BRIDGING CLASSICAL AND QUANTUM TURBULENCE, JUL 2023 CARGÈSE, CORSICA

Are turbulent puffs and slugs (in pipes) and turbulent plugs (in He-II counterflow) similar ?

Anna Pomyalov

Weizmann Institute of Science, Israel

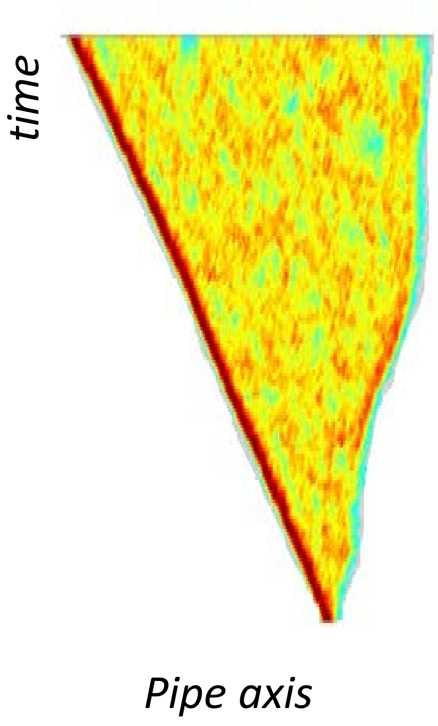
PHYS. REV. B **101**, 134515 (2020)

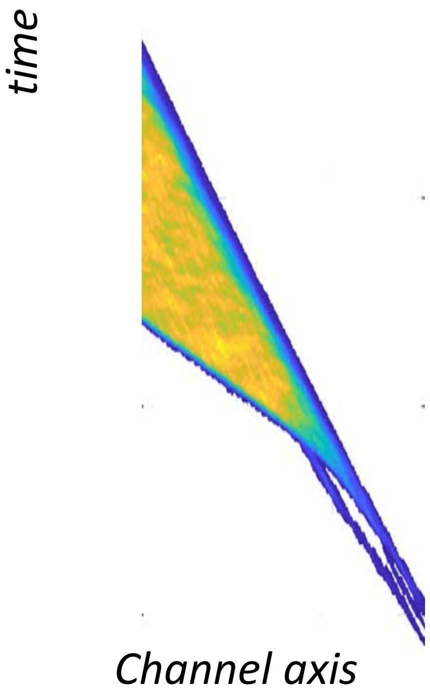
In classical fluid in pipes and ducts:

The puffs are localized turbulent patches that do not expand
The slugs are localized turbulent patches that expand

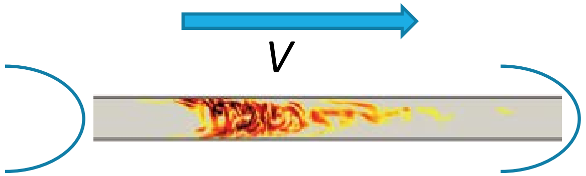
In superfluid He-4

The plugs are localized patches of vortex line density –
the turbulent state of superfluid He component

