

Patterns of Turbulence

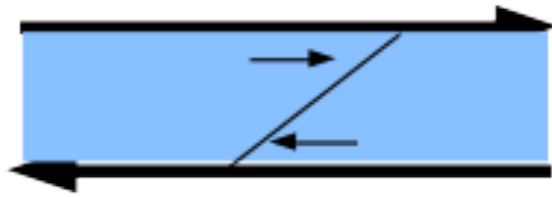
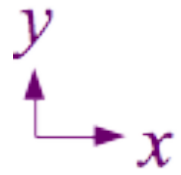
Laurette Tuckerman, PMMH-ESPCI-CNRS

Dwight Barkley, University of Warwick

Matthew Chantry, Oxford University

Sébastien Gomé, PMMH-CNRS-ESPCI-Sorbonne

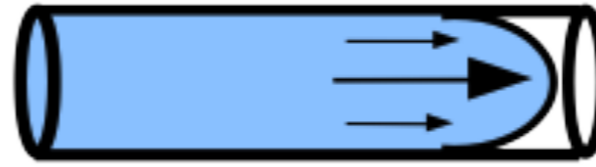
Wall-bounded shear flows



Couette

linear instability: $Re = \infty$ (Romanov)

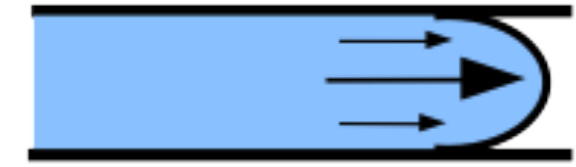
transition to turbulence: $Re \approx 300$



Pipe

$Re = \infty$

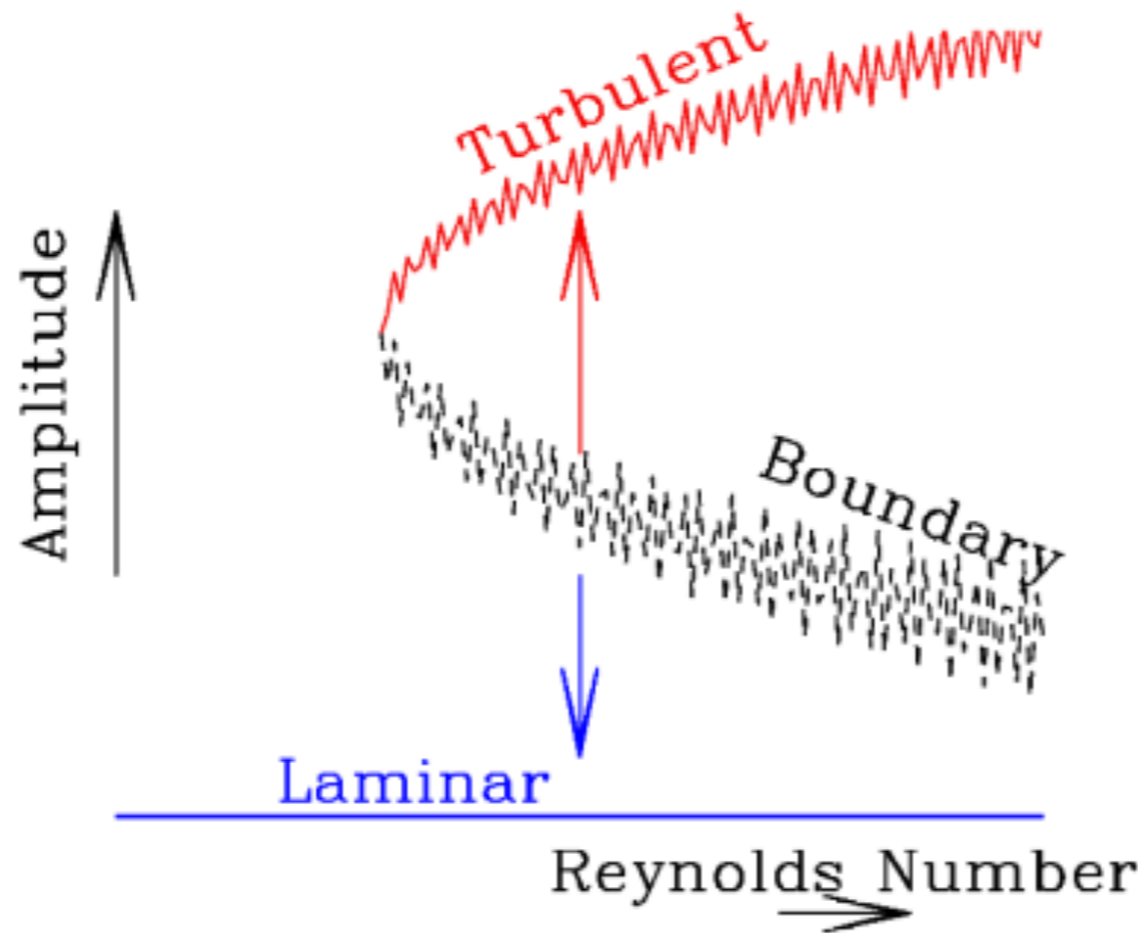
$Re \approx 2000$



Poiseuille

$Re = 5772$ (Orszag)

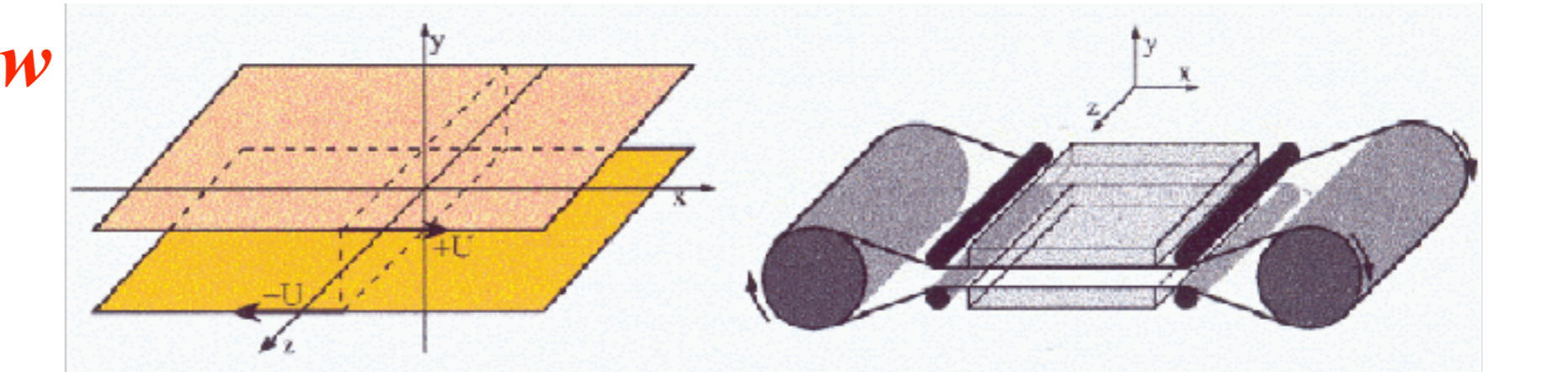
$Re \approx 1000$



Experiments at CEA/Saclay by Prigent, Dauchot (2000-3)

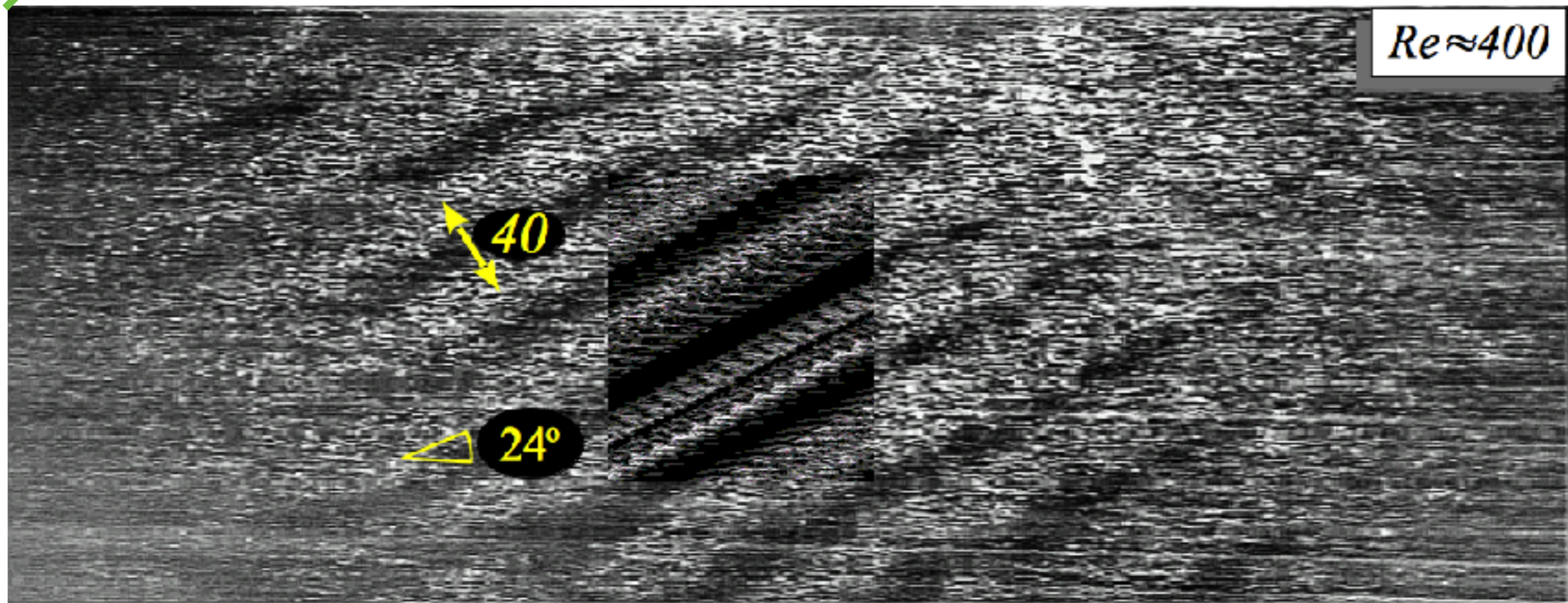
Plane Couette Flow

$$Re_{\text{Cou}} \equiv \frac{\Delta U \Delta Y}{4\nu}$$



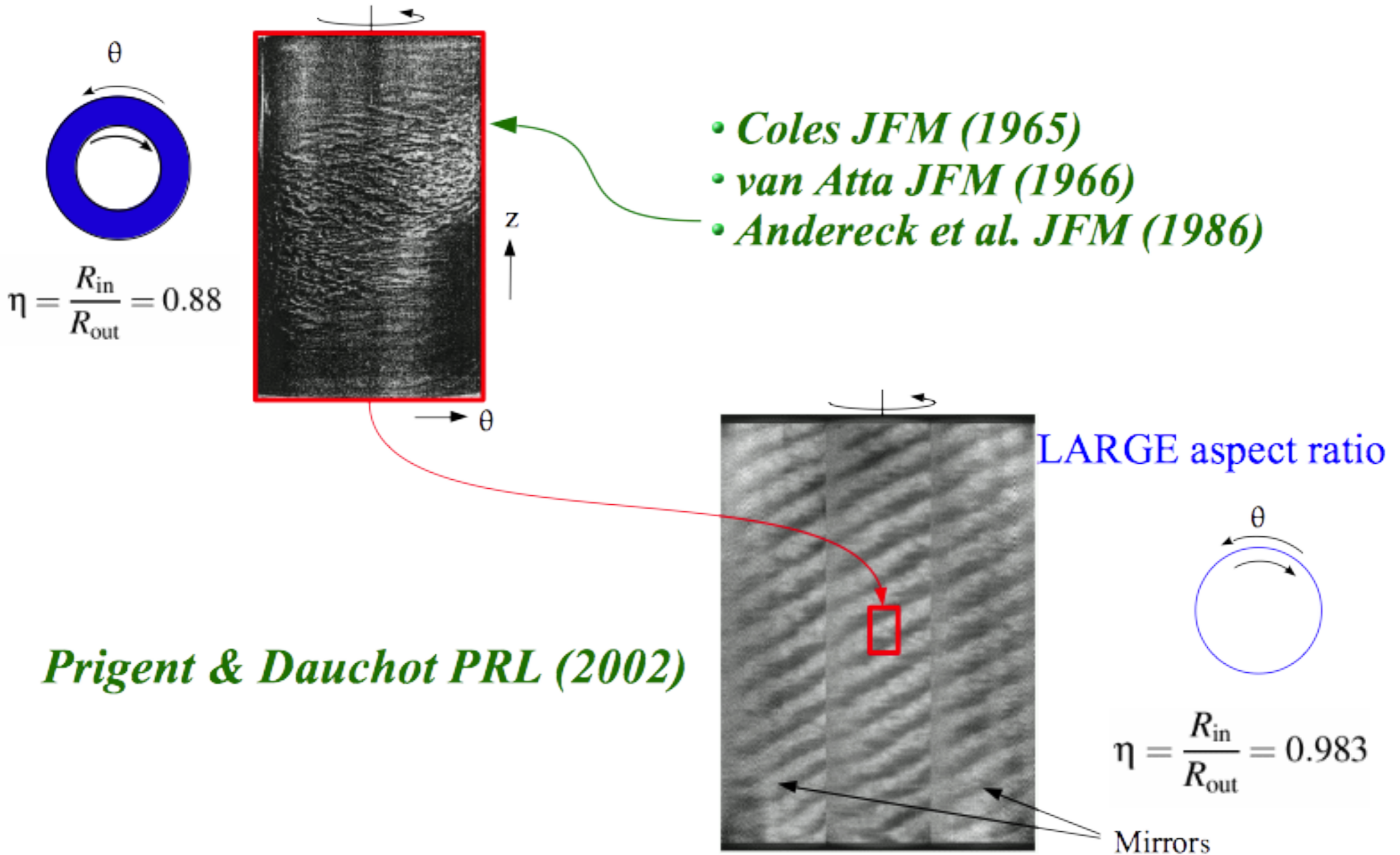
Gap 2/

Spanwise



Length 770 half-gaps Streamwise →

Spiral Turbulence in counter-rotating Taylor-Couette Flow



Between rotating and stationary disks

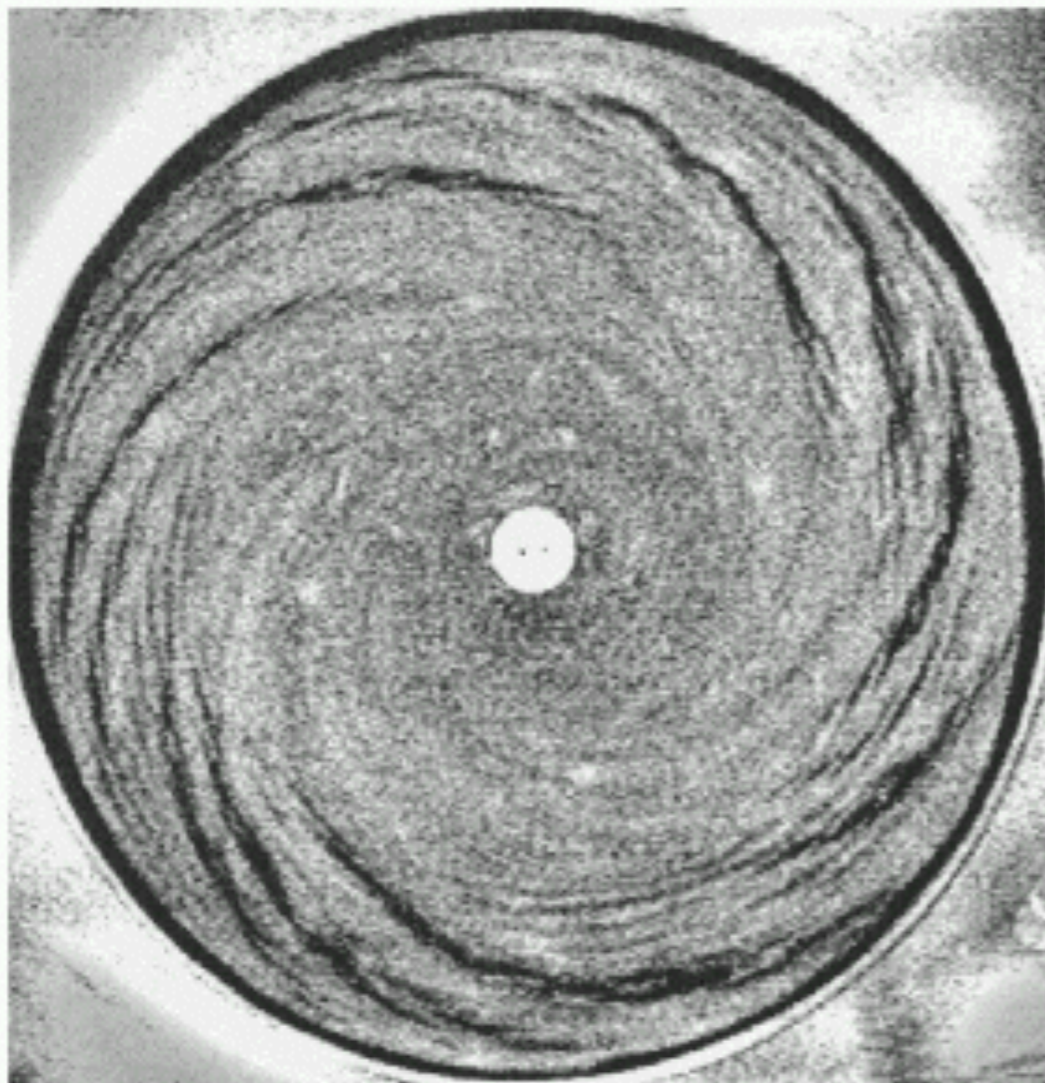


FIG. 5. Turbulent spirals for $h=2.2$ mm and $\Omega=52$ rpm clockwise ($Re=106\,700$).

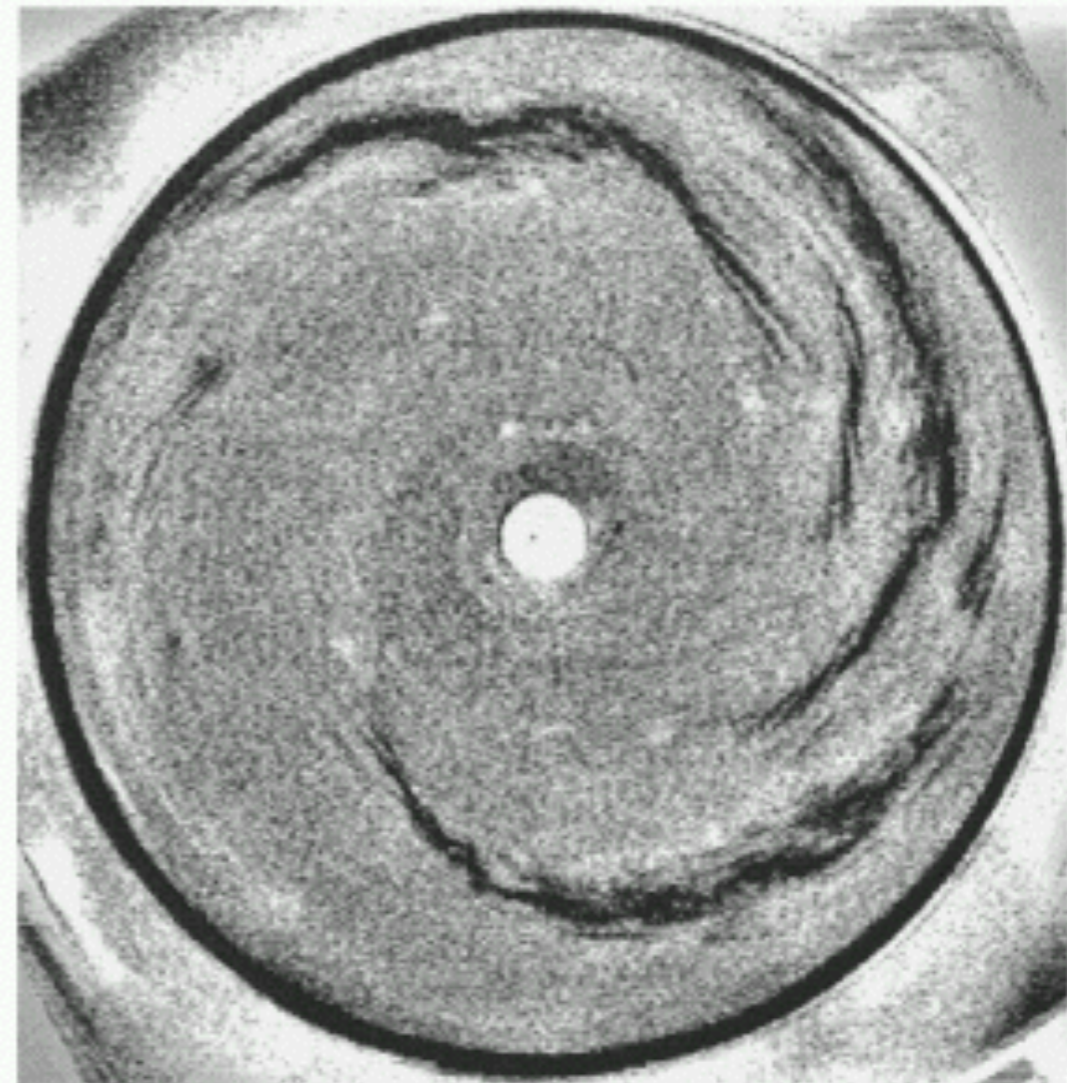
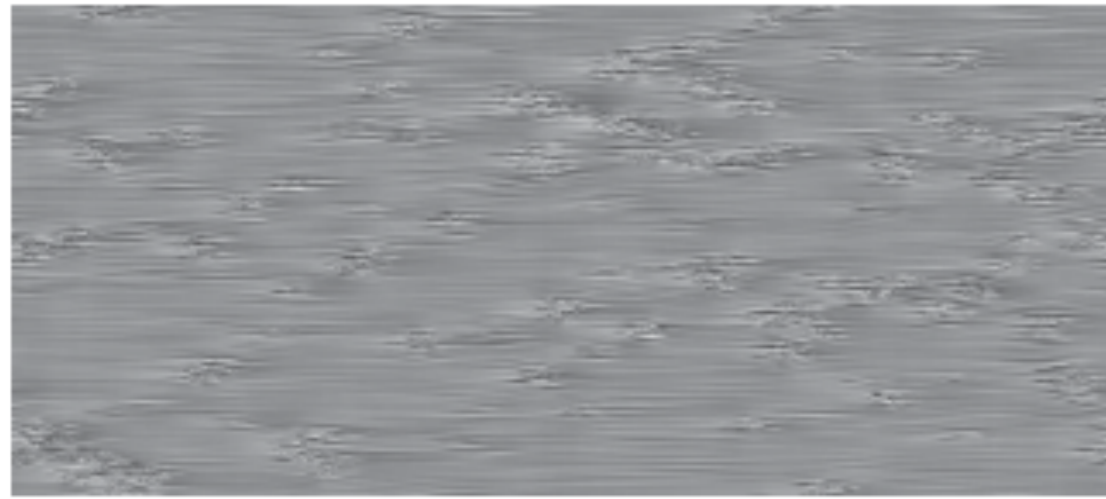


FIG. 7. Turbulent spirals for $h=2.2$ mm and $\Omega=74$ rpm clockwise ($Re=151\,900$). One spot is visible on the left side of the disk.

