/12+0.03751\epsilon^2$
γ_p	2.277730(5)	1.60	1.25	$1+\epsilon/6+0.06683\epsilon^2$

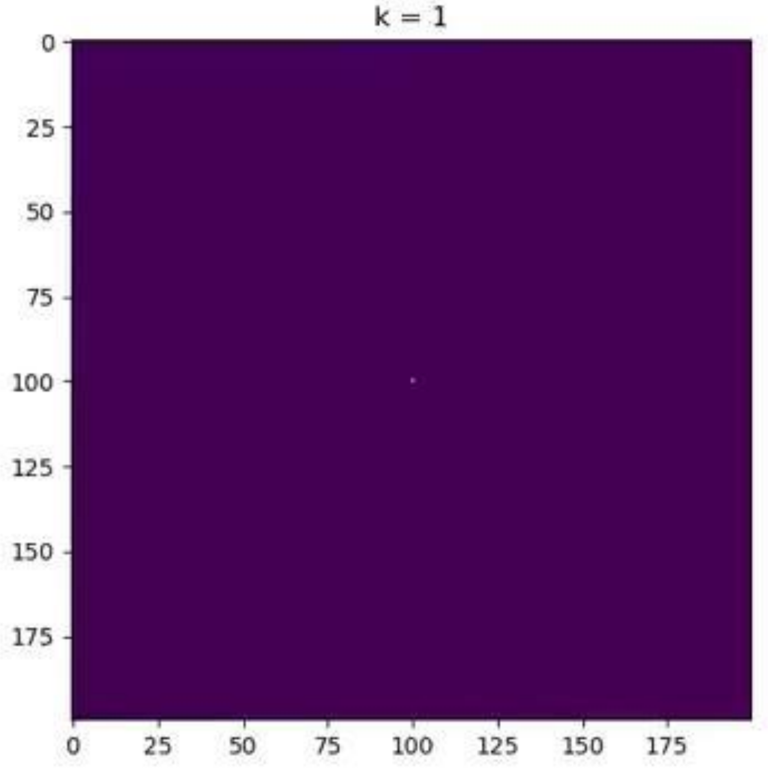
For a points on a lattice, “activation” spreads to nearest neighbours with probability *p* per time step (*directed percolation*)



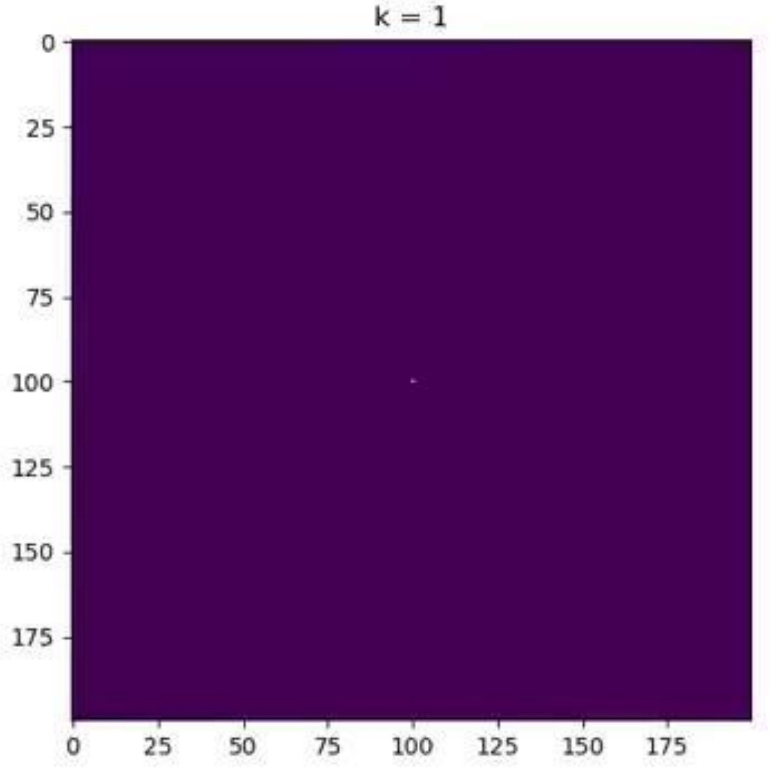
For low spreading probabilities the clusters die