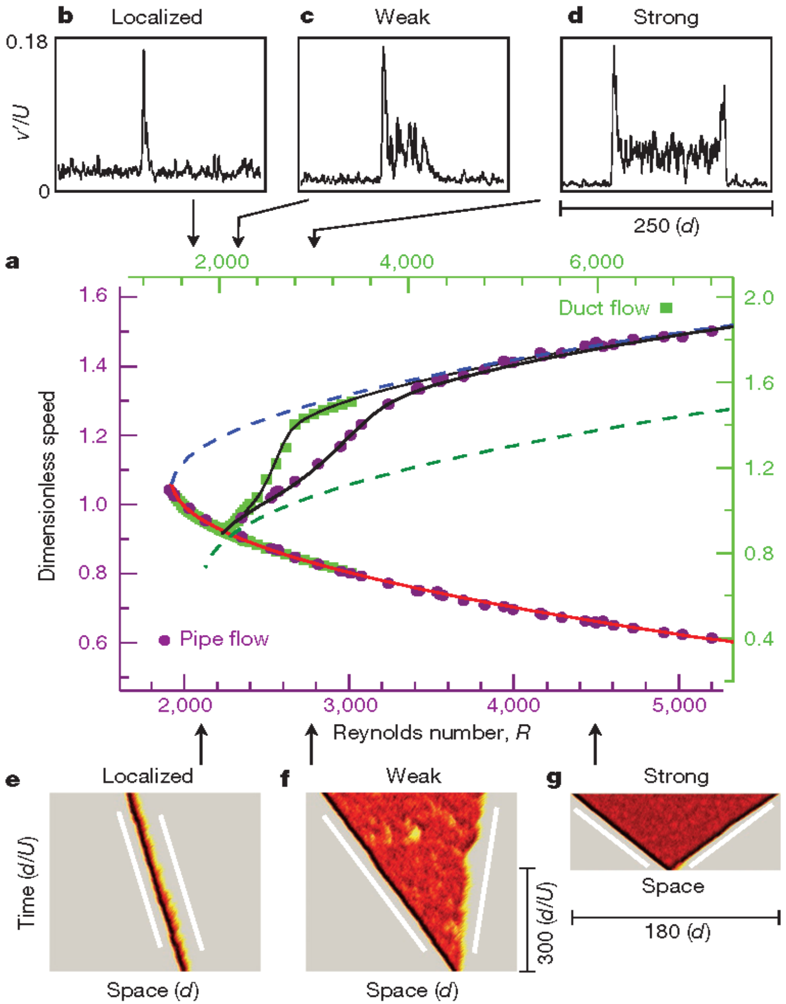




Turbulent fronts in pipes and ducts



- The turbulent intensity peaks at the leading (upstream) front
- At low *Re* – localized puffs, both fronts have the same speed
- At intermediate *Re* – growing slugs, mostly due to the upstream (leading) front lagging behind the advecting velocity
- At high *Re* – both fronts take energy from the flow, front speeds are symmetric around the advection speed
- Nonlinear dependence of on *Re*



Barkley *et al*, Nature, 15701 (2015)

\+ many more Barkley JFM **803**, P1 (2016)

Song *et al*, JFM **813**, 1045 (2017)

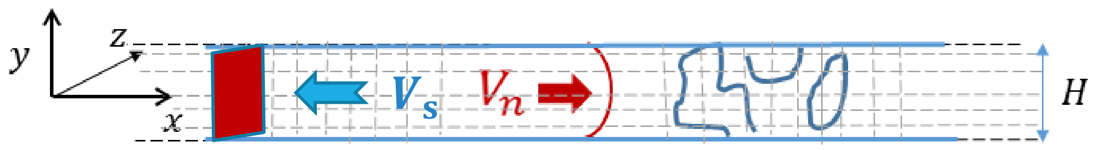
He-II channel counterflow – parameters and geometry

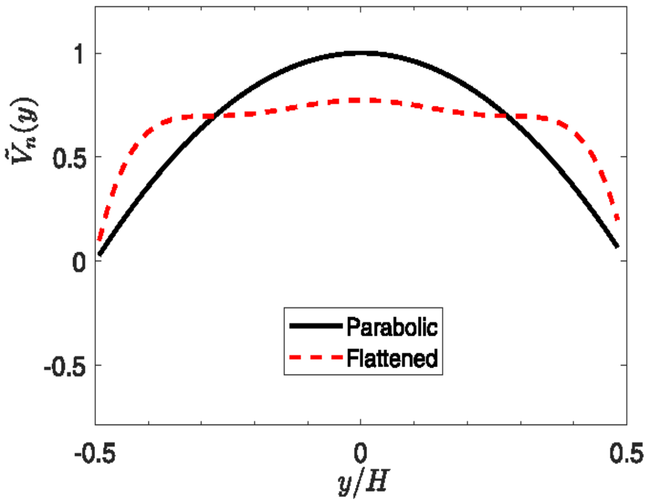
- Long planar channel of a square cross-section
- Imposed laminar normal fluid profile
- Flow in the channel is generated by a thermal gradient
- Zero net mass flux $\rho_n\langle V_n\rangle + \rho_s\langle V_s\rangle = 0$ condition leads to a counterflow – opposite movement of the normal fluid and superfluid
  At high *T* the superfluid is faster
  At low *T* the normal fluid is faster

- Fully non-local Vortex filament method for quantum vortex dynamics, line resolution $\Delta\xi = 0.001\text{cm}$