(a)



J. Fluid Mech. (2010), vol. 650, pp. 119–129. © Cambridge University Press 2010
doi:10.1017/S0022112010000297

Formation of turbulent patterns near the onset of transition in plane Couette flow

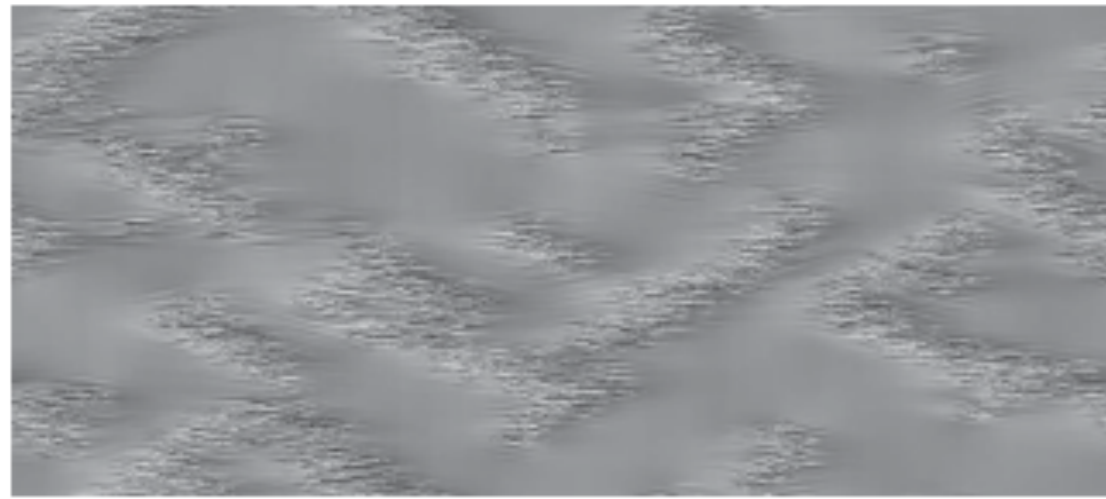
Y. DUGUET^{1,2†}, P. SCHLATTER¹ AND D. S. HENNINGSON¹

¹Linné Flow Centre, KTH Mechanics, SE-10044 Stockholm, Sweden

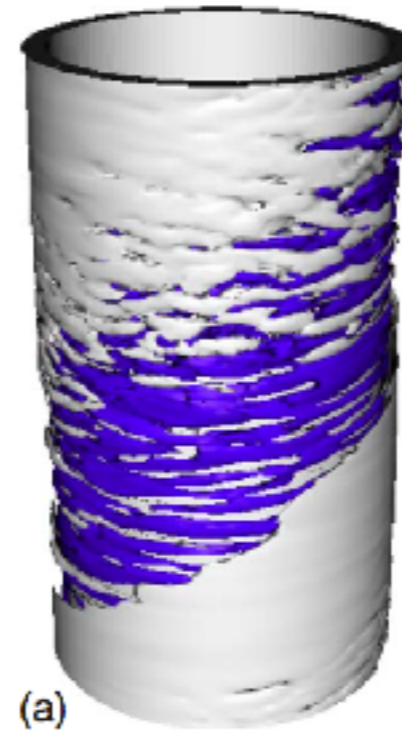
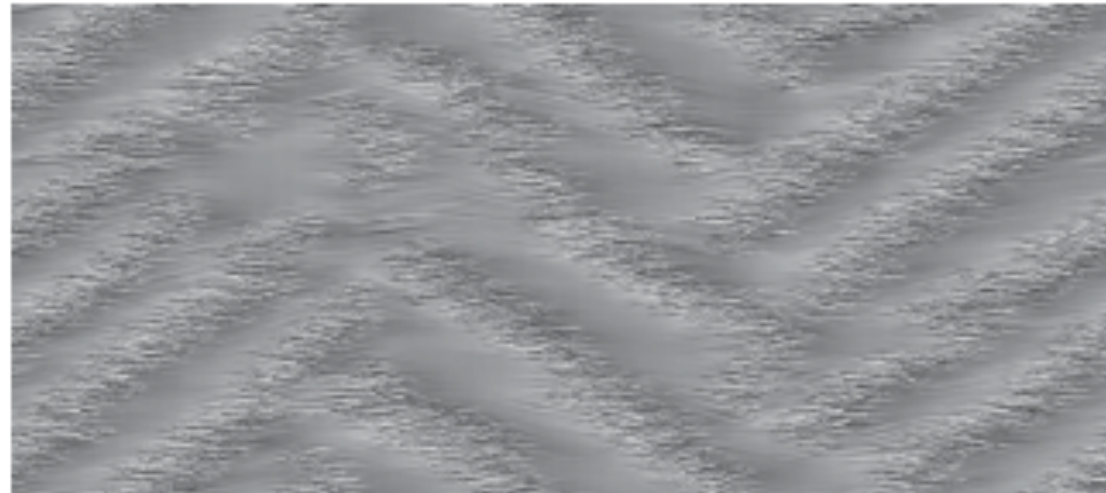
²LIMSI-CNRS, UPR 3251, 91403 Orsay, France



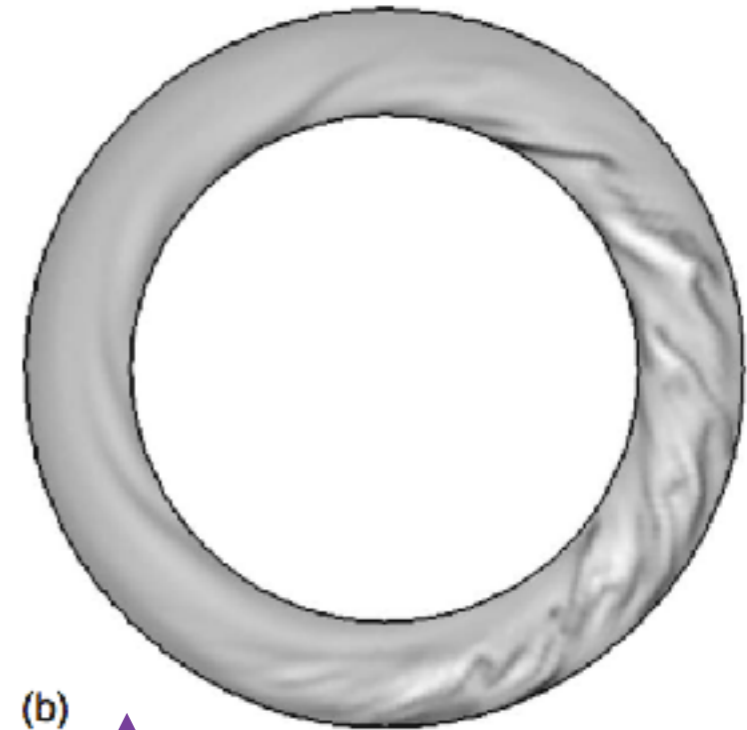
(b)



(c)



(a)



(b)



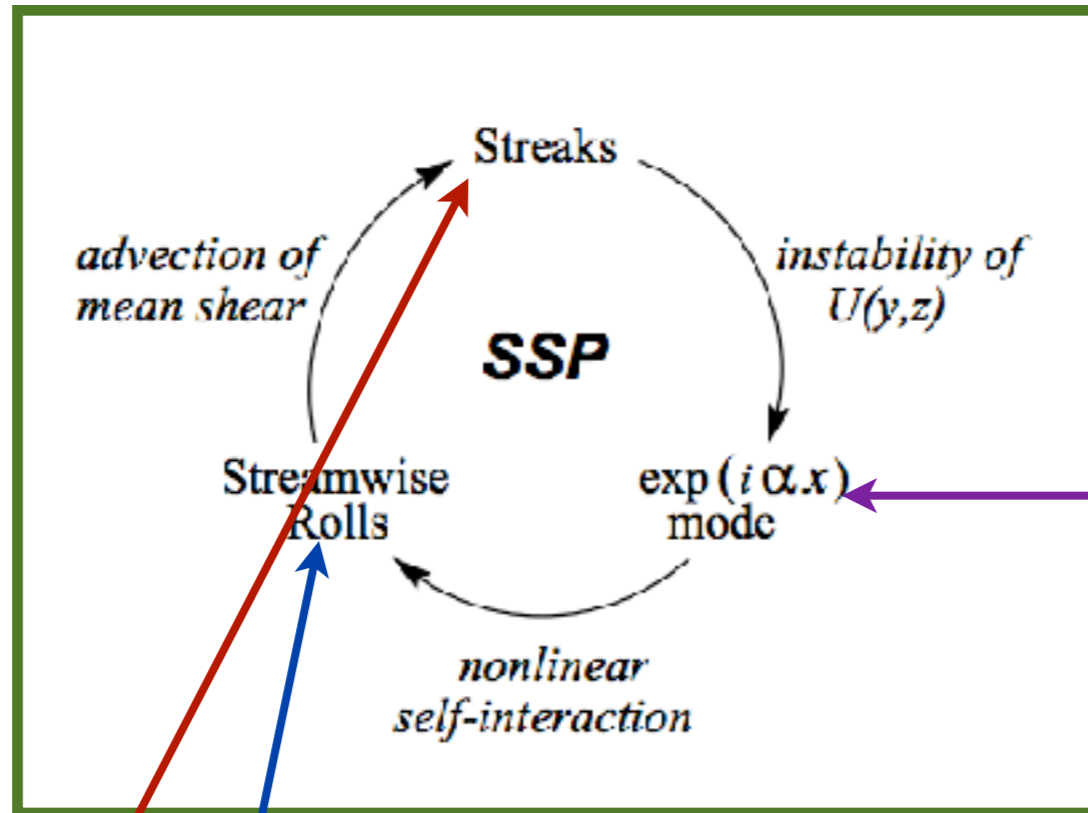
PHYSICAL REVIEW E 80, 046315 (2009)

Instability mechanisms and transition scenarios of spiral turbulence in Taylor-Couette flow

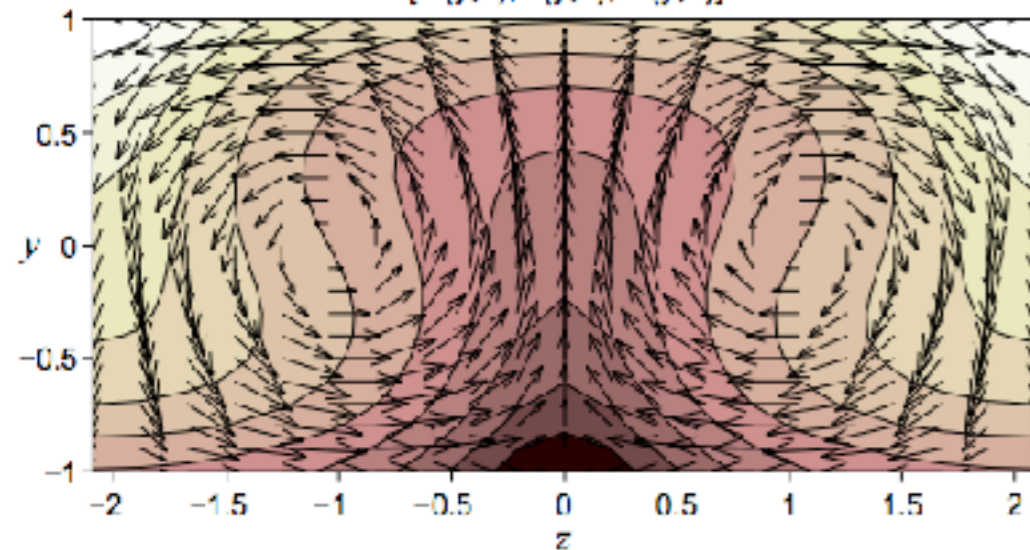
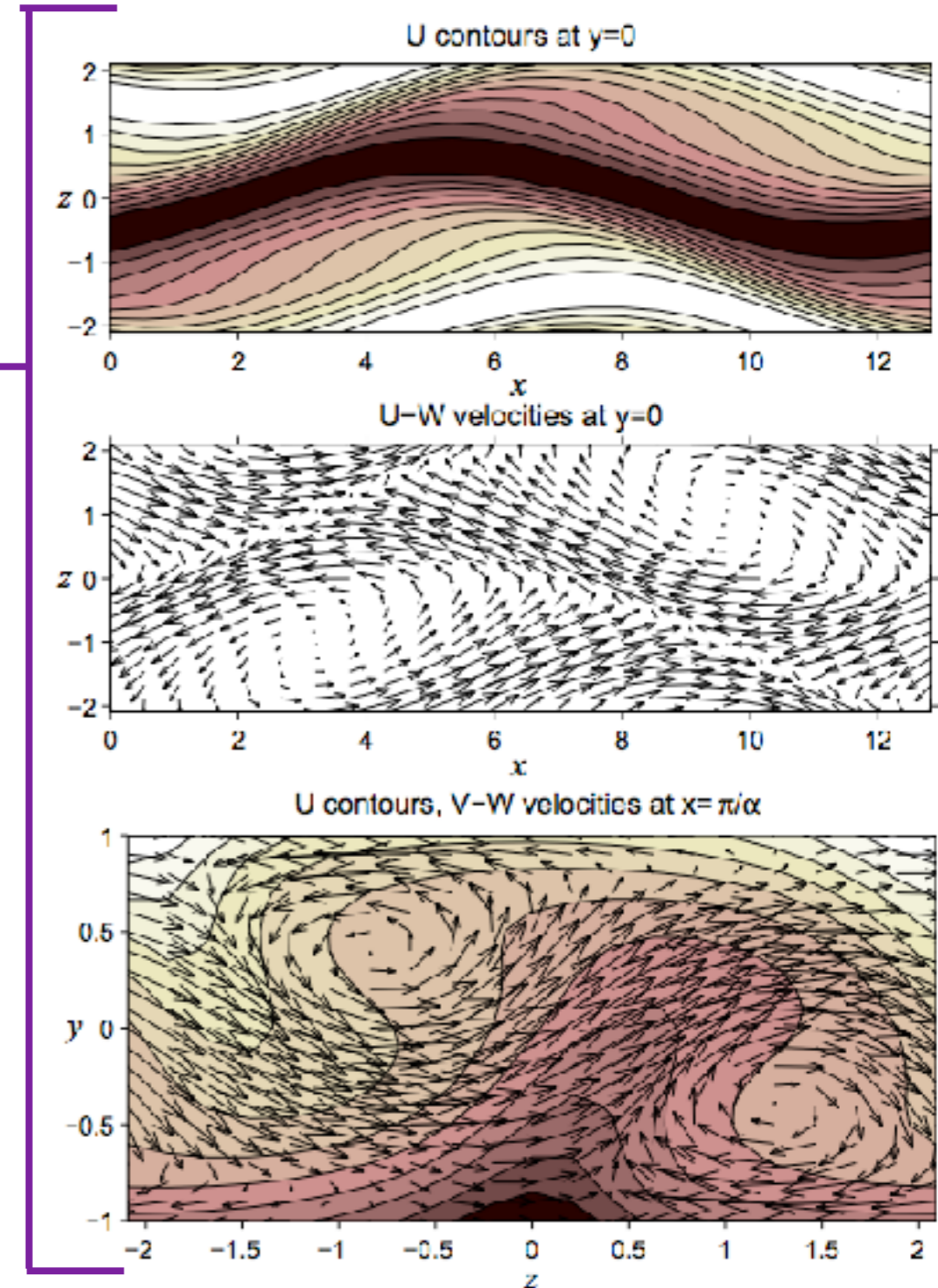
Alvaro Meseguer,^{1,*} Fernando Mellibovsky,¹ Marc Avila,² and Francisco Marques¹
¹Departament de Física Aplicada, Universitat Politècnica de Catalunya, 08034 Barcelona, Spain
²Max Planck Institute for Dynamics and Self-Organization, 37073 Göttingen, Germany

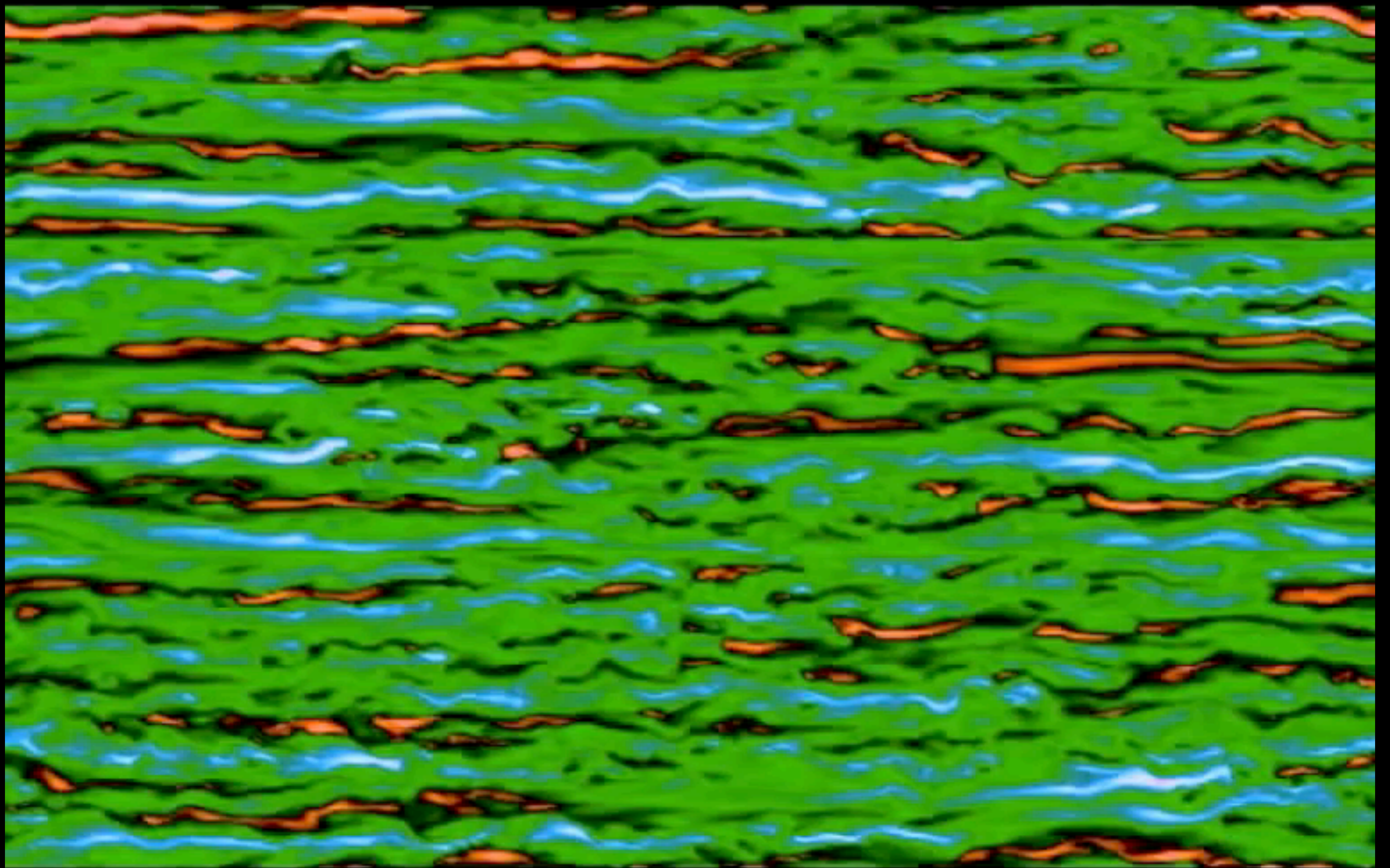
Waleffe: self-sustaining process (SSP)

F. Waleffe & J. Kim, How streamwise rolls and streaks sustain in a shear flow: Part 2, AIAA paper 98-2997 (Albuquerque, June 1998)
 F. Waleffe, On a self-sustaining process in shear flows, Phys. Fluids 9, 883-900 (1997)



$[U(y,z), V(y,z), W(y,z)]$

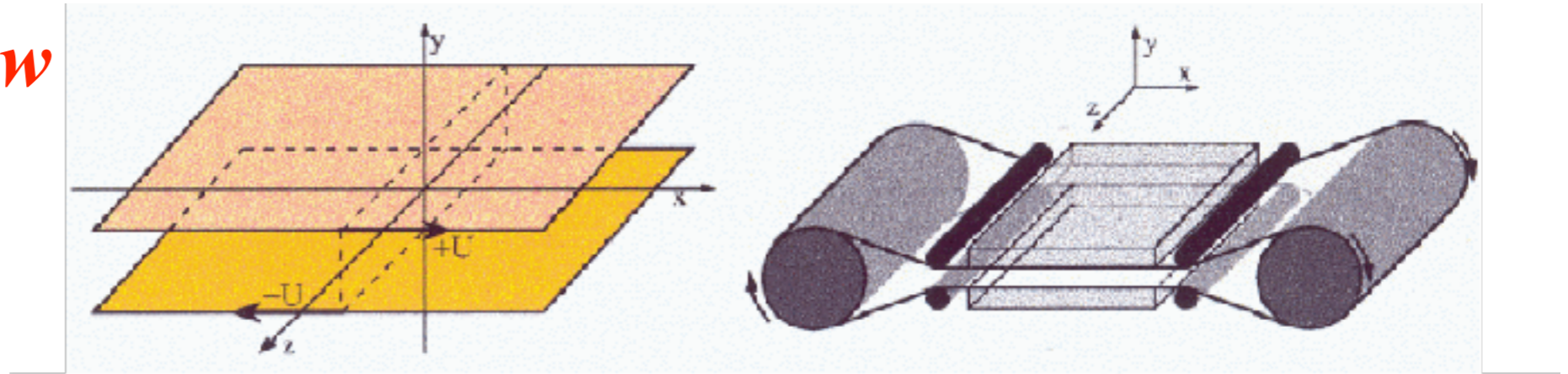




Experiments at CEA/Saclay by Prigent, Dauchot (2000-3)