As the probability increases, clusters live longer

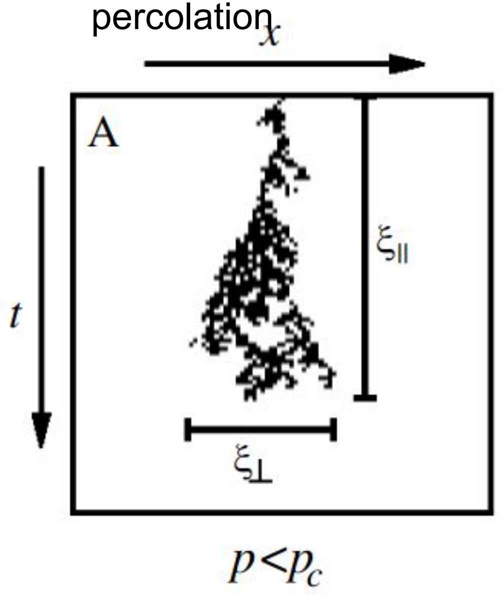


And eventually they start spreading to infinity



The exponent is compatible with 2D directed percolation

(1+1)D directed percolation



Haye Hinrichsen (2000) Non-equilibrium critical phenomena
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Advances in Physics, 49:7, 815-958

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Géza Ódor: Universality classes in nonequilibrium lattice systems