- Initial conditions: 8 rings, 4 at the walls, 4 in the bulk, $R_0 \ll H$

- Boundary conditions:
  open conditions in *x*-direction
  periodic conditions in *z*-direction
  slip conditions at the solid walls in y-direction

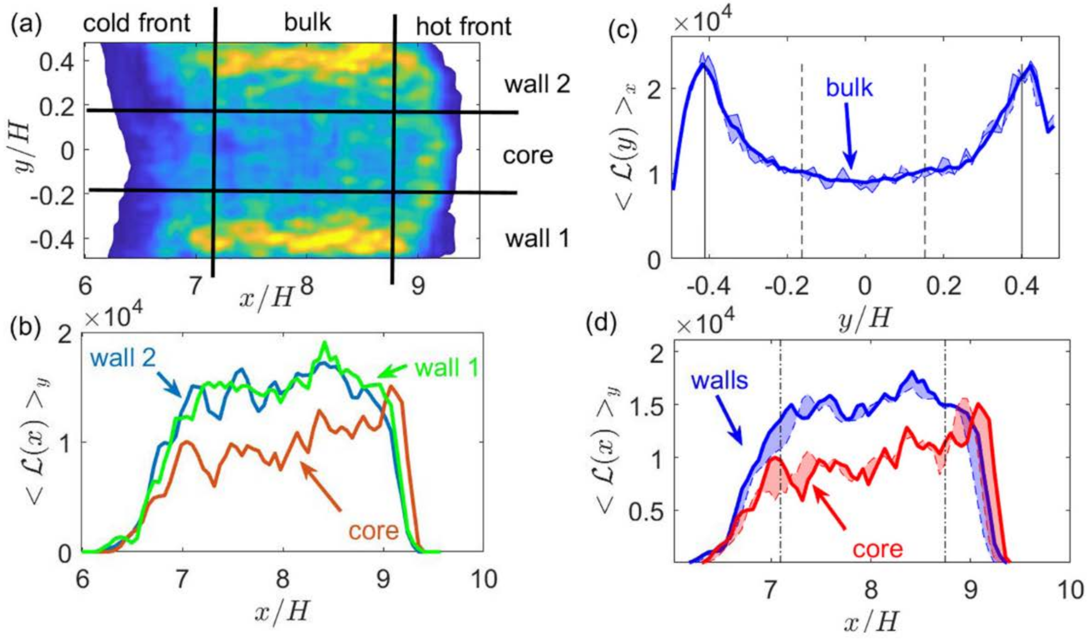


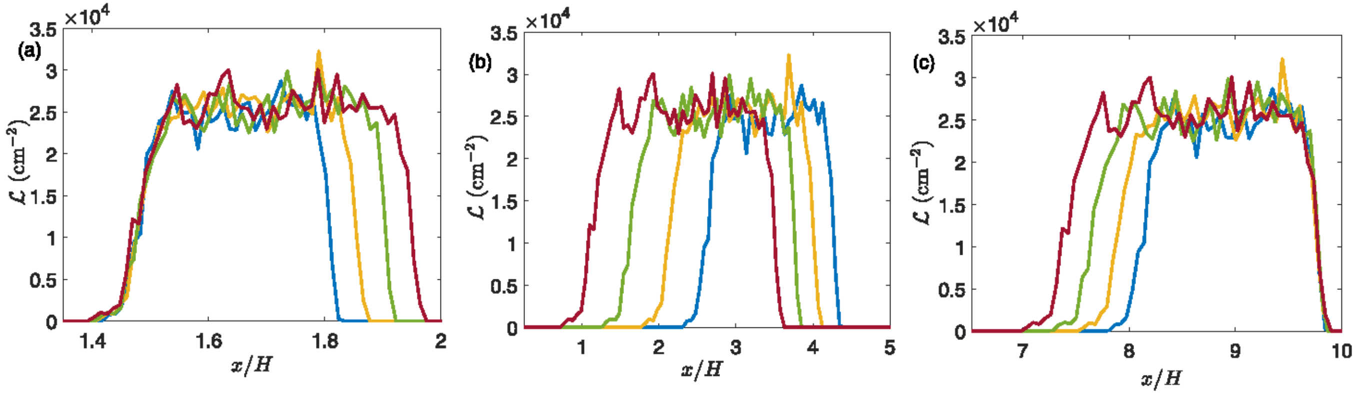


Back reaction of superfluid on the mean normal fluid profile – flattening of $V_n(y)$ profile