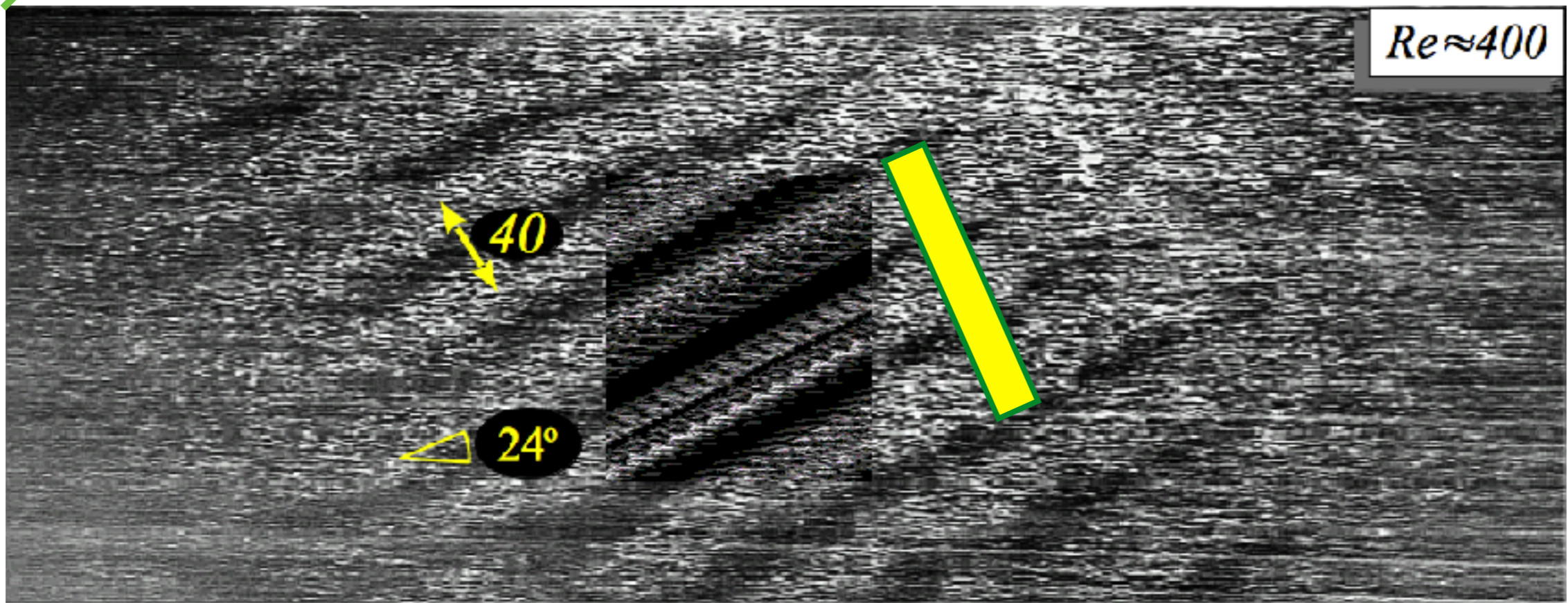
Plane Couette Flow

$$Re_{\text{Cou}} \equiv \frac{\Delta U \Delta Y}{4\nu}$$



Gap 2/

Spanwise

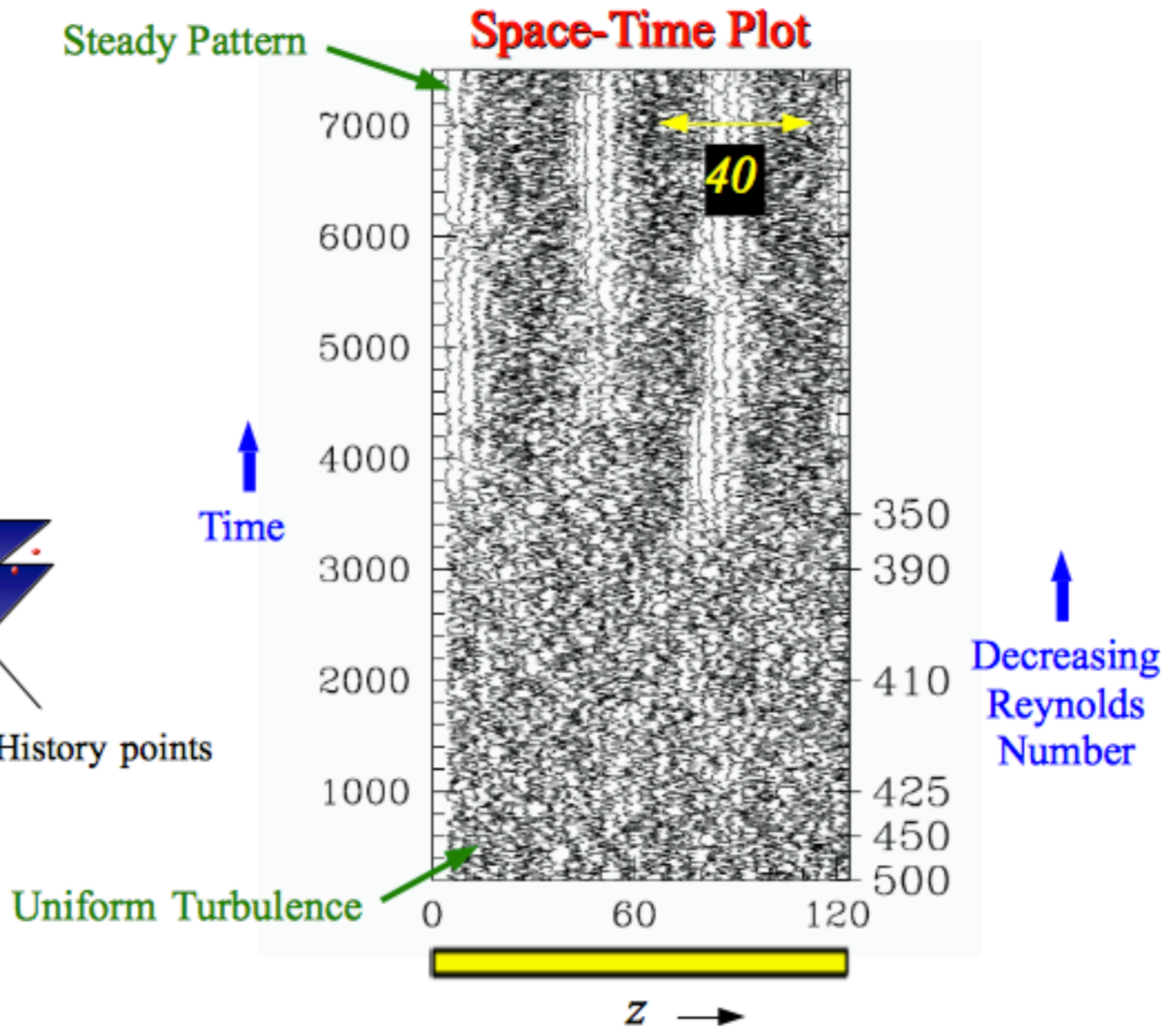
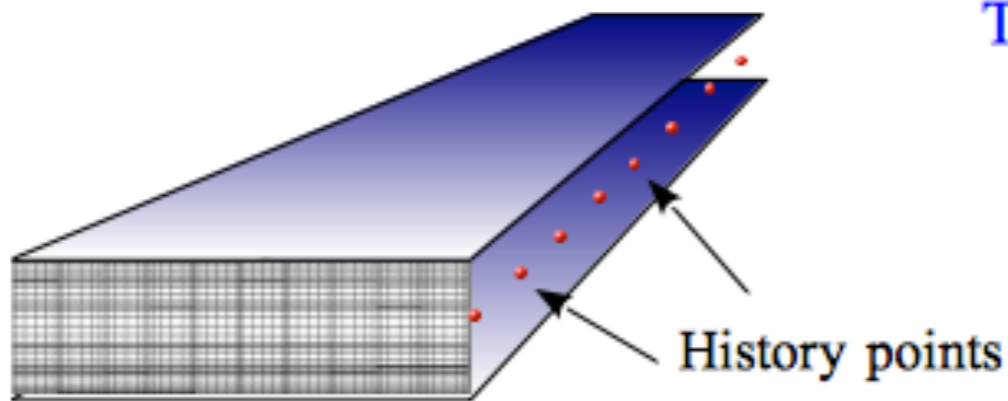


Length 770 half-gaps Streamwise

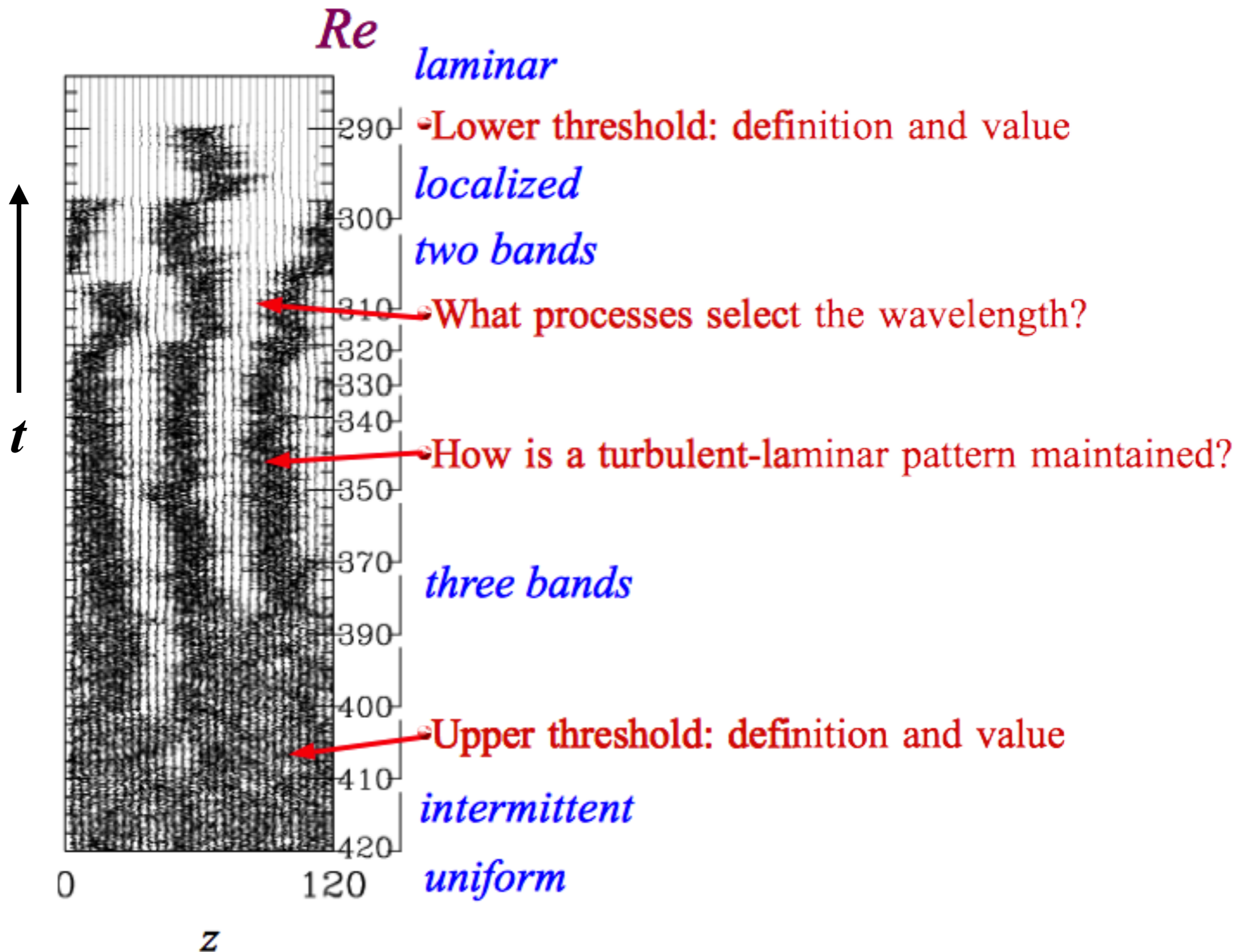
Couette Flow: spatio-temporal scan

For each domain:

- Start at $Re = 500$
- Obtain turbulent flow
- Decrease Re
- Monitor turbulence



Couette Flow: spatio-temporal scan

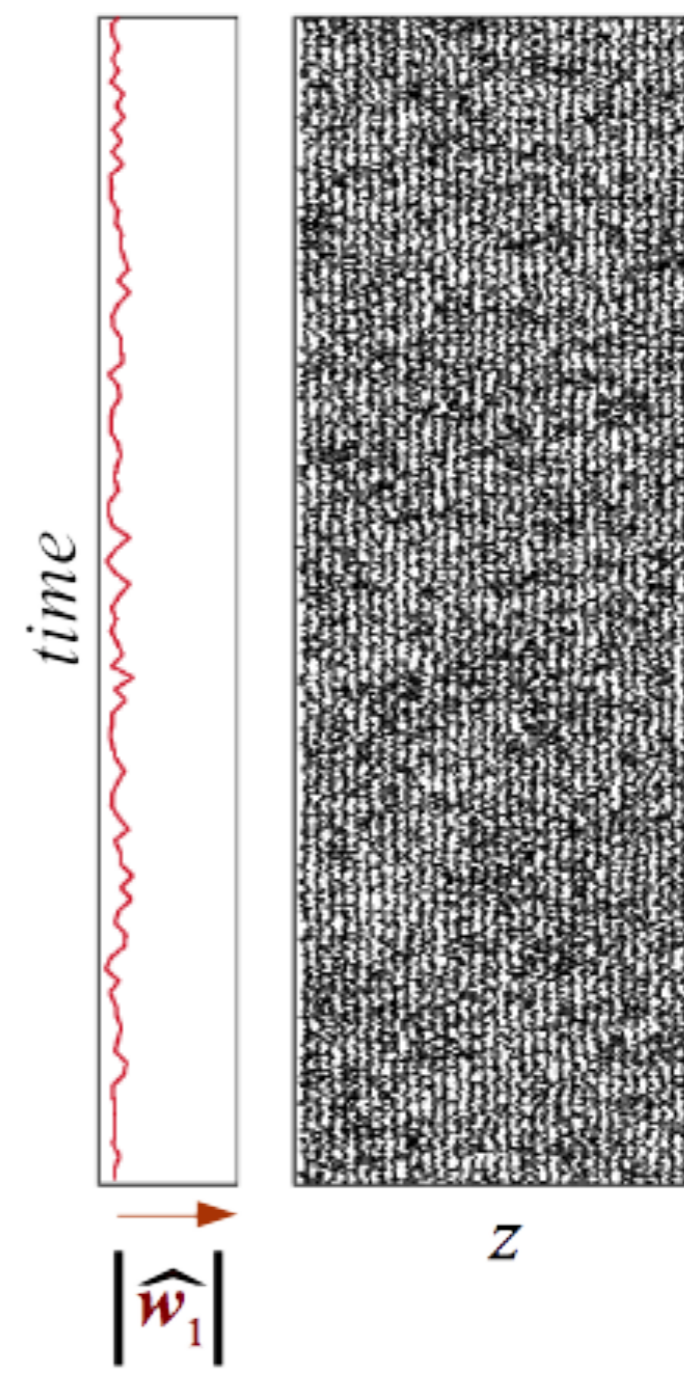
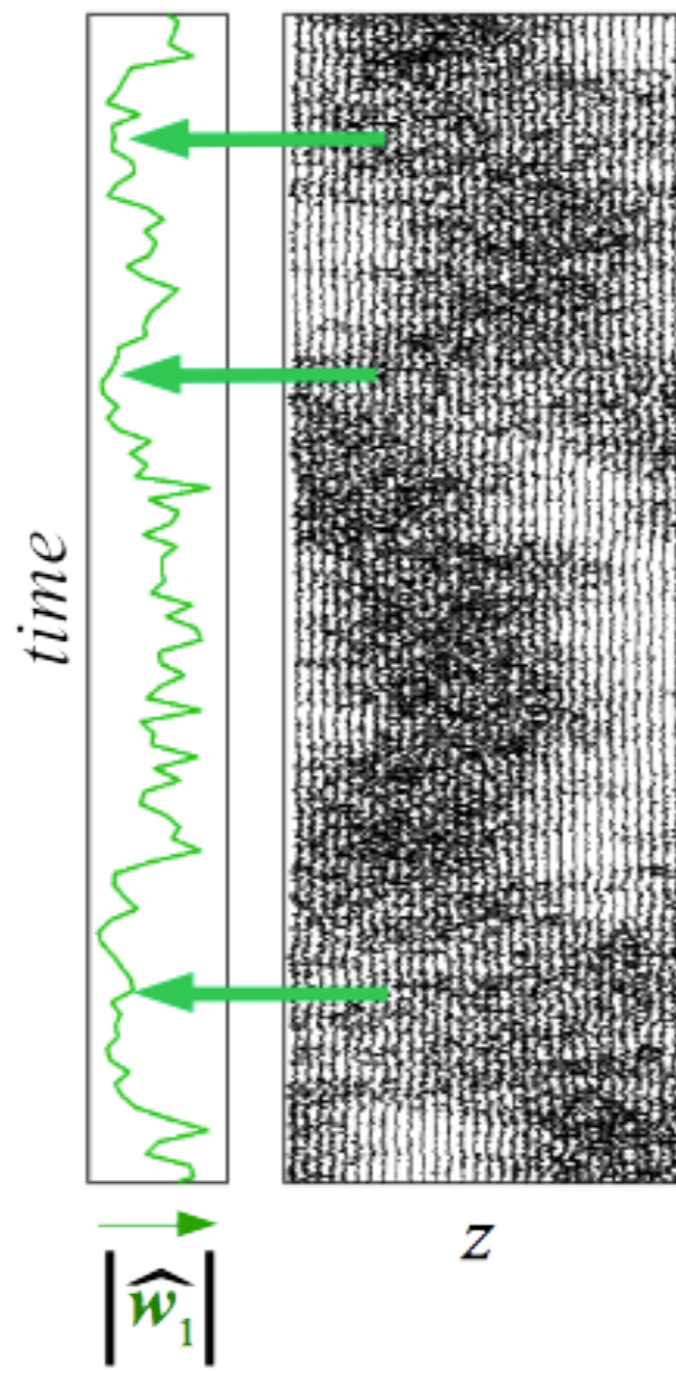
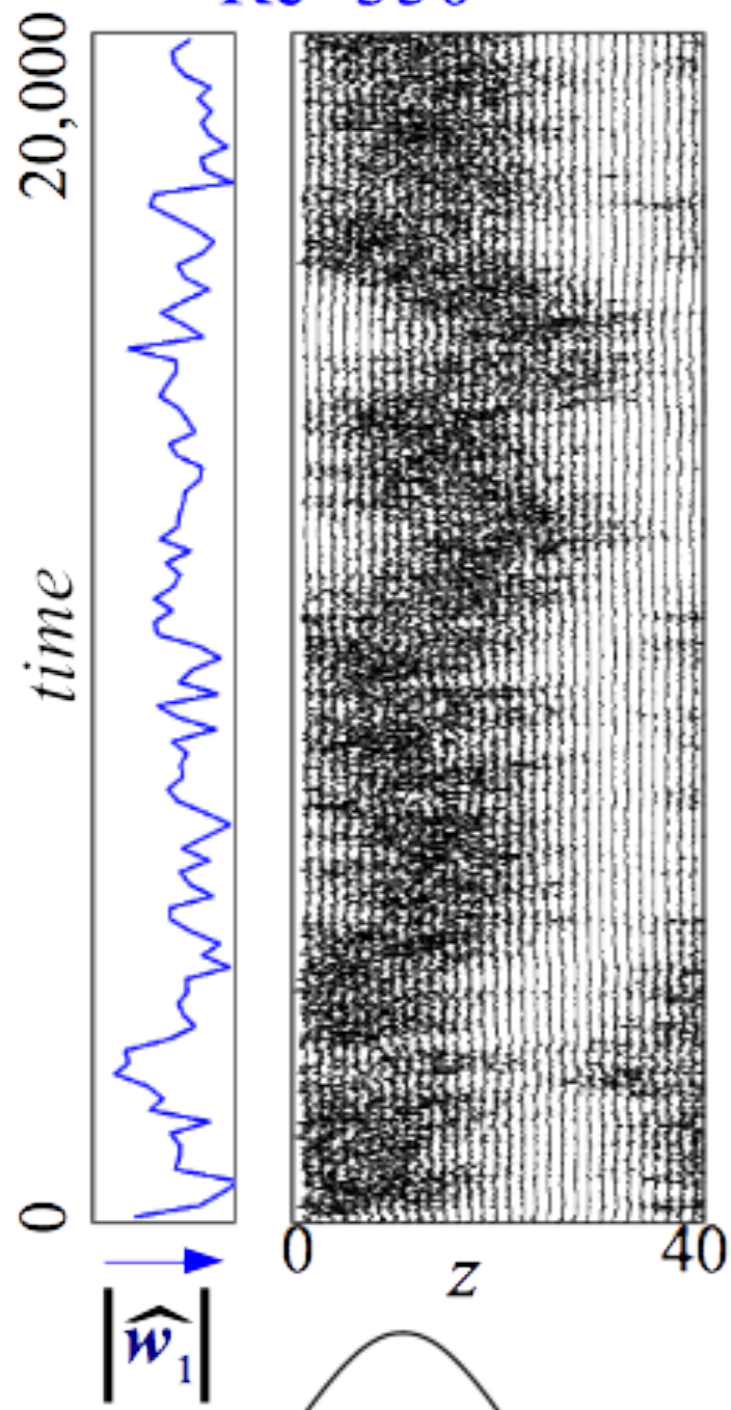