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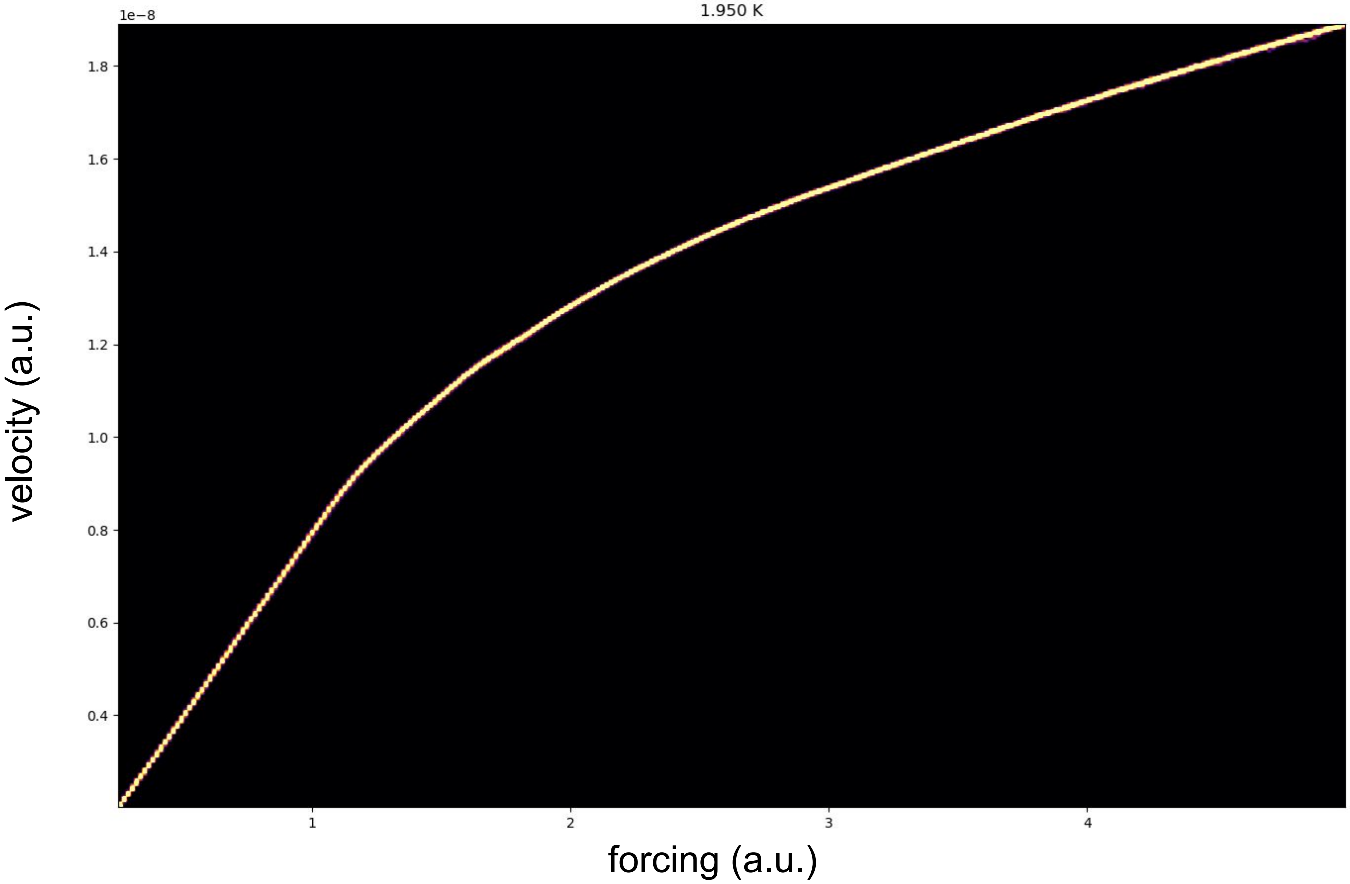
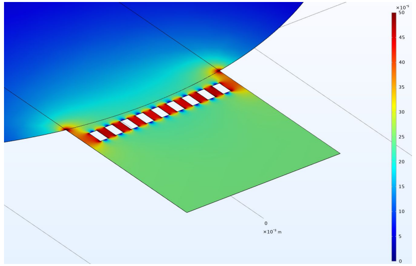
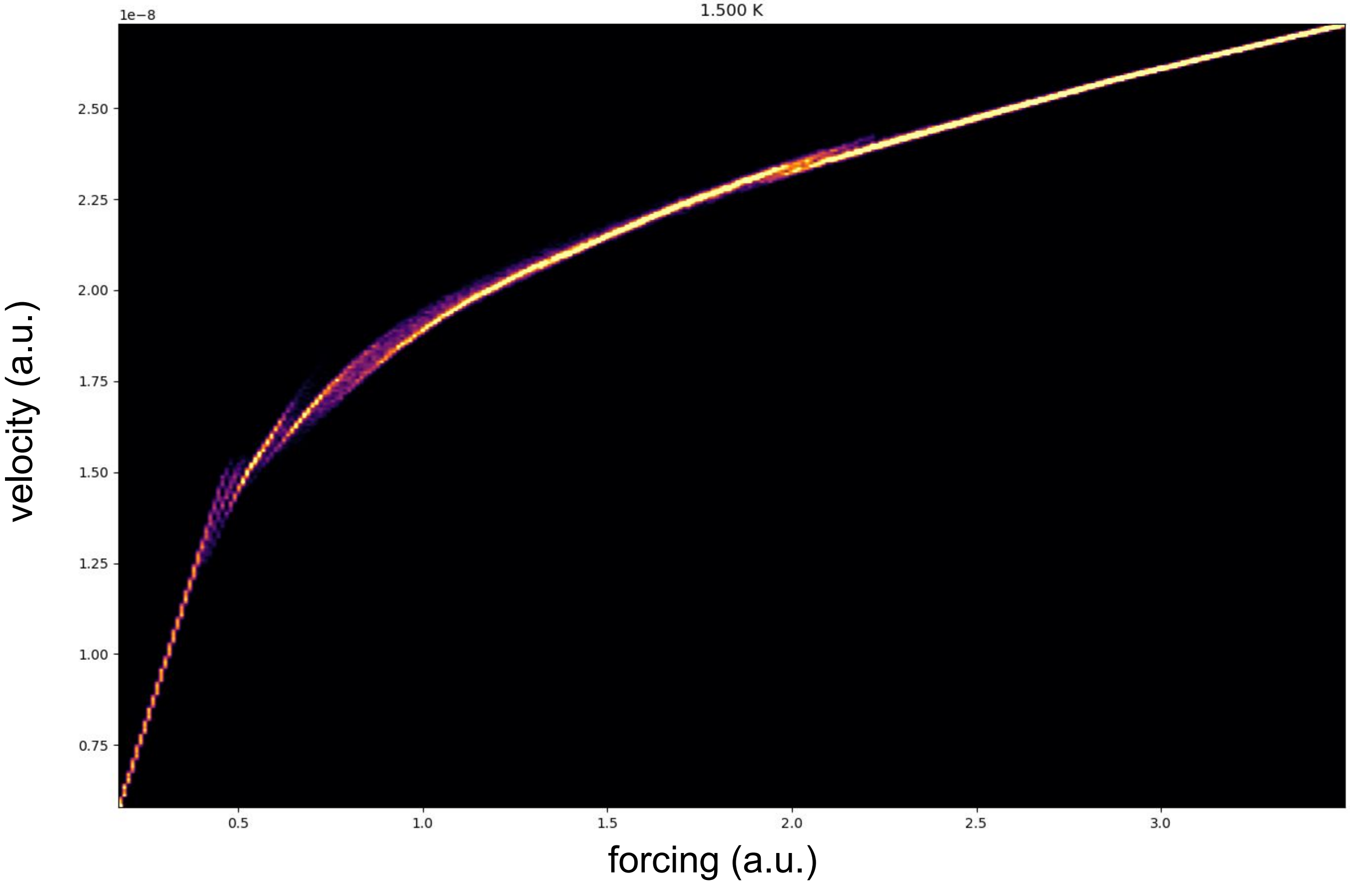
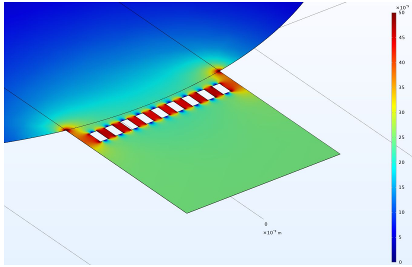