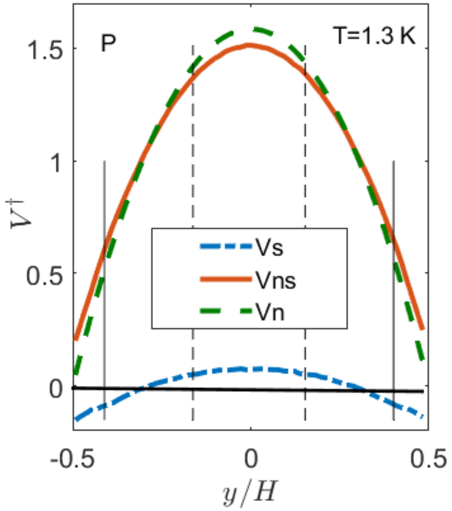
Vortex Tangle characterization

- 2D time-dependent maps of various properties are calculated on a course-grained grid $\Delta x = 0.011$ cm, $\Delta y = 0.0015$ cm by integrating over the volume Δx x Δy x H (H=0.1, 0.15, 0.2).
- The profiles are calculated by integrating the maps over the bulk (for wall-normal profiles) and over core or wall regions (for stream-wise profiles).
- Front shapes and speeds are obtained by collapse of the tangle edges of last 1 s of evolution.



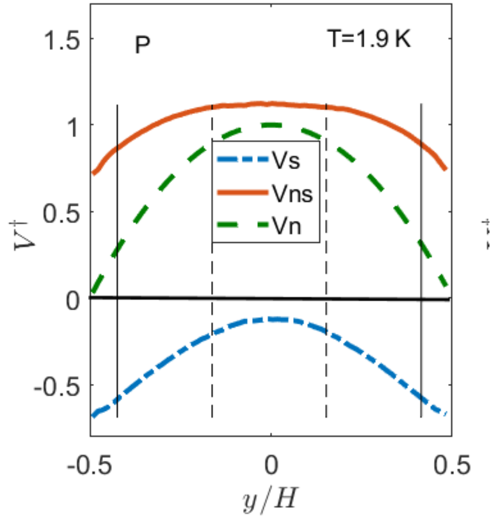


Vortex Line Density dynamics



At low T the tangle grows faster in the direction of V_n

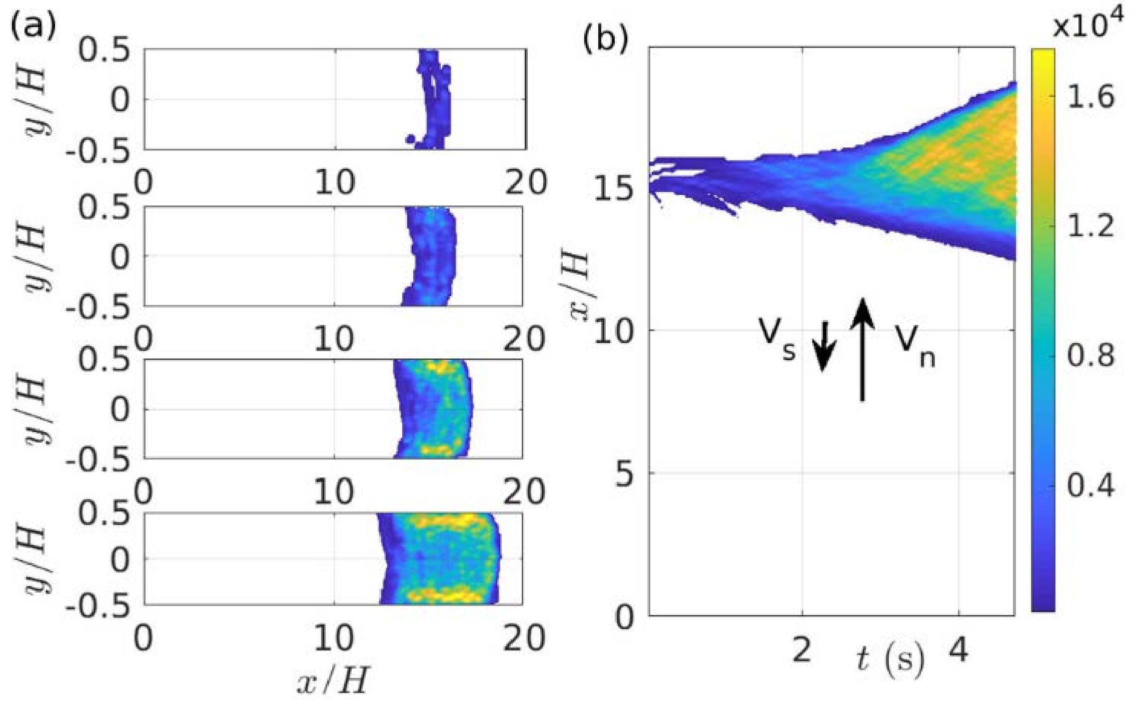
At high T the tangle grows faster in the direction of V_s