Turbulent Timeseries

Banded
Re=350

Intermittent
Re=410

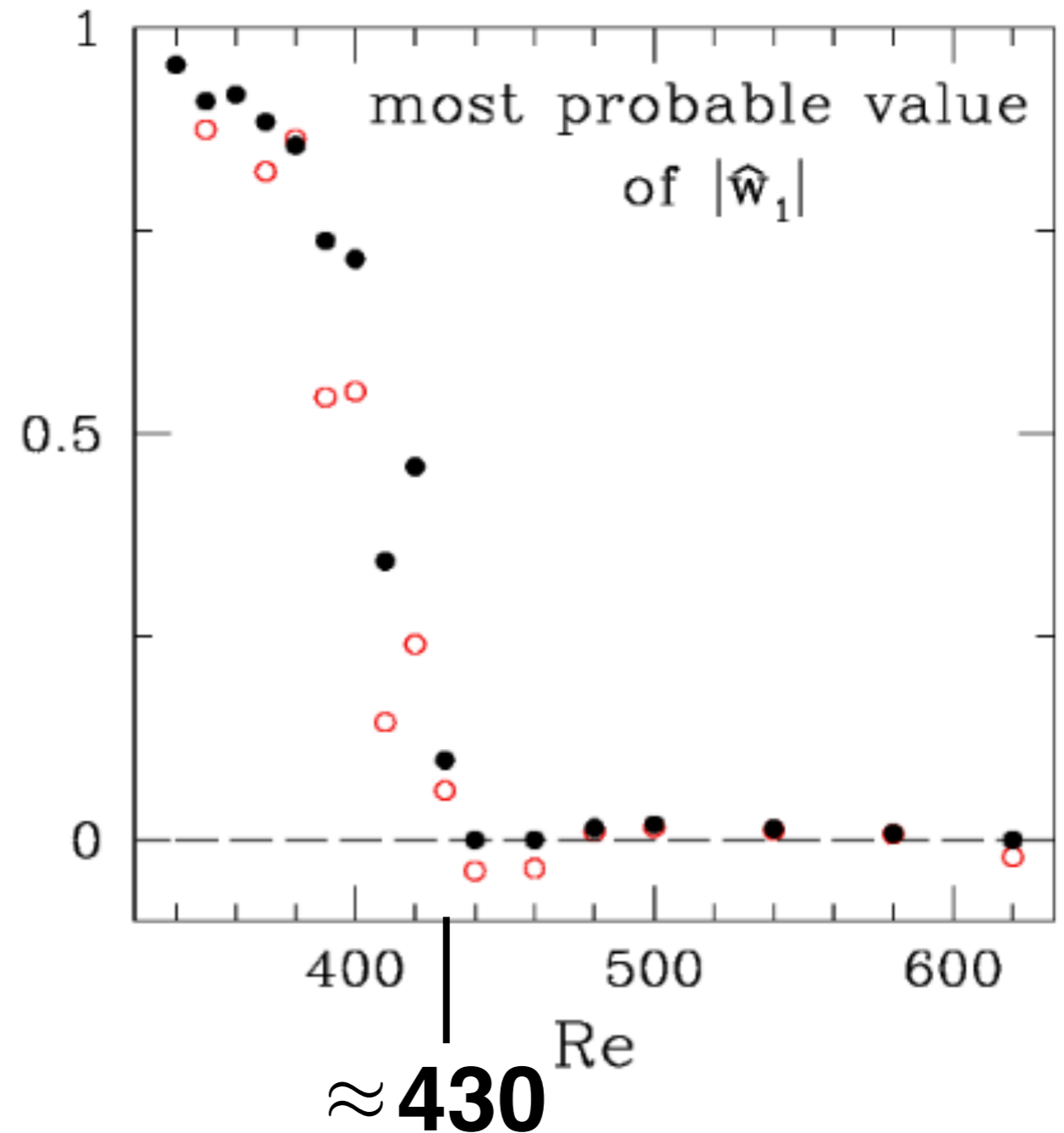
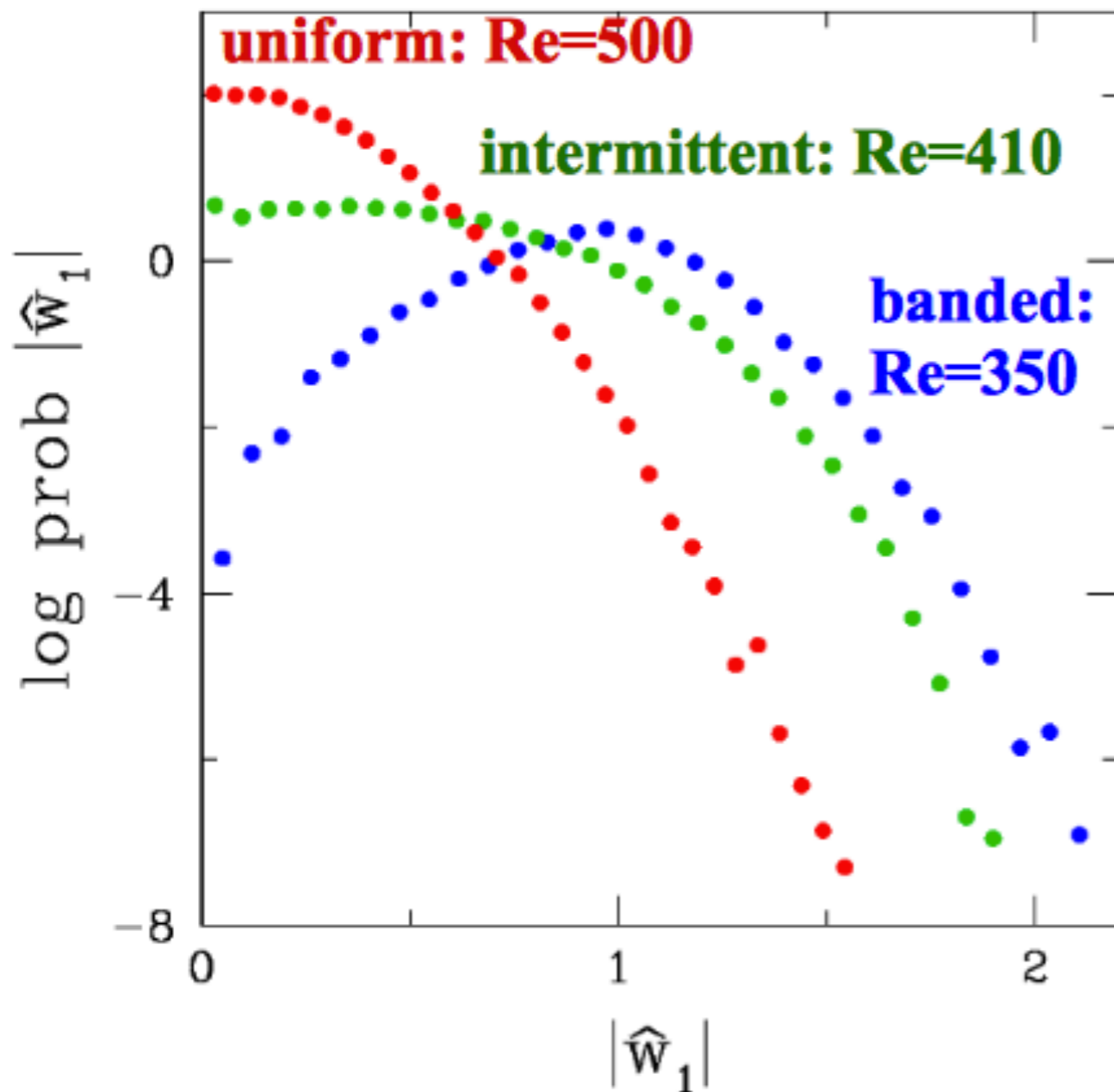
Uniform
Re=500



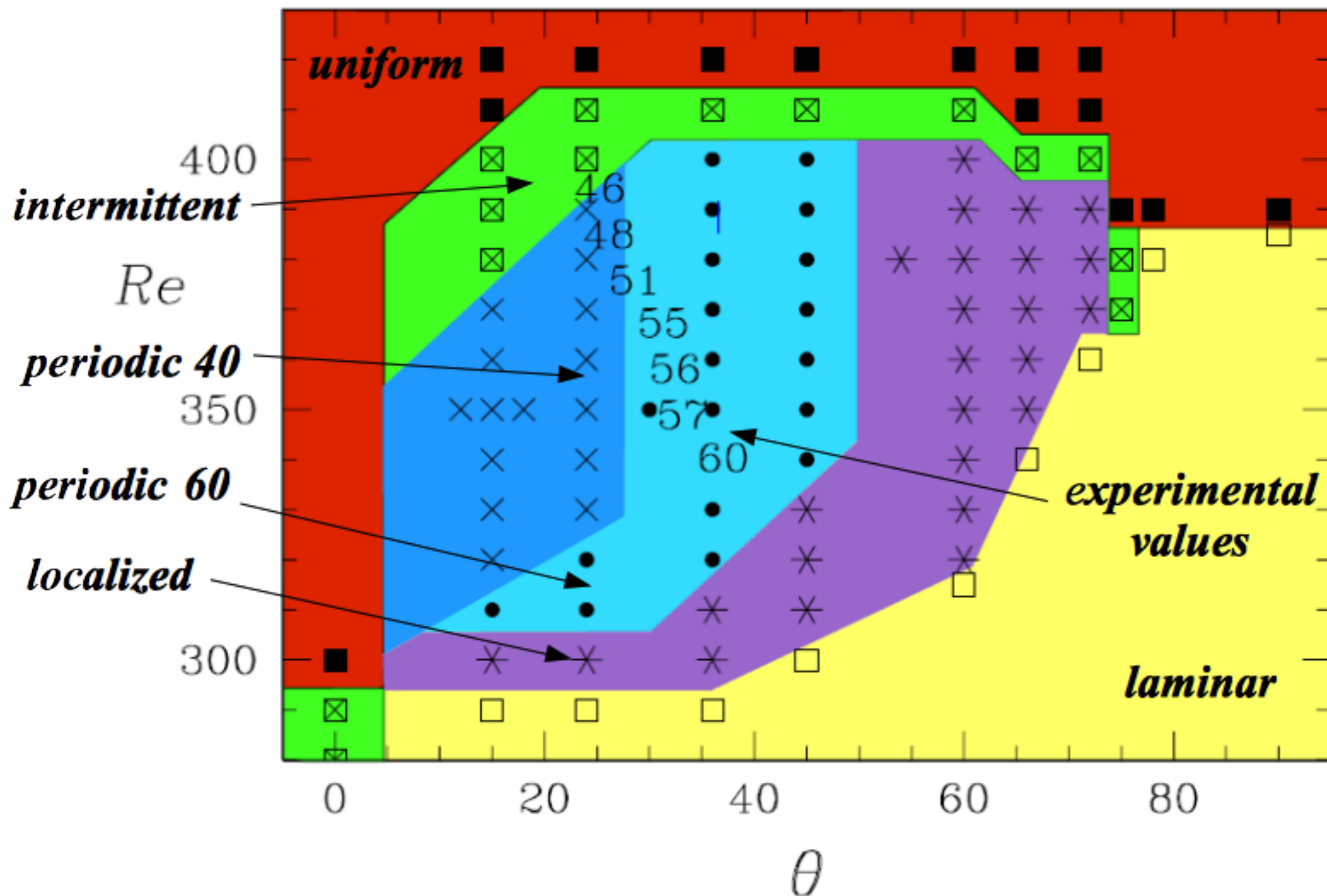
$$w(x=0, y=0, z, t) \xrightarrow{\text{z Fourier transform}} \widehat{w}_1 \longrightarrow |\widehat{w}_1|$$

Upper Threshold

*Probability Distribution Function of $|\hat{w}_1|$
(modulus of $m=1, \lambda=40$ component of spanwise velocity)*



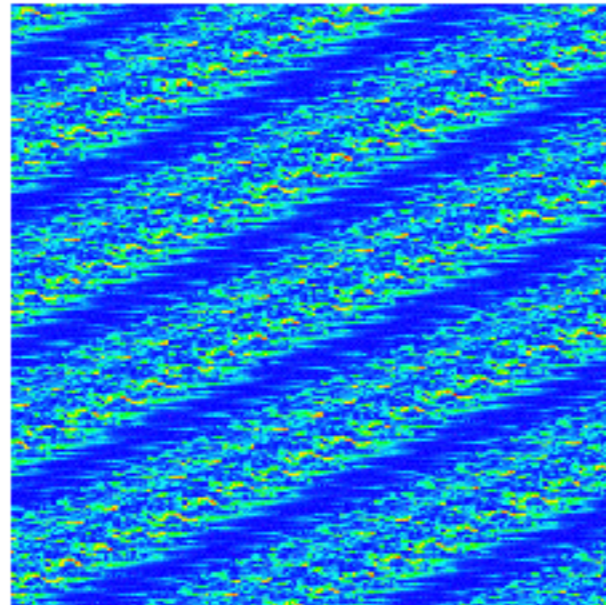
Varying angle: Regimes as a function of θ , Re



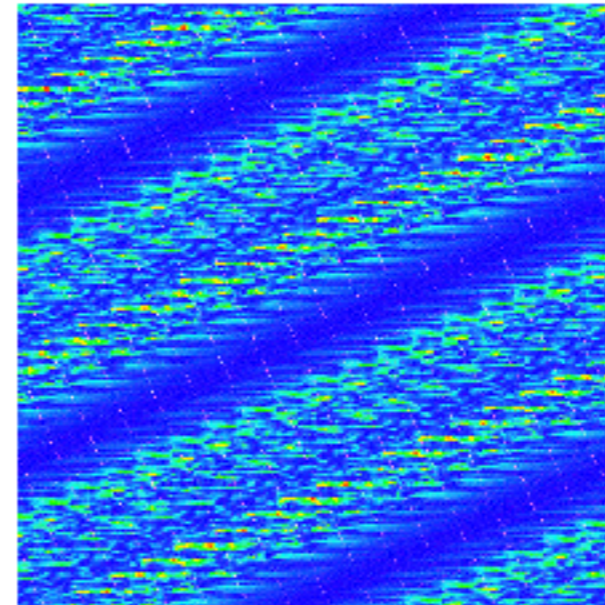
Extreme Wavelengths and Tilt Angles

Fixed $Re = 350$

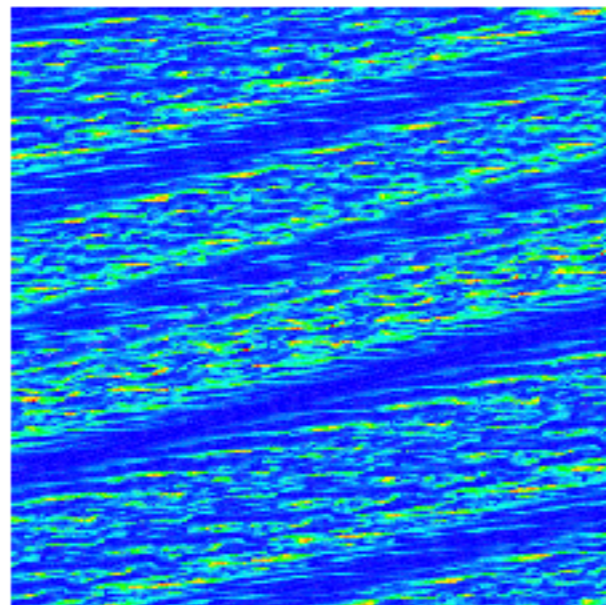
Minimum
Wavelength 35
at $\theta = 24^\circ$



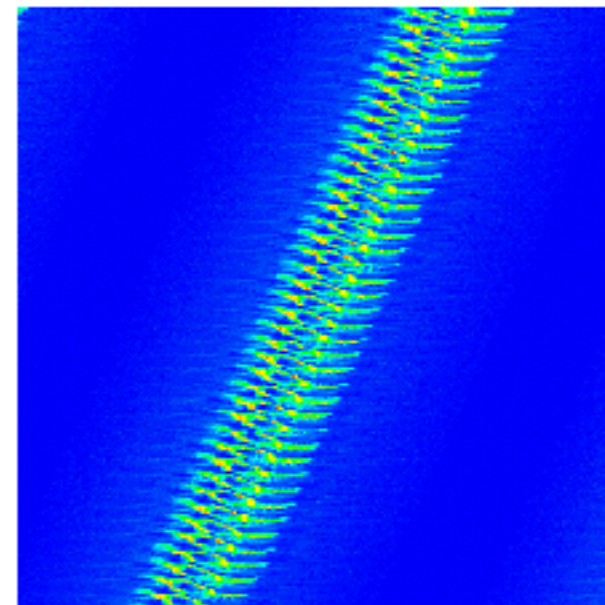
Maximum
Wavelength 65
at $\theta = 24^\circ$