1.950 K
1e−8
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
1
2
3
4
velocity (a.u.)
forcing (a.u.)

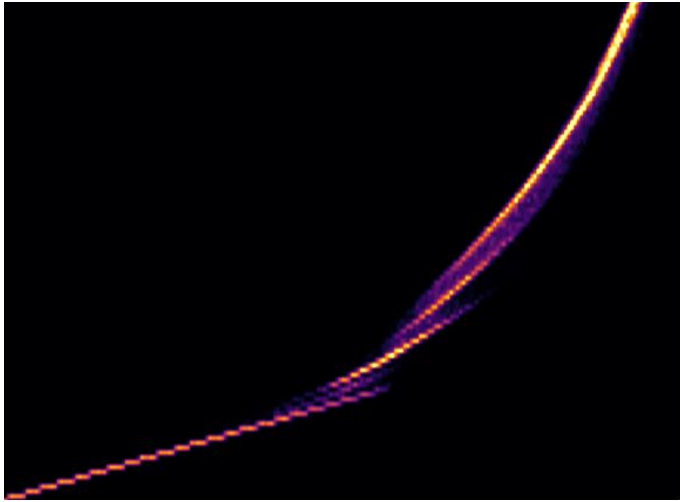


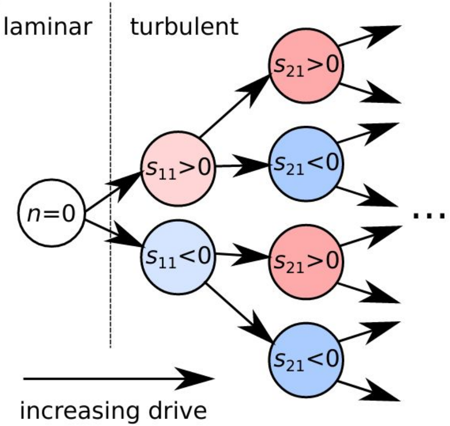


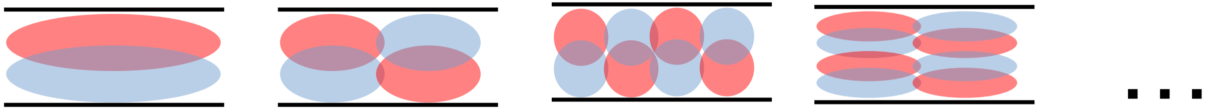
1.500 K
1e−8
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.5
1.0
1.5
2.0
2.5
3.0
velocity (a.u.)
forcing (a.u.)



Multiple turbulent states possibly correspond to multiple types of polarization of the flow







$$
\begin{aligned}
\dot{n} &= f_0(n, s_1, s_2, ...) \\
\dot{s}_1 &= f_1(n, s_1, s_2, ...) \\
\dot{s}_2 &= f_2(n, s_1, s_2, ...) \\
&\;\;\cdots
\end{aligned}
$$

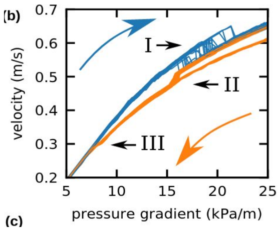
Can we break the symmetry between positive and negative vortices?

Soon

also D. Schmoranzer, L. Skrbek

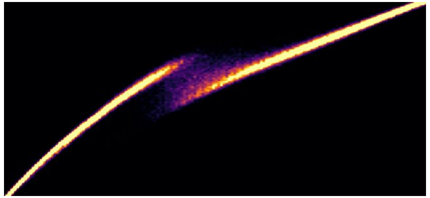
Conclusions

2D turbulence can spontaneously develop large scales



Quasi-2D quantum turbulence can support more than one large-scale stable state

Transition to turbulence possibly shows some critical phenomena



Thank you for your attention!