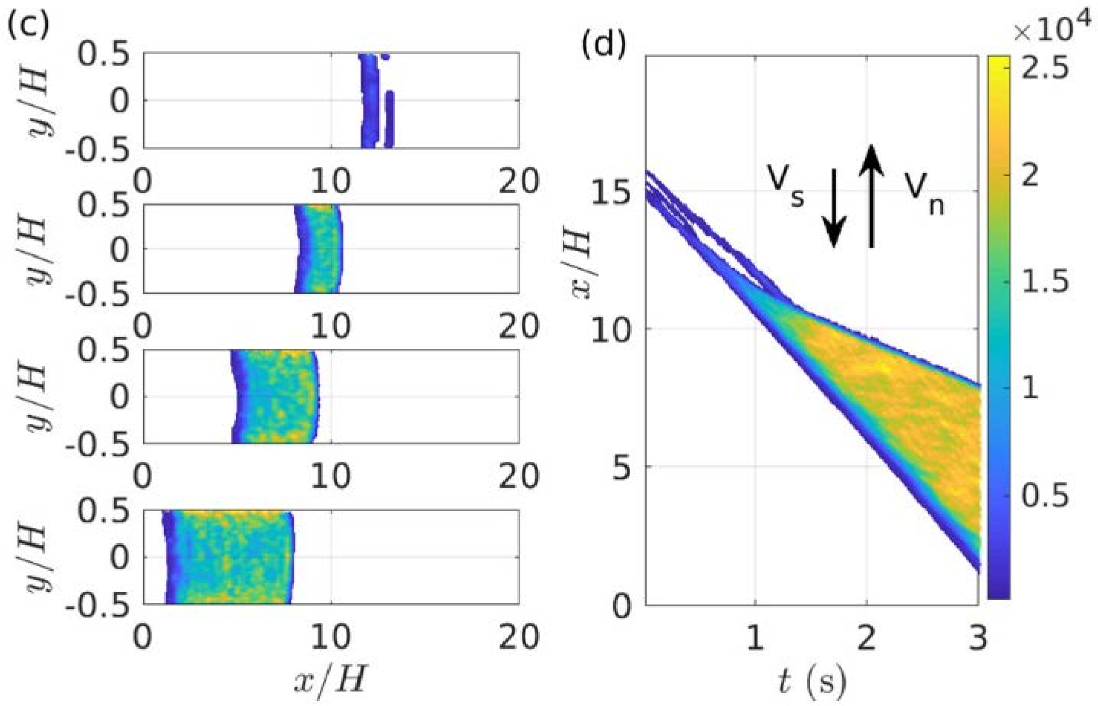


Which direction is upstream and which downstream?

The front moving along V_n - hot front

The front moving along V_s - cold front





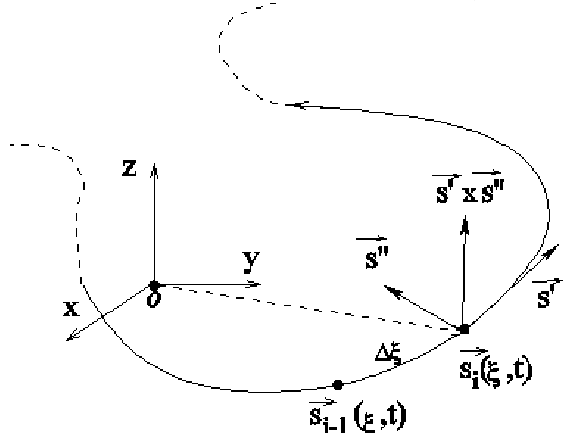
Advection-Diffusion-Reaction Dynamics for VLD

$$\frac{\partial L(\boldsymbol{r},t)}{\partial t} + \nabla[\mathcal{J}(\boldsymbol{r},t)] = D\,\nabla^2 L(\boldsymbol{r},t) + F[L(\boldsymbol{r},t)]$$