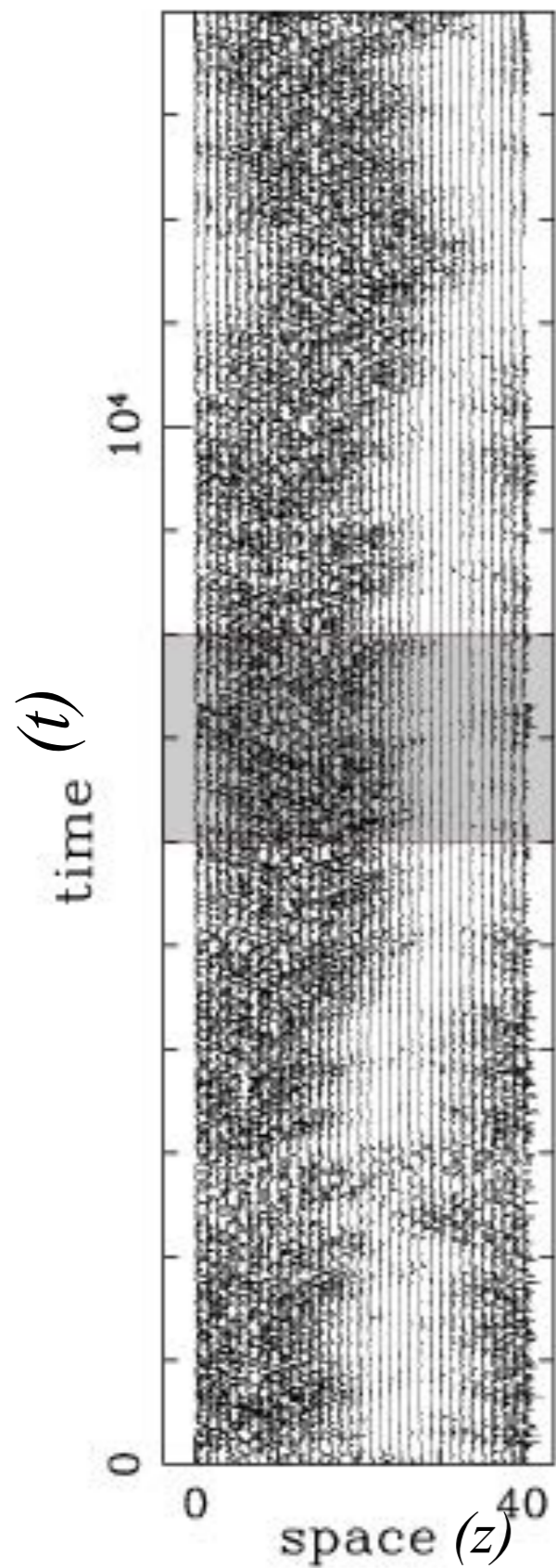
Minimum tilt 15°
at $L_z = 120$



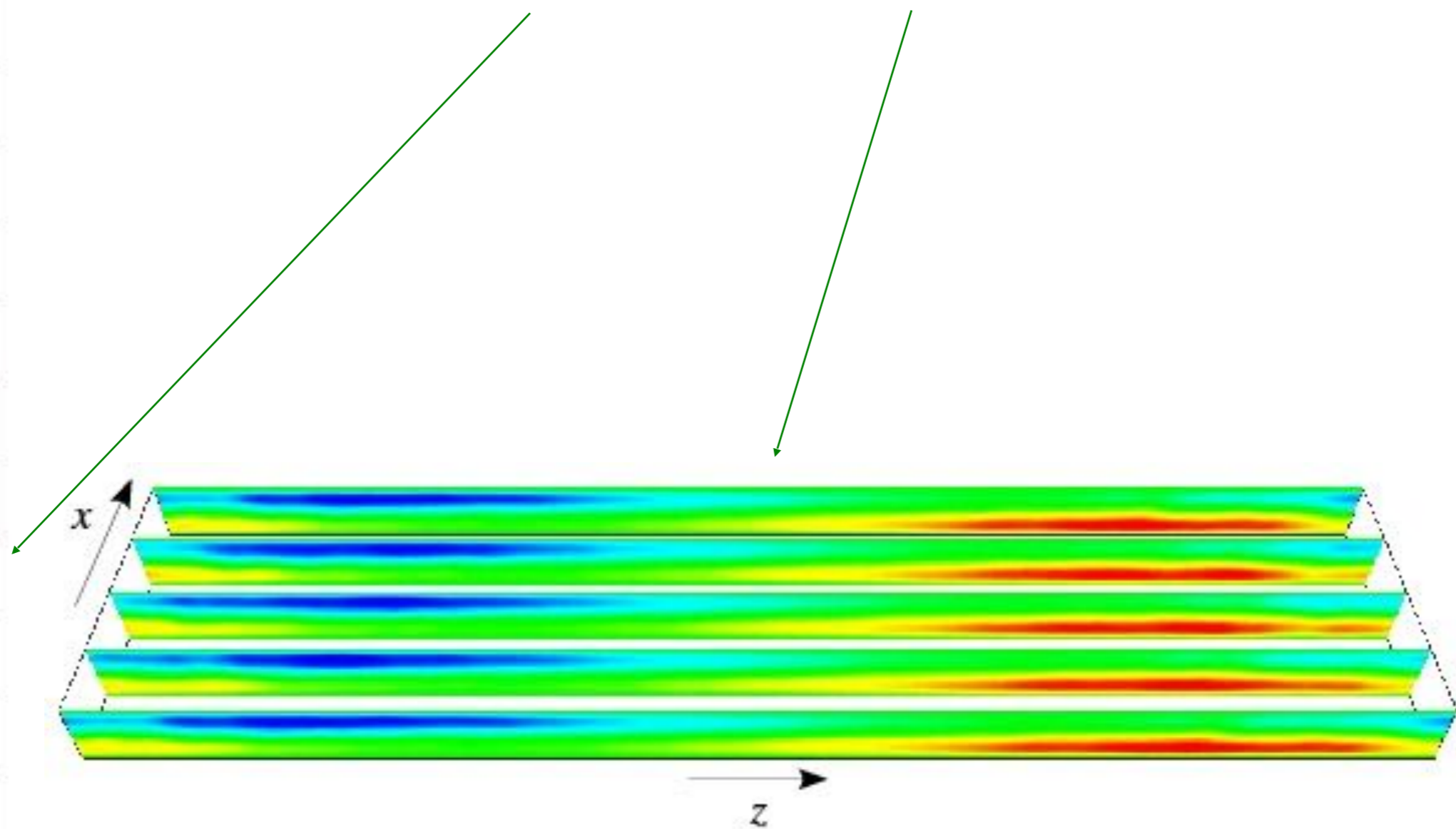
Maximum tilt 66°
at $L_z = 120$



Average: in time, then in x



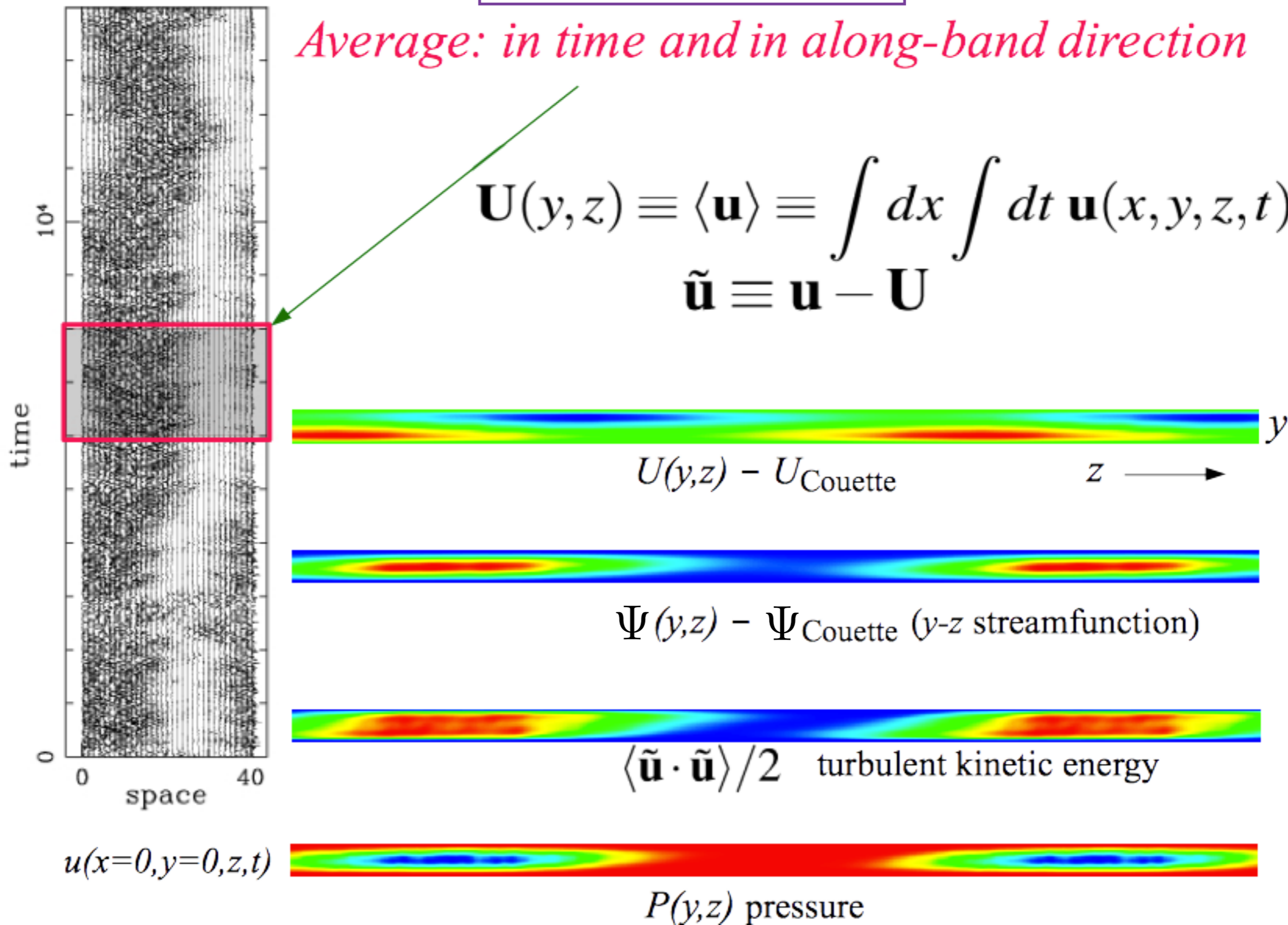
$$u(x=0, y=0, z, t)$$



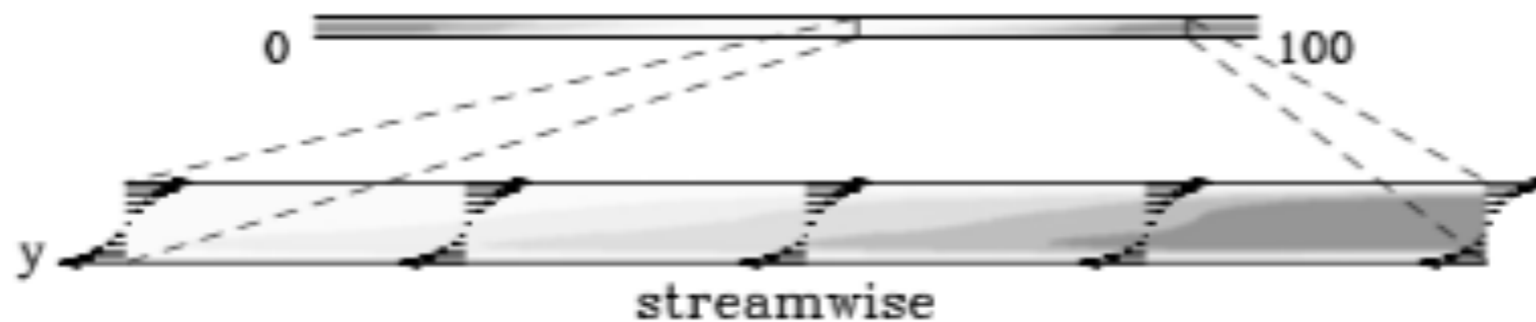
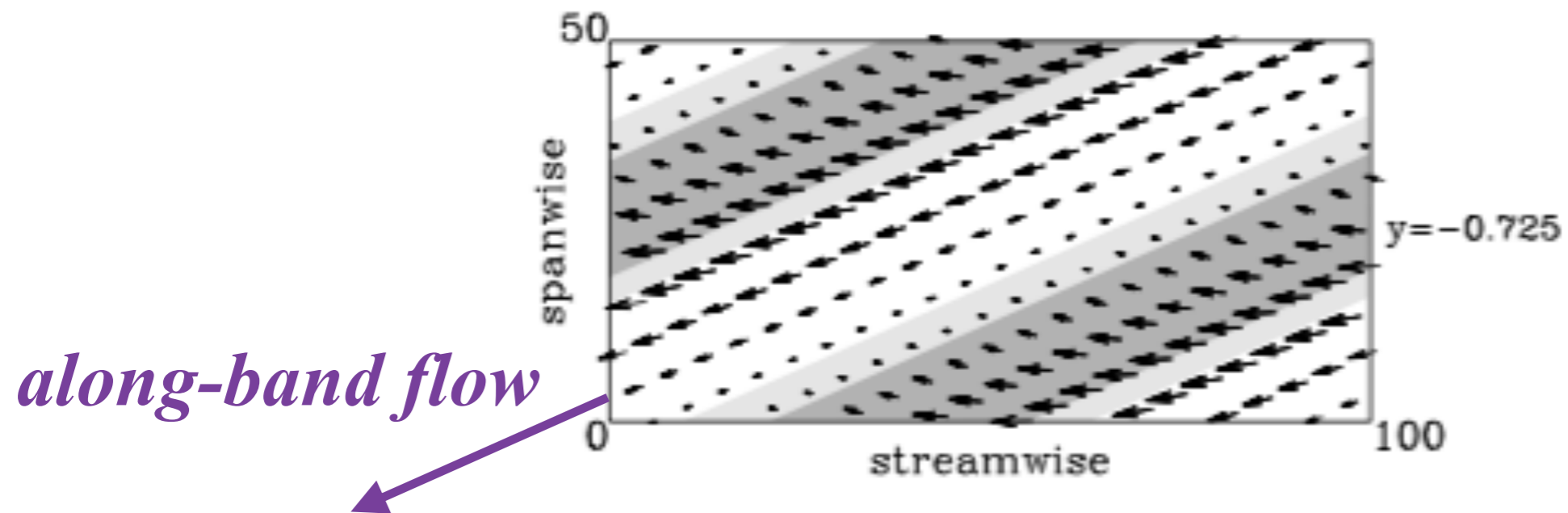
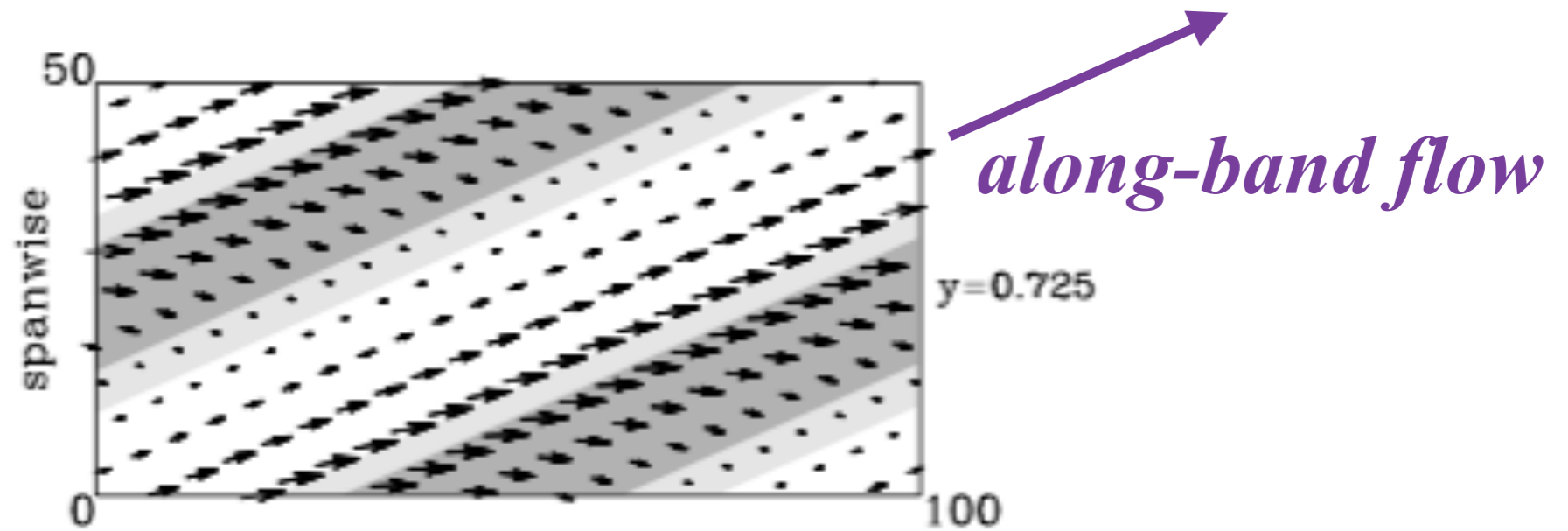
Mean flow

Average: in time and in along-band direction

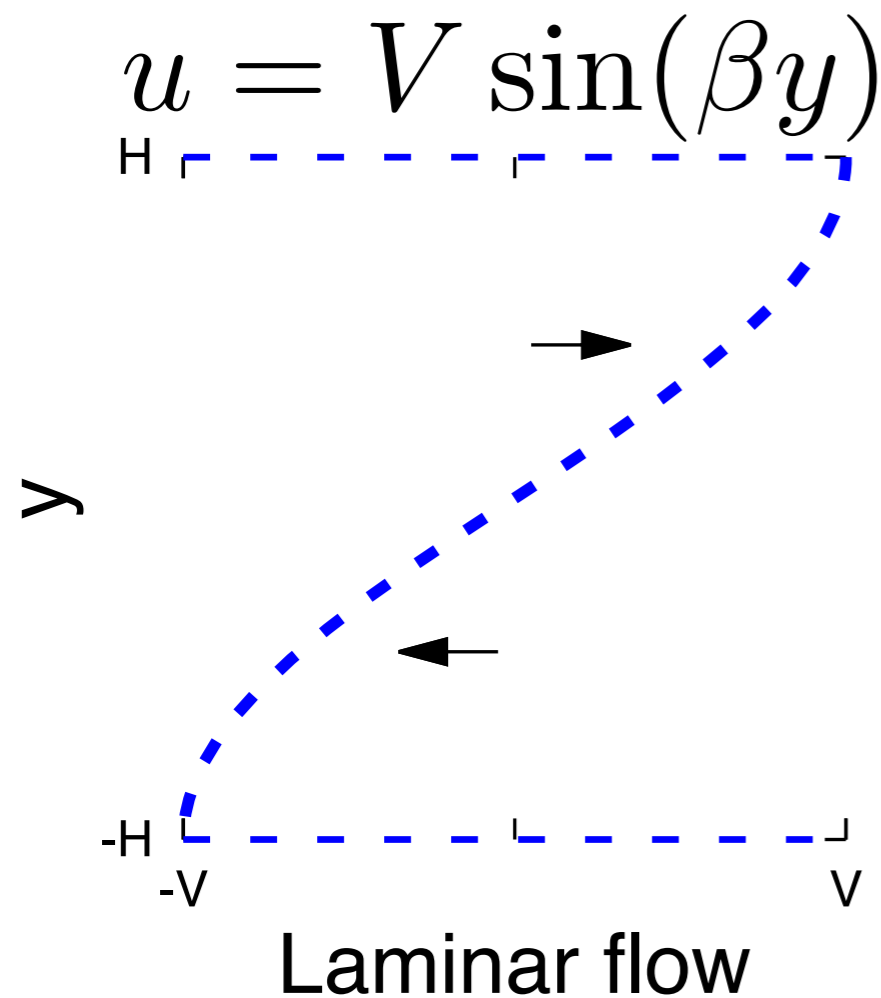
$$\mathbf{U}(y, z) \equiv \langle \mathbf{u} \rangle \equiv \int dx \int dt \mathbf{u}(x, y, z, t)$$
$$\tilde{\mathbf{u}} \equiv \mathbf{u} - \mathbf{U}$$



Mean flow (seen in horizontal plane)



Waleffe flow



Toy model of plane Couette flow

Stress free boundaries in y

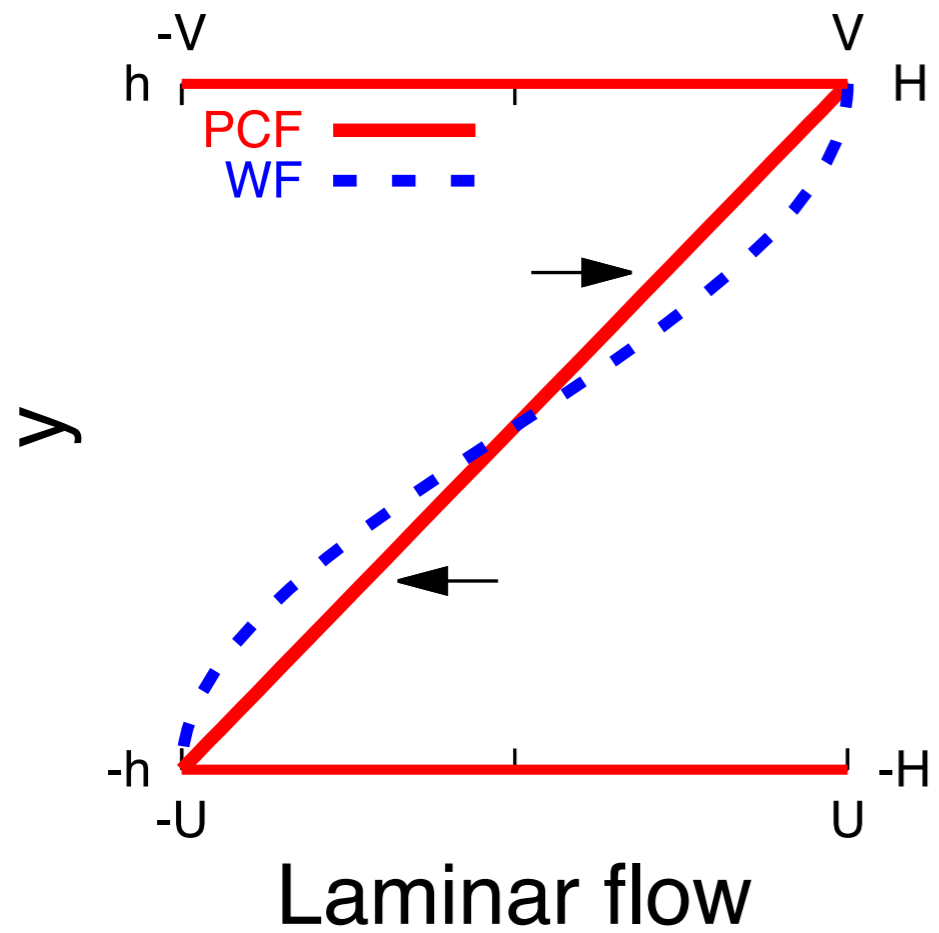
Body forced, $F = C \sin\left(\frac{\pi}{2} \frac{y}{H}\right)$

Demonstration of self-sustaining process.

Confined Kolmogorov flow

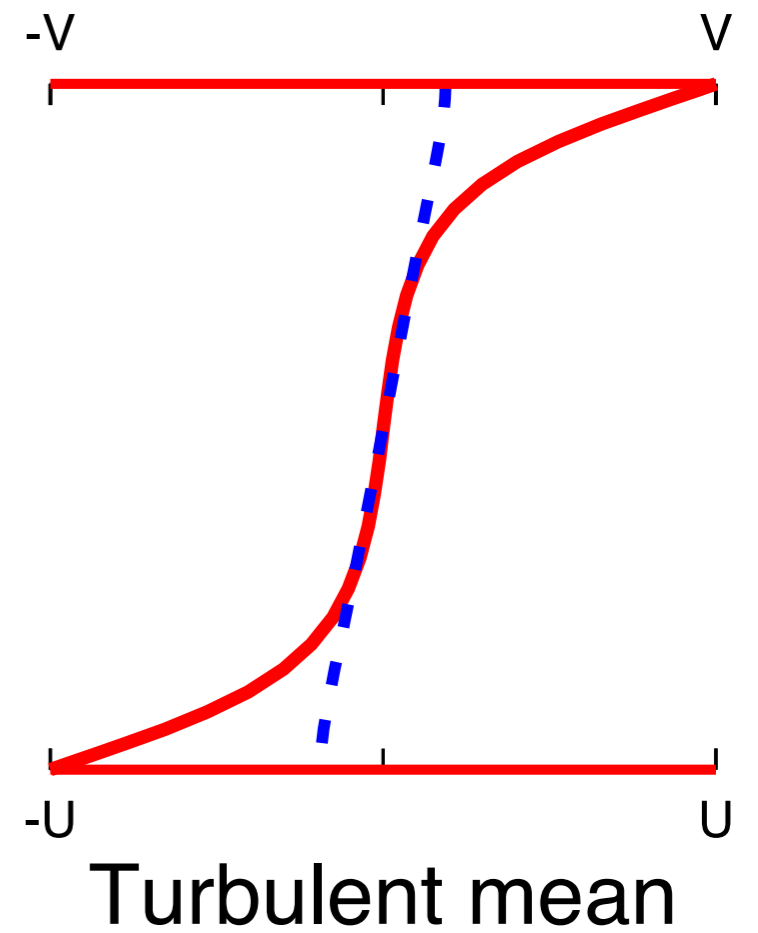
Do the same structures emerge?

PCF vs Waleffe flow



PCF

Linear laminar profile

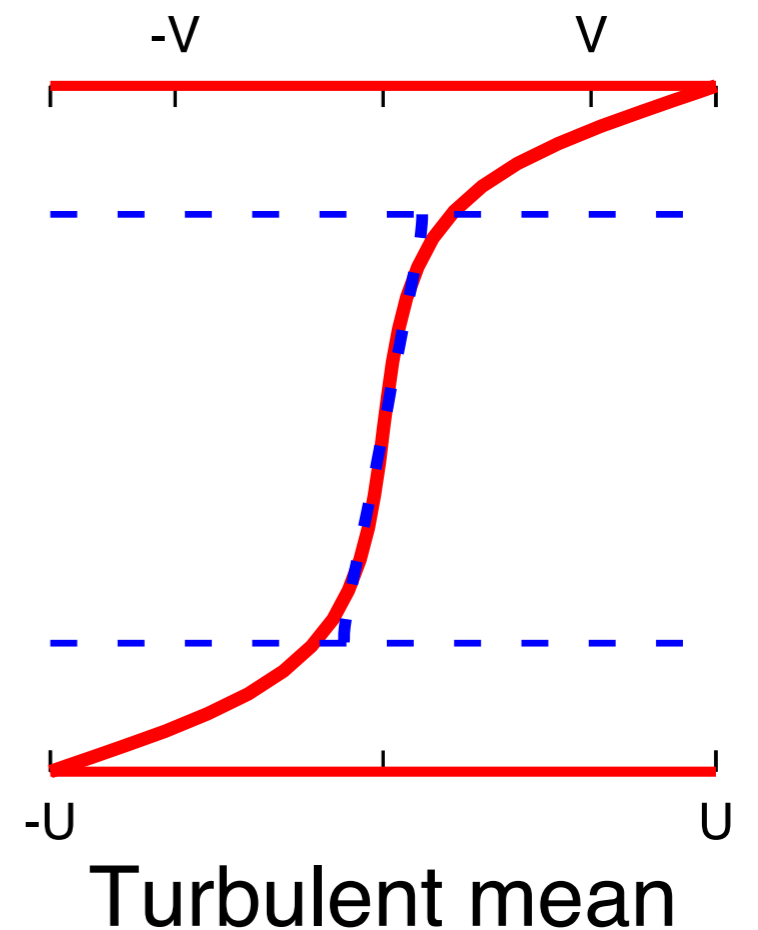
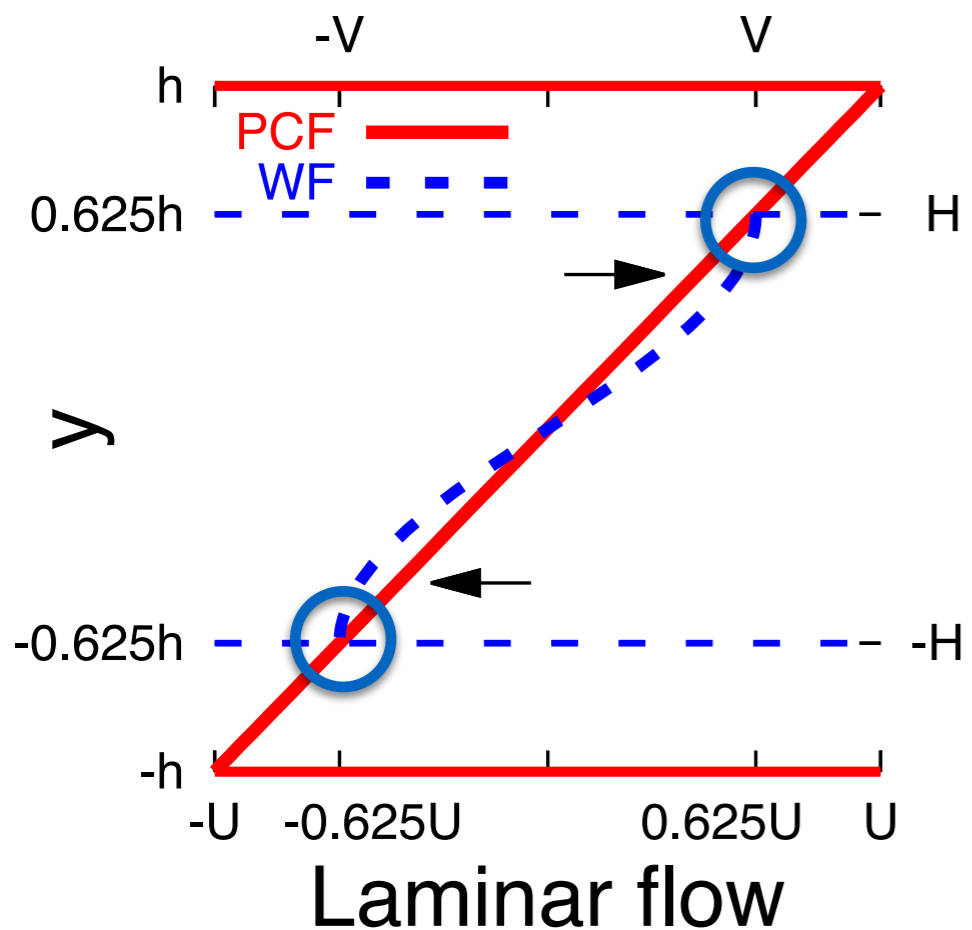


Waleffe flow

Linear turbulent profile

PCF vs Waleffe flow

Waleffe flow captures the interior of PCF

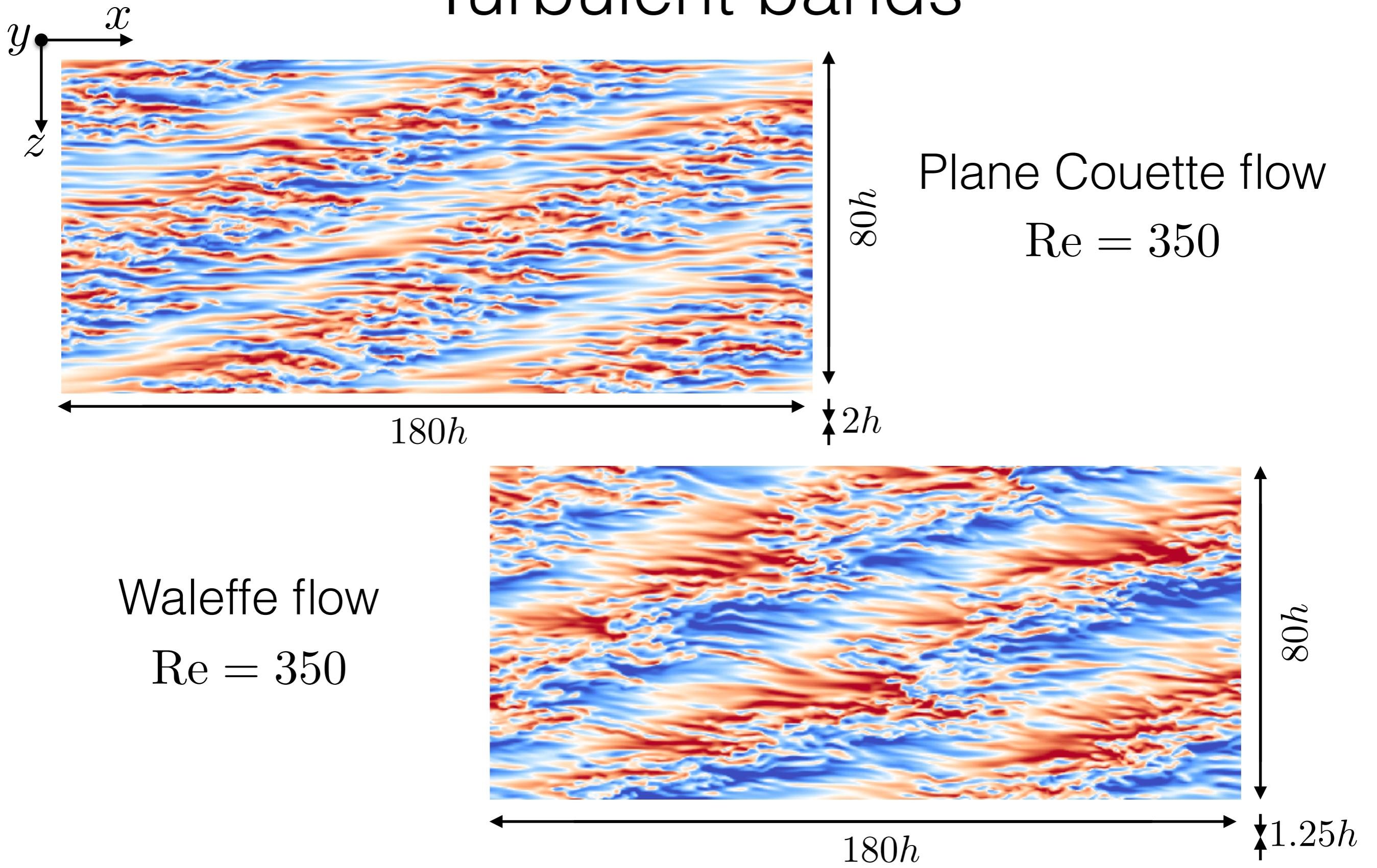


Rescale relative to PCF

$$H = 0.625h, \quad V = 0.625U$$

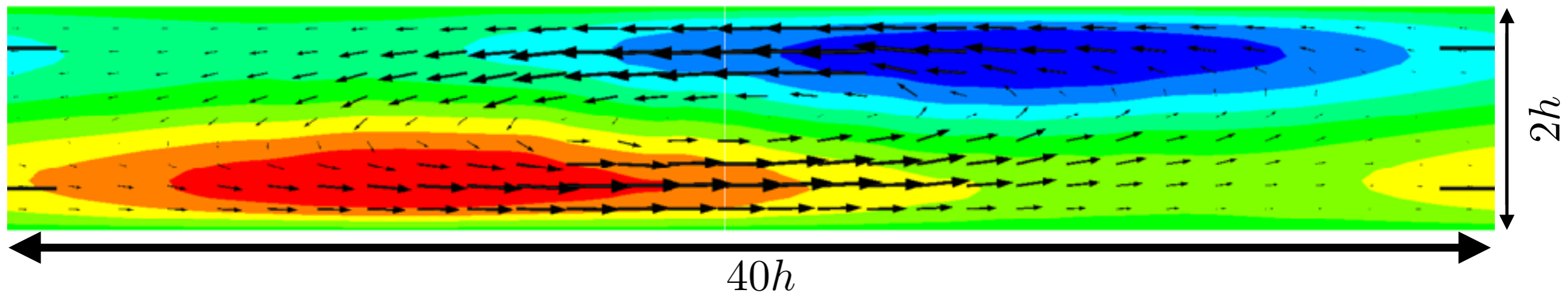
$$\text{Re} = \frac{Uh}{\nu} = \frac{VH}{0.625^2\nu}$$

Turbulent bands

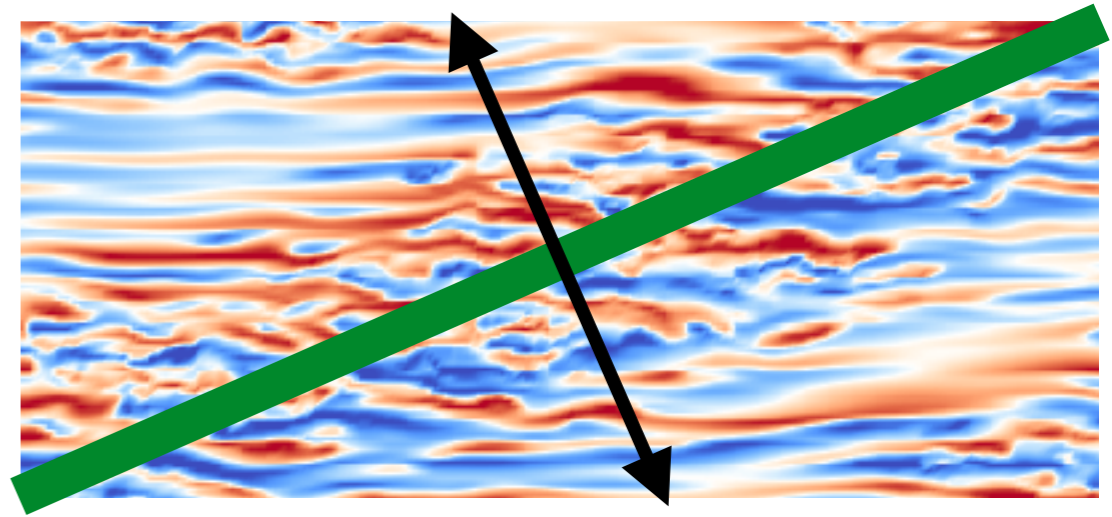


Mean structure

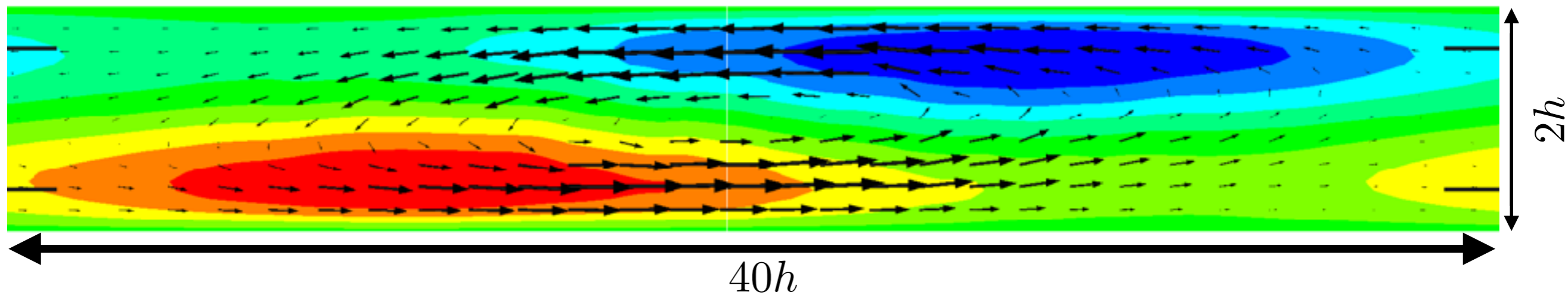
Plane Couette flow, $Re = 350$



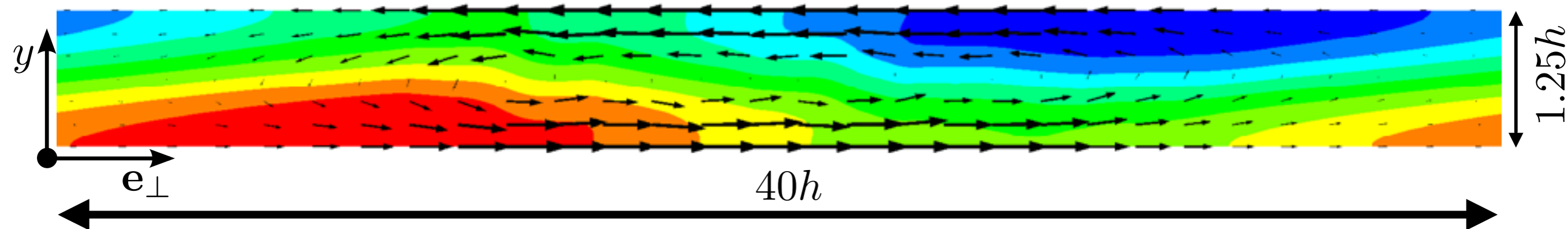
Mean structure



Plane Couette flow, $Re = 350$

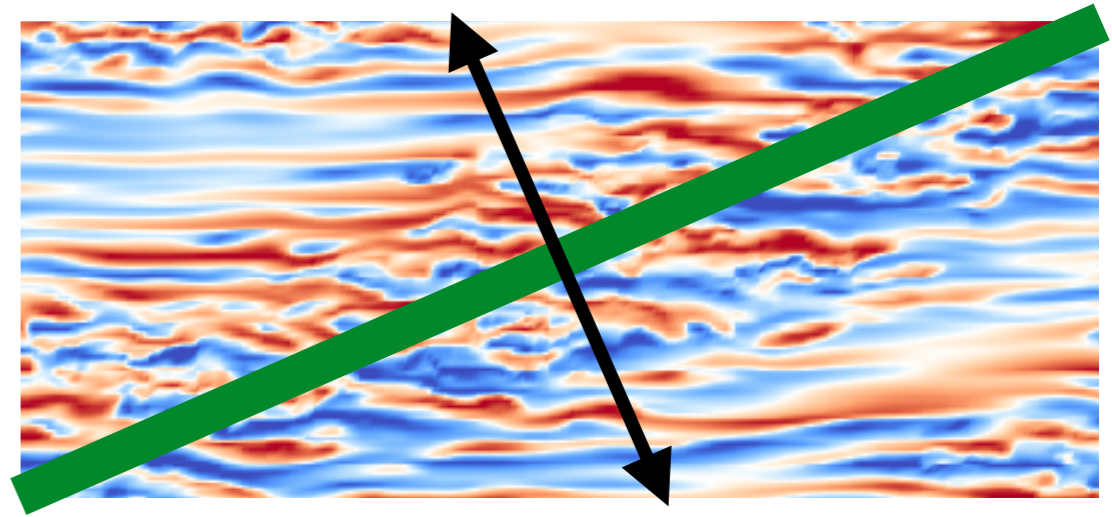


Waleffe flow, $Re = 350$

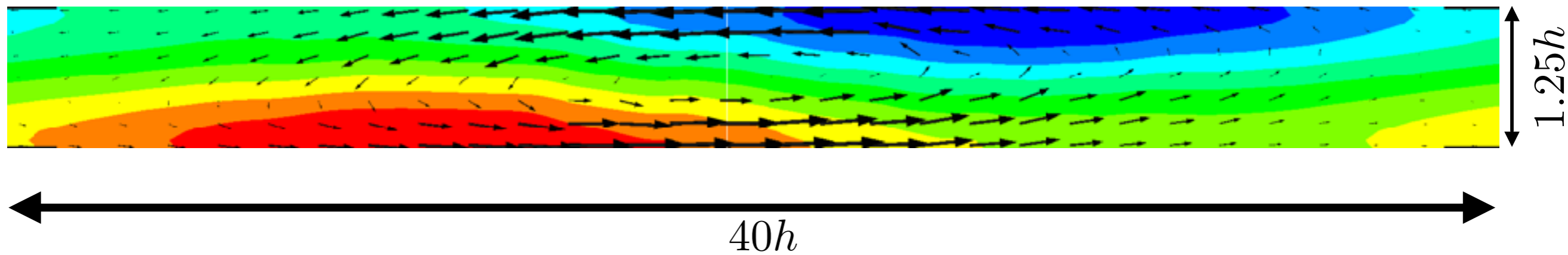


Existence range: PCF $Re \in [325, 420]$
WF $Re \in [260, 640]$

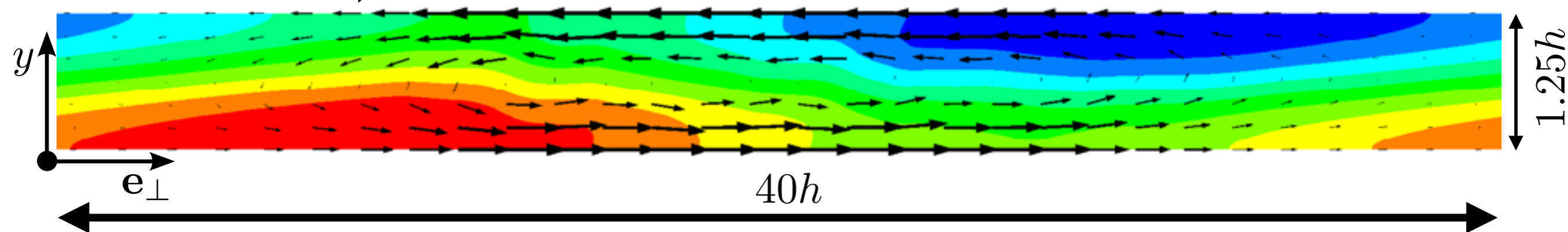
Mean structure



Plane Couette flow, $Re = 350$

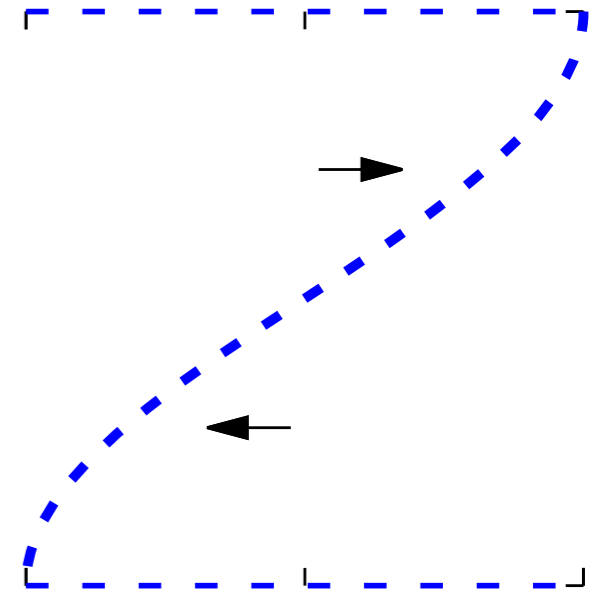


Waleffe flow, $Re = 350$



Existence range: PCF $Re \in [325, 420]$
WF $Re \in [260, 640]$

Modelling Waleffe flow



Using leading 4 Fourier modes that satisfy BC

$$u(x, y, z) = u_0(x, z) + u_1(x, z) \sin(\beta y) \\ + u_2(x, z) \cos(2\beta y) + u_3(x, z) \sin(3\beta y),$$

Result: 7 PDEs in (t, x, z)

Also requires only half the resolution in x, z

Plane Couette flow

$90h$

$40h$

Waleffe flow

$90h$

$40h$

PCF, $Re=500$

WF, $Re = 750$

$t = 0$

MWF, $Re = 250$

Model Waleffe flow

$90h$

$40h$

Defining and characterizing the lower bound



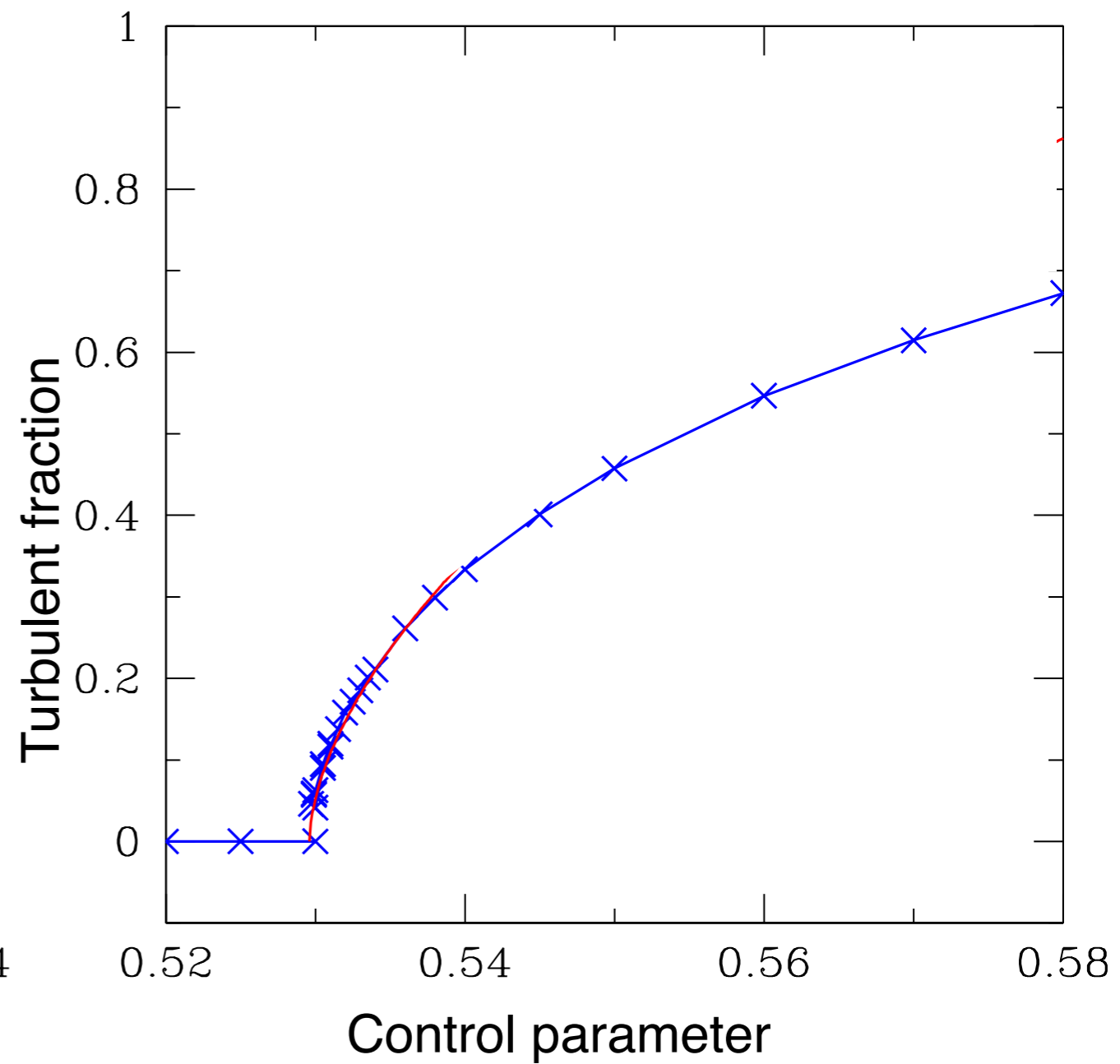
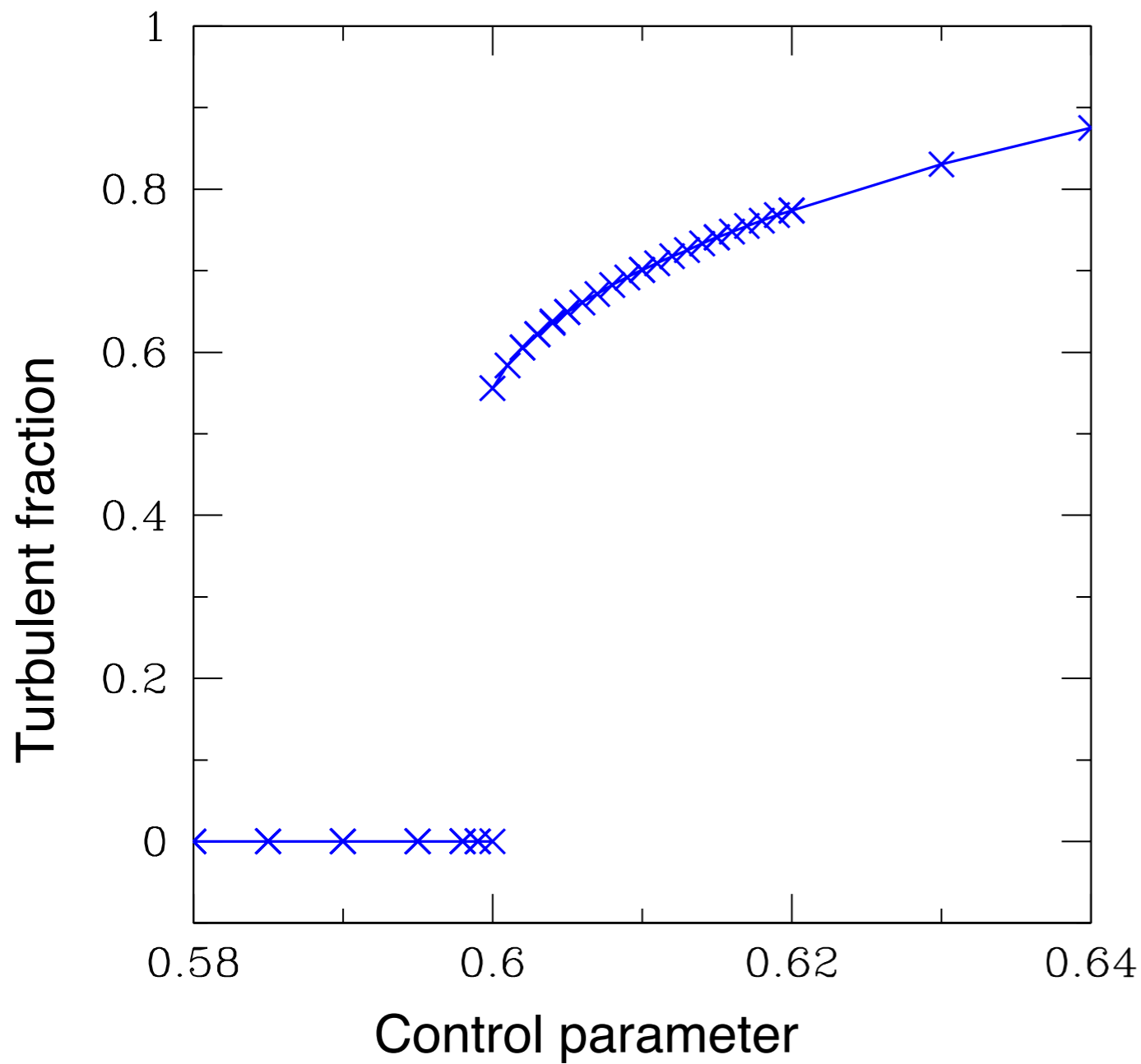
How low does turbulence go?