Vortex line density normalized by the bulk VLD

$$L(\boldsymbol{r},t) = \mathcal{L}(\boldsymbol{r},t)/\mathcal{L}_0$$

Vortex line is parameterized by a directional curve $\boldsymbol{s}(\xi,t)$



$\boldsymbol{s}'$ ~ local tangent

$\boldsymbol{s}''$ ~local curvature

K. W. Schwarz, Phys. Rev. B, 31, 5782 (1985) ,
Phys. Rev. B, 38, 2398 (1988)

Contributions to the line point velocity

Velocity due to intra-tangle interactions

$$\boldsymbol{V}_{\mathrm{BS}}(\boldsymbol{s},t) = \frac{\kappa}{4\pi}\int_{\Omega}\frac{\boldsymbol{s}-\boldsymbol{s_1}}{|\boldsymbol{s}-\boldsymbol{s_1}|^3}\times d\boldsymbol{s_1} = \boldsymbol{V}_{\mathrm{loc}} + \boldsymbol{V}_{\mathrm{nl}},$$

$$V_{\mathrm{loc}} = \beta(\boldsymbol{s}'\times\boldsymbol{s}''),\ \beta = \frac{\kappa}{4\pi}\ln\left(\frac{1}{a_0\,|s''|}\right)$$

Velocity due to “mutual friction”

$$V_{\mathrm{mf}} = (\alpha - \alpha'\boldsymbol{s}')\times\boldsymbol{s}'\times V_{\mathrm{ns}}$$

Counterflow velocity

$$V_{\mathrm{ns}}^0 = \langle V_{\mathrm{n}}\rangle_y(1+\rho_{\mathrm{n}}/\rho_{\mathrm{s}})$$

Advection-Diffusion-Reaction Dynamics for VLD

$$\frac{\partial L(\boldsymbol{r},t)}{\partial t} + \nabla[\mathcal{J}(\boldsymbol{r},t)] = D\,\nabla^2 L(\boldsymbol{r},t) + F[L(\boldsymbol{r},t)]$$

$$F = \mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3 - \mathcal{D}$$

$\boldsymbol{s}'$ ~ local tangent

$\boldsymbol{s}''$ ~local curvature

$$\mathcal{P}_1 = \frac{\alpha}{\mathcal{L}_0 V'} \int_{\Omega'} (\boldsymbol{V}^0_{\rm ns} - \boldsymbol{V}_{\rm nl}) \cdot (\boldsymbol{s}' \times \boldsymbol{s}'')\, d\xi,$$

$$\mathcal{P}_2 = \frac{1}{\mathcal{L}_0 V'} \int_{\Omega'} \boldsymbol{s}' \cdot \boldsymbol{V}'_{\rm nl}\, d\xi,$$

$$\mathcal{P}_3 = -\frac{\alpha'}{\mathcal{L}_0 V'} \int_{\Omega'} \boldsymbol{s}'' \cdot \boldsymbol{V}_{\rm ns}\, d\xi,$$

VLD production

$$\mathcal{D} = \frac{\alpha}{\mathcal{L}_0 V'} \int_{\Omega'} \boldsymbol{V}_{\rm loc} \cdot (\boldsymbol{s}' \times \boldsymbol{s}'') d\xi.$$

VLD decay

$$\mathcal{J} = \frac{1}{\mathcal{L}_0 V'} \int_{\Omega'} \boldsymbol{V}_{\rm drift}\, d\xi = \boldsymbol{V}^0_{\rm s} L + \frac{1}{\mathcal{L}_0 V'} \int_{\Omega'} (\boldsymbol{V}_{\rm BS} + \boldsymbol{V}_{\rm mf})\, d\xi \ldots$$