Continuous or discontinuous?

Universal or not?

First asked by Pomeau 1986

Discontinuous or Continuous?



Supersize me

$$Re > Re_c$$

Bottin et al. 1998

Prigent et al. 2002

Duguet et al. 2010

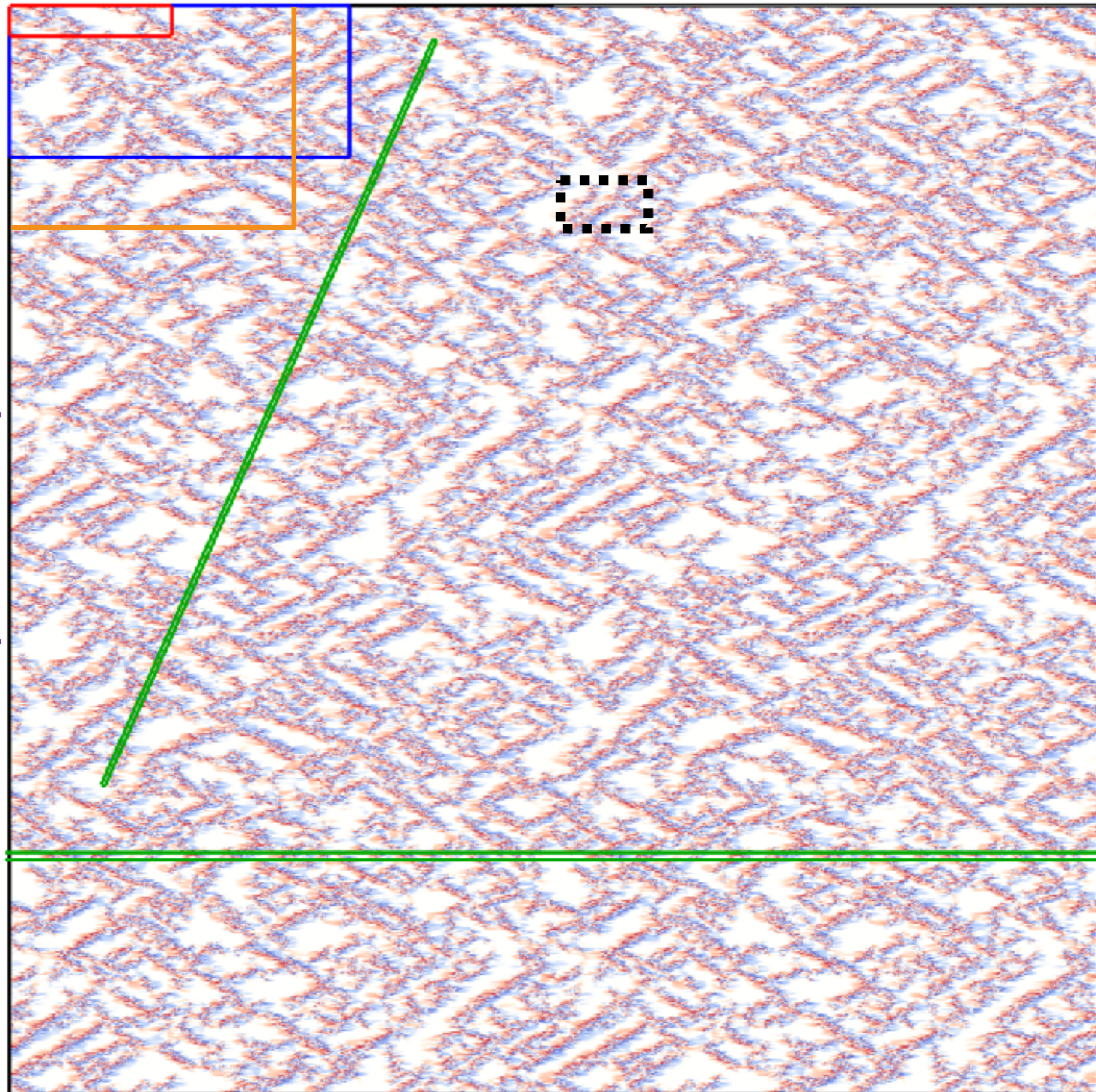
K. Avila 2013

Sano & Tamai 2016

Lemoult et al. 2016

Our system

[2560, 2560]



Supersize me

$$Re \approx Re_c$$

Bottin et al. 1998

Prigent et al. 2002

Duguet et al. 2010

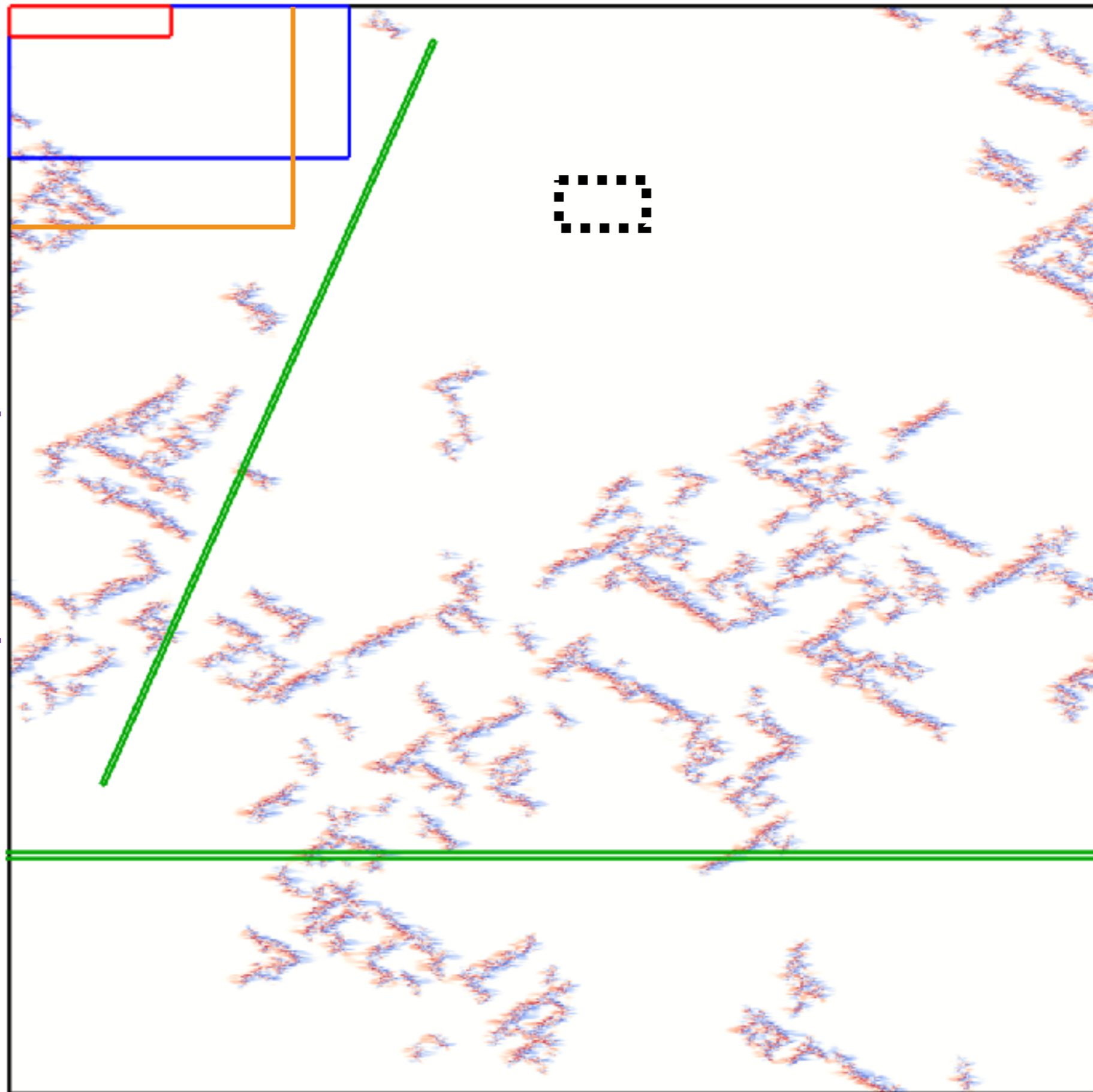
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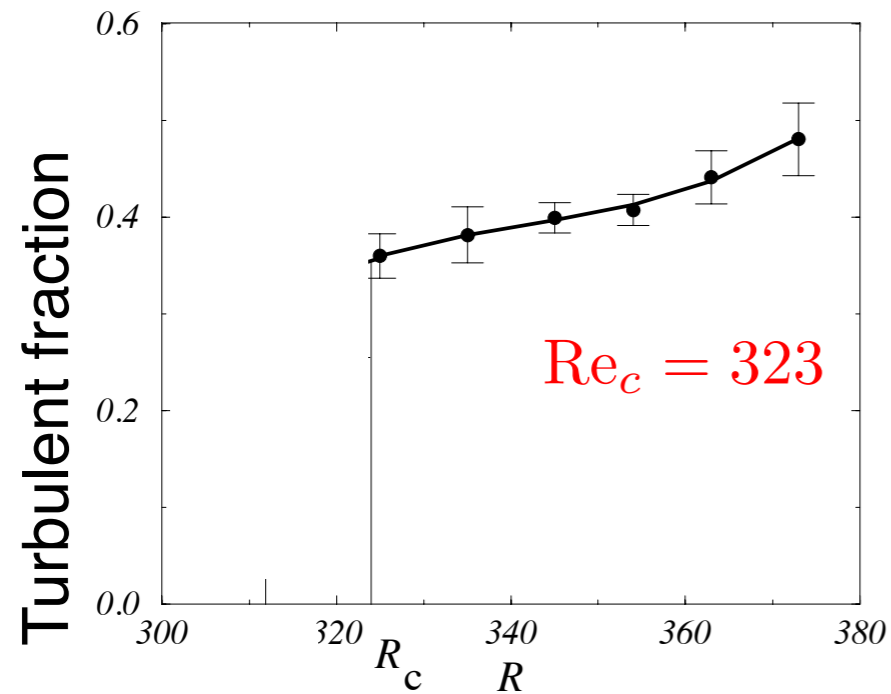
Lemoult et al. 2016

Our system

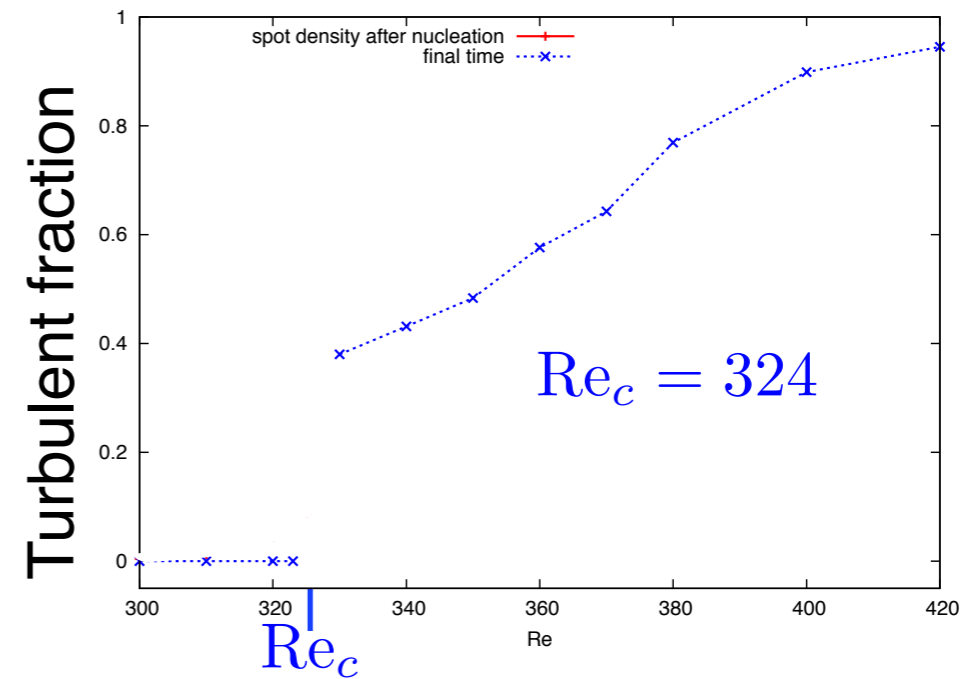
[2560, 2560]



Previous studies: discontinuous

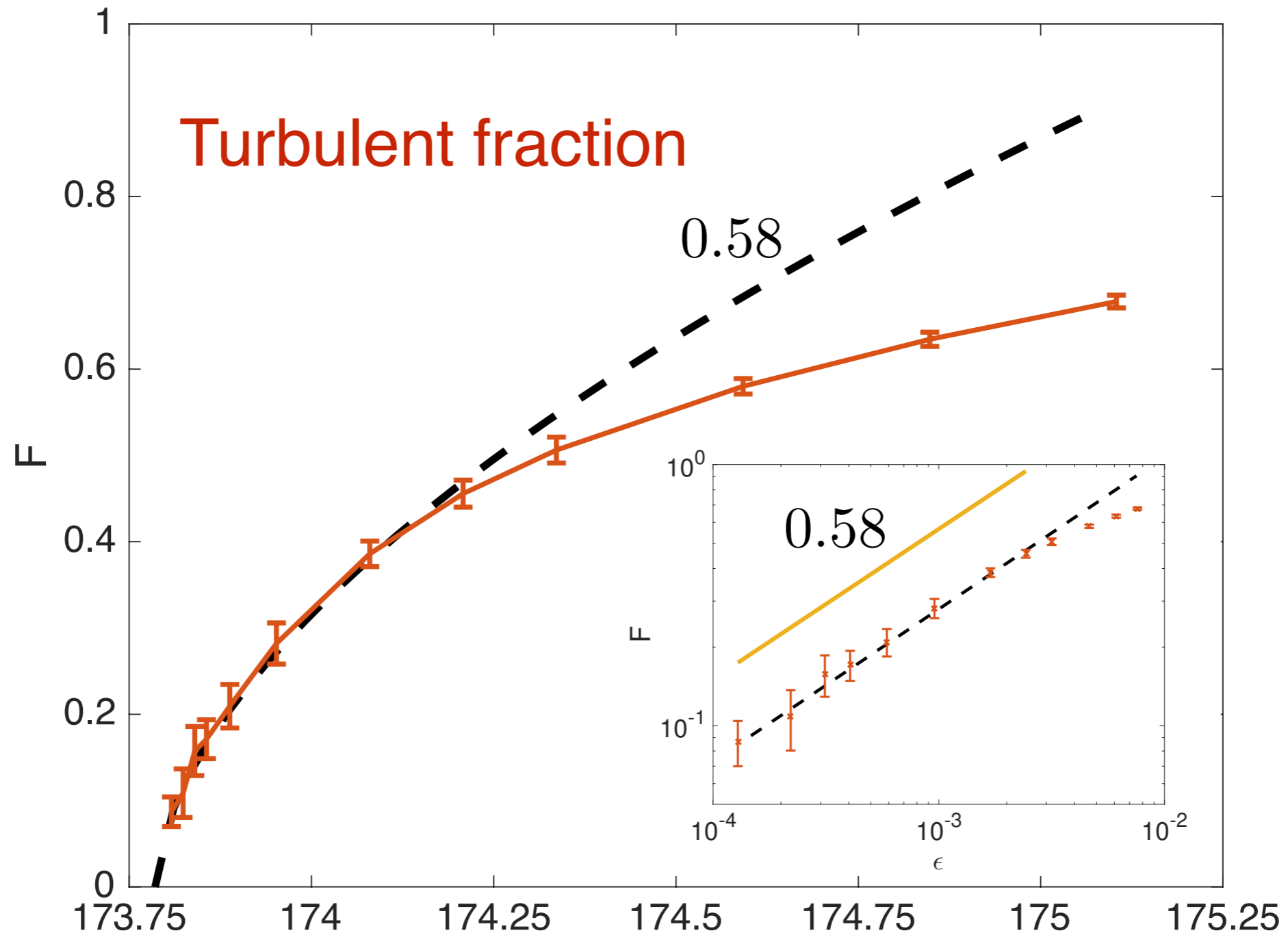


Bottin & Chaté 1998

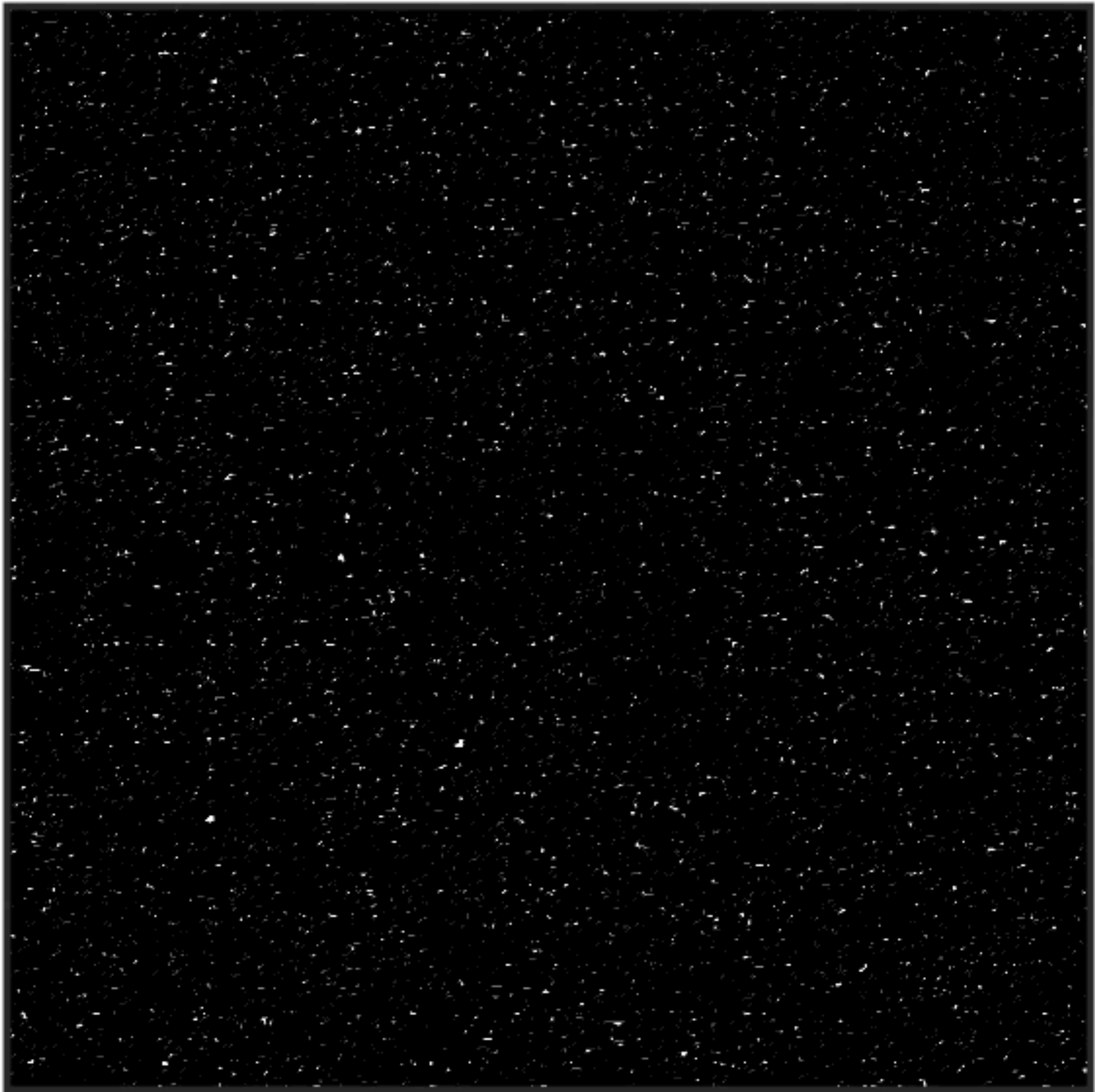


Duguet et al. 2010

Our result: **Continuous**

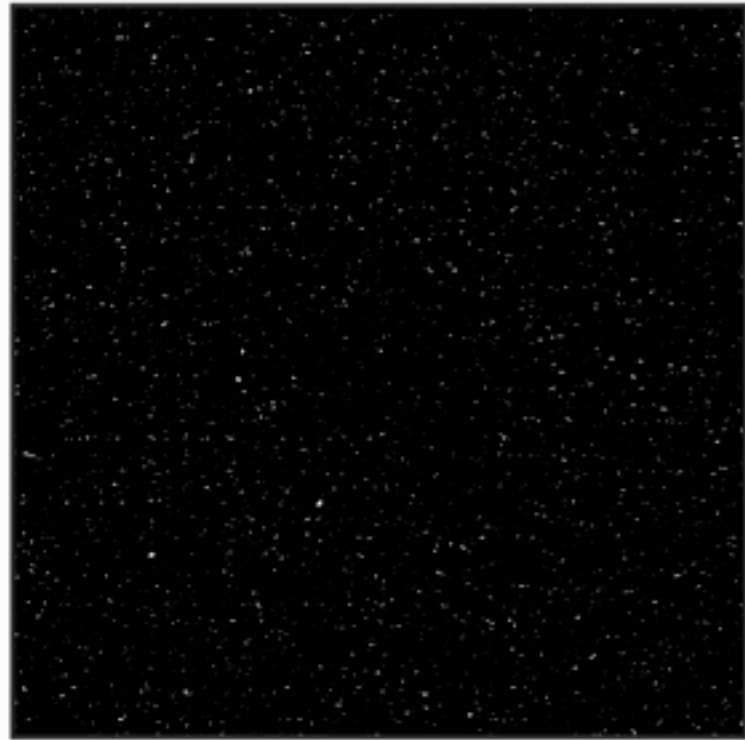


Re for Truncated Waleffe flow

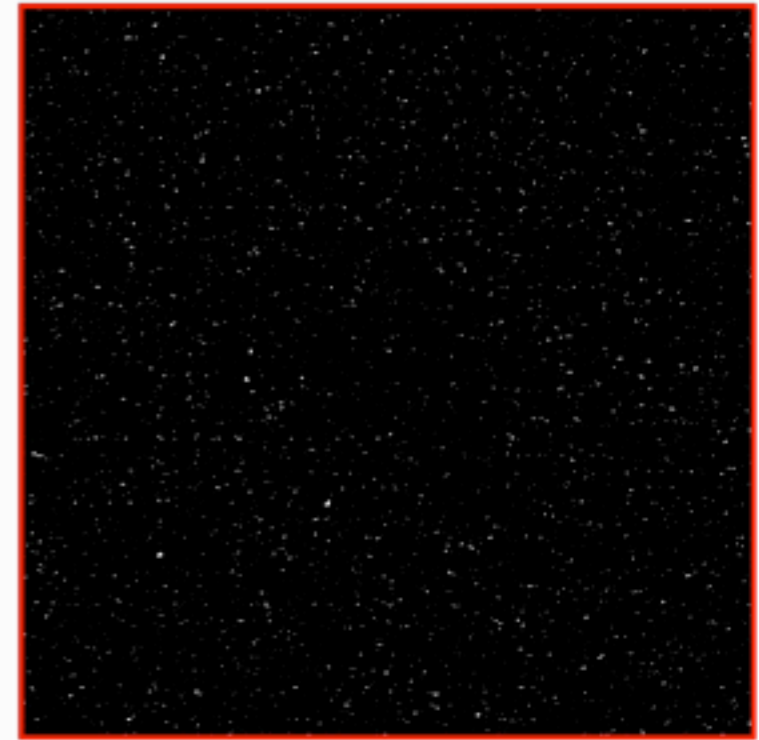
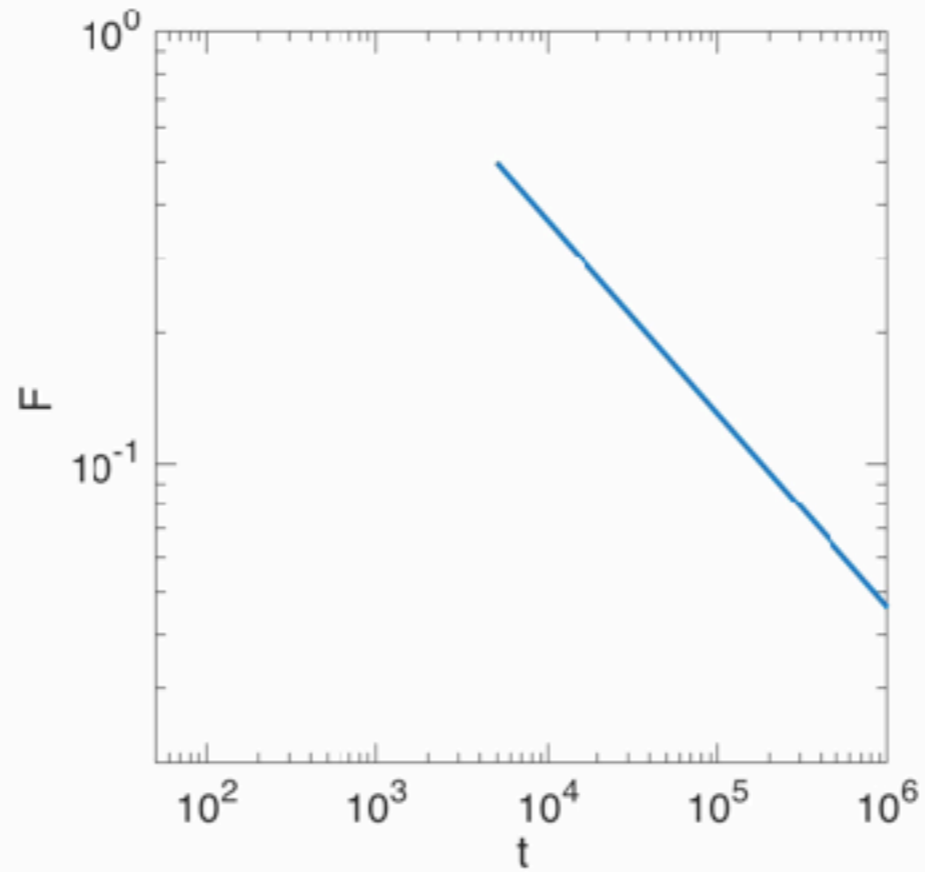


1 sec = $10^4 h/U$

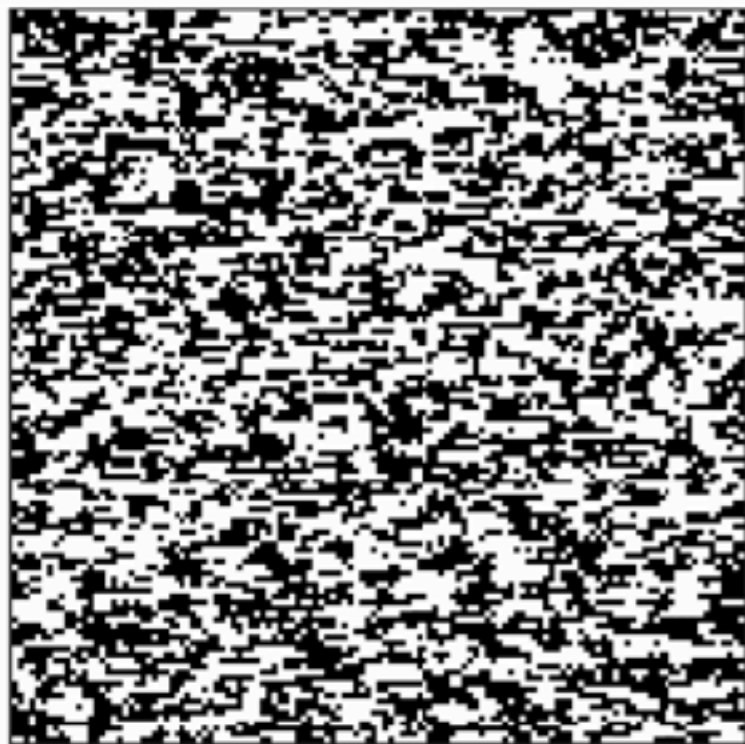
Shear flow



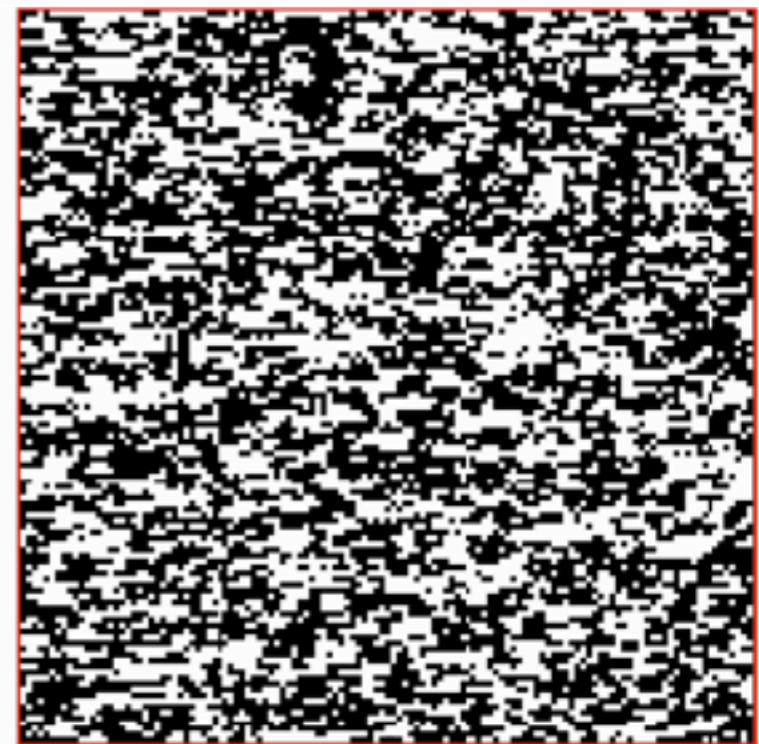
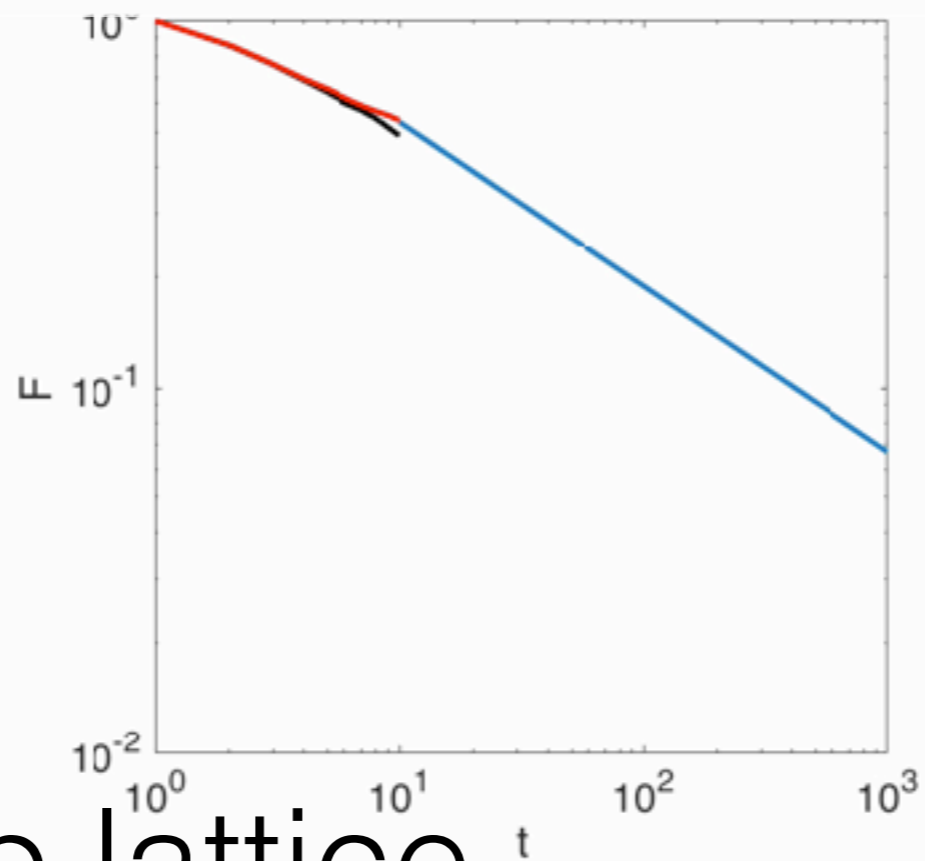
$Re \approx Re_c$

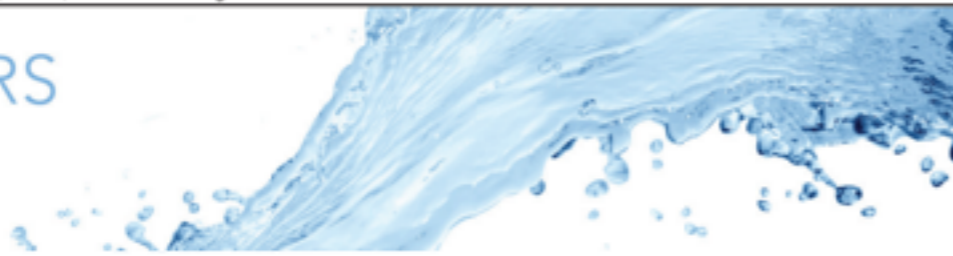


$Re > Re_c$



Coupled map lattice





Patterns in transitional shear turbulence. Part 1

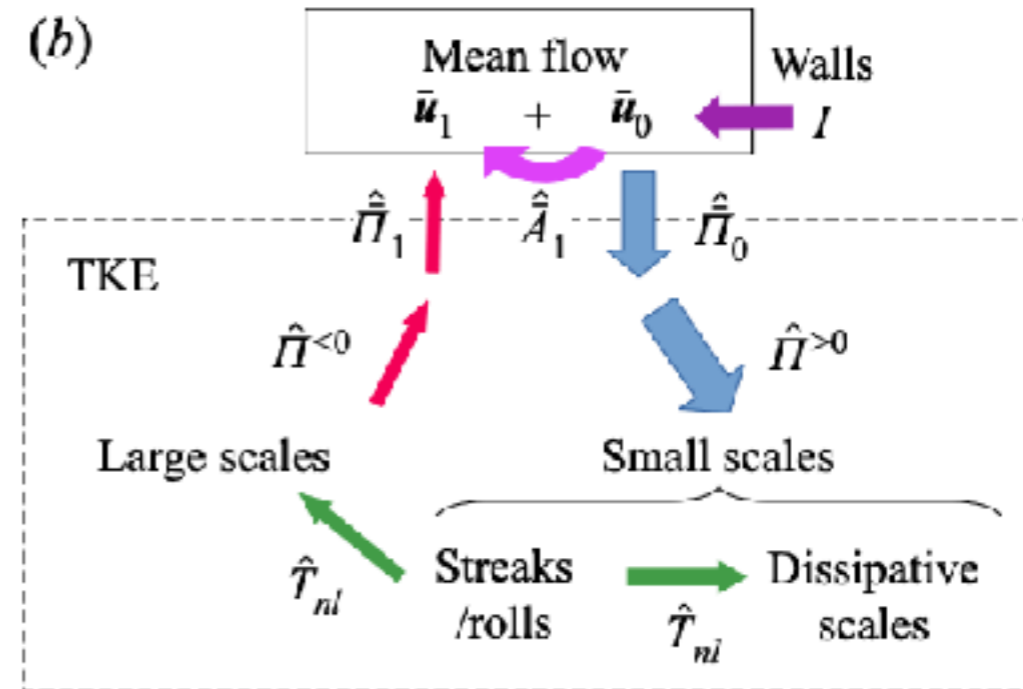
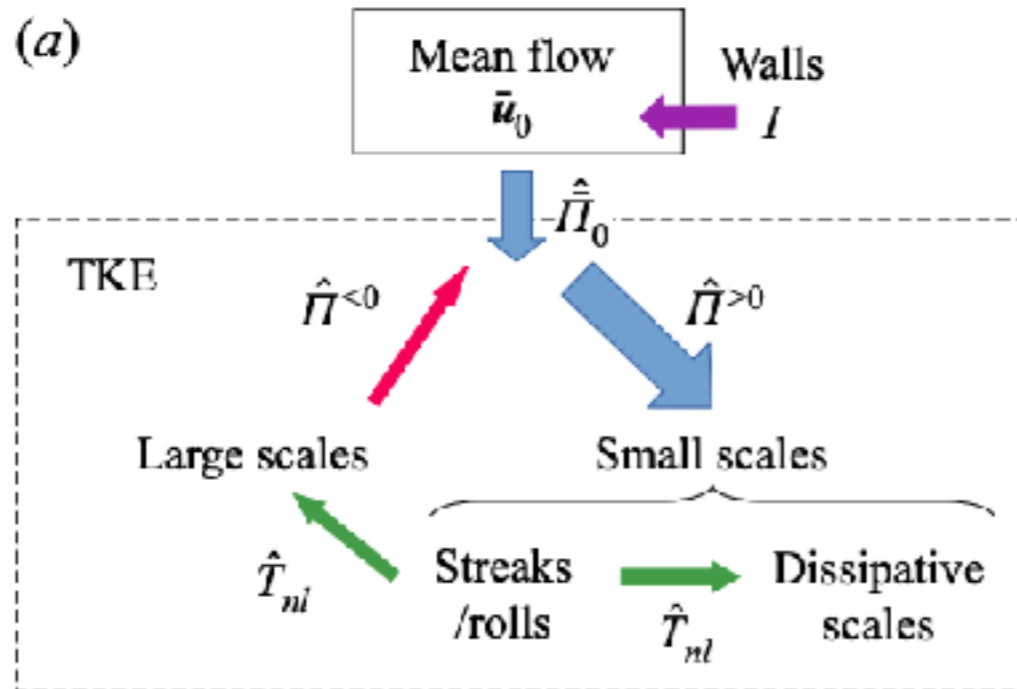
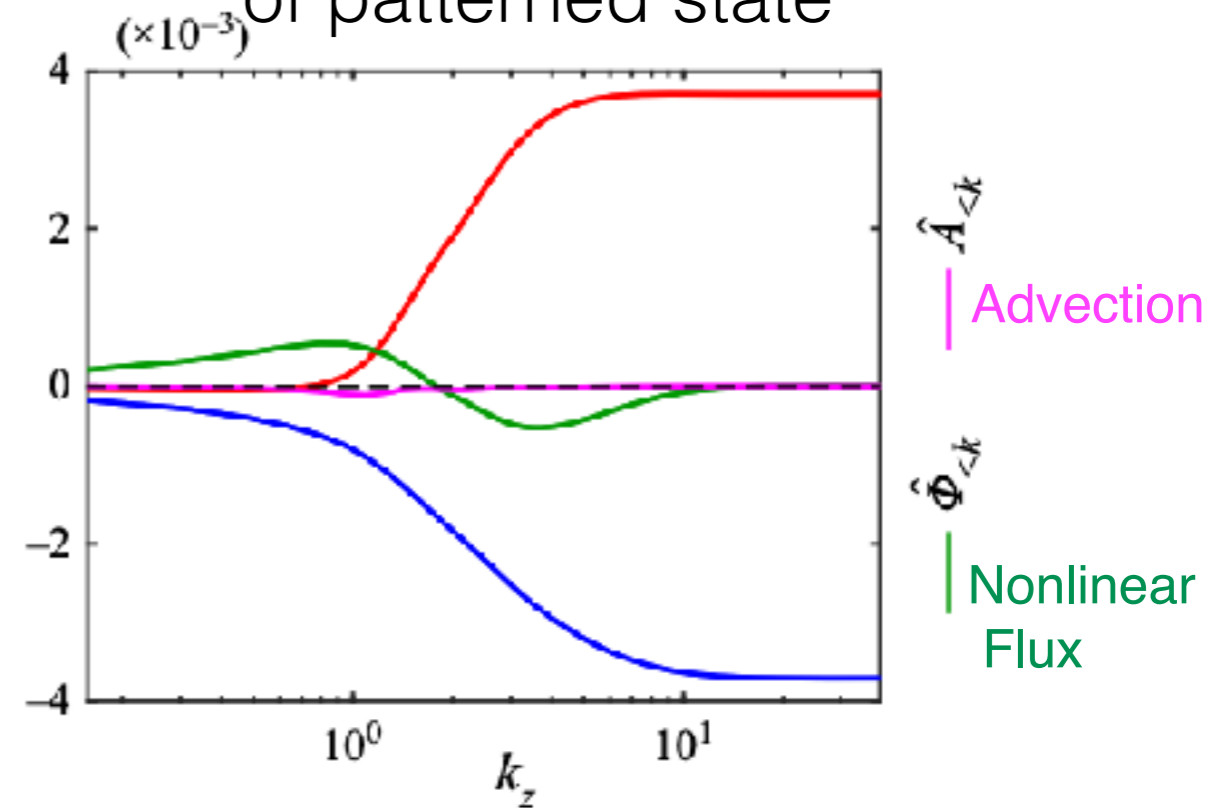
Energy transfer and mean-flow interaction

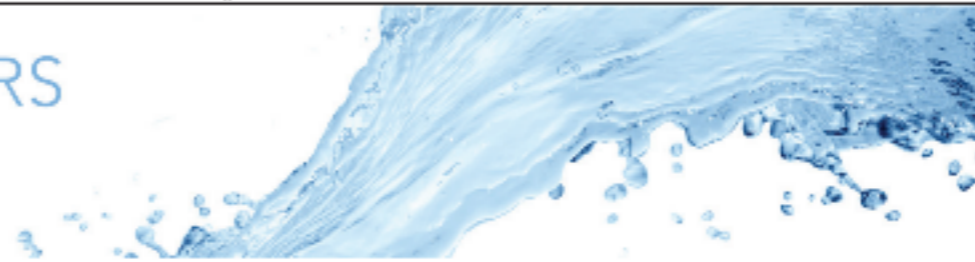
Sébastien Gomé^{1,†}, Laurette S. Tuckerman¹ and Dwight Barkley²

Production $\hat{\Pi}_{<k}$

Dissipation $-\hat{D}_{<k}$

Spectral energy balance of patterned state





Patterns in transitional shear turbulence. Part 2. Emergence and optimal wavelength

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