VLD Flux

Contributions to the line point velocity

Velocity due to intra-tangle interactions

$$\boldsymbol{V}_{\rm BS}(\boldsymbol{s},t) = \frac{\kappa}{4\pi} \int_{\Omega} \frac{\boldsymbol{s} - \boldsymbol{s_1}}{|\boldsymbol{s} - \boldsymbol{s_1}|^3} \times \boldsymbol{ds_1} = \boldsymbol{V}_{\rm loc} + \boldsymbol{V}_{\rm nl},$$

$$V_{\rm loc} = \beta(\boldsymbol{s}' \times \boldsymbol{s}''),\ \ \beta = \frac{\kappa}{4\pi}\ln\left(\frac{1}{a_0\, |s''|}\right)$$

Velocity due to "mutual friction"

$$V_{\rm mf} = (\alpha - \alpha'\boldsymbol{s}') \times \boldsymbol{s}' \times V_{\rm ns}$$

Counterflow velocity

$$V^0_{\rm ns} = \langle V_{\rm n} \rangle_y (1 + \rho_{\rm n}/\rho_{\rm s})$$

$$\frac{\partial L(\boldsymbol{r},t)}{\partial t} + \nabla[\mathcal{J}(\boldsymbol{r},t)] = D\,\nabla^2 L(\boldsymbol{r},t) + F[L(\boldsymbol{r},t)]$$

3D tangle structure, 2D interface

Separate core and wall regions ⇨ 1D equations

$$\frac{\partial L^j(x,t)}{\partial t} + \nabla[\mathcal{J}^j(x,t)] = D^j\,\nabla^2 L^j(x,t) + \tilde{F}^j\,[L(x,t)]$$

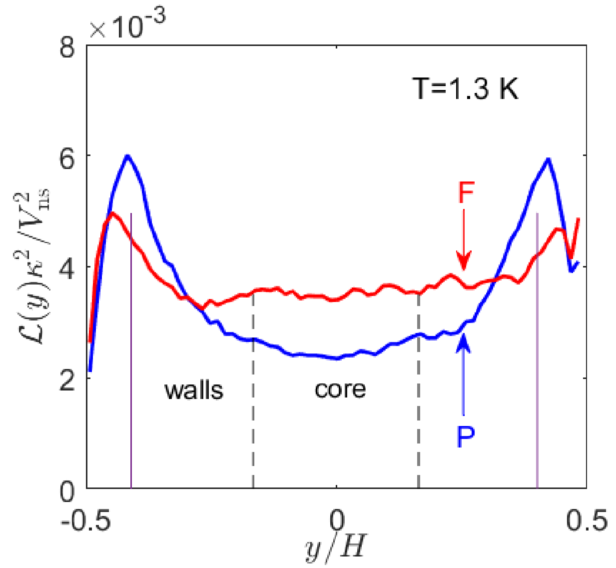
$$j = \text{core or wall}$$

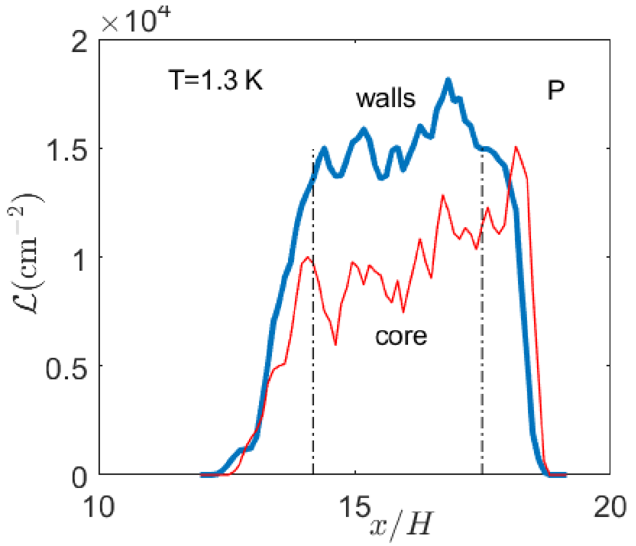
$\mathcal{J}_x{}^j(x,t) = V_s^x L^j(x,t)$ ⇦ Accounts for the flux along the tangle

$$\tilde{F}^j[L(x,t)] = F^j[L(x,t)] - \frac{d\mathcal{J}_y{}^j(x,t)}{dy}$$

⇧

Transverse VLD flux accounts
for the exchange between the core and the walls





the core and the wall regions
have different but well-defined properties

Closure for $\tilde{F}^j[L(x,t)]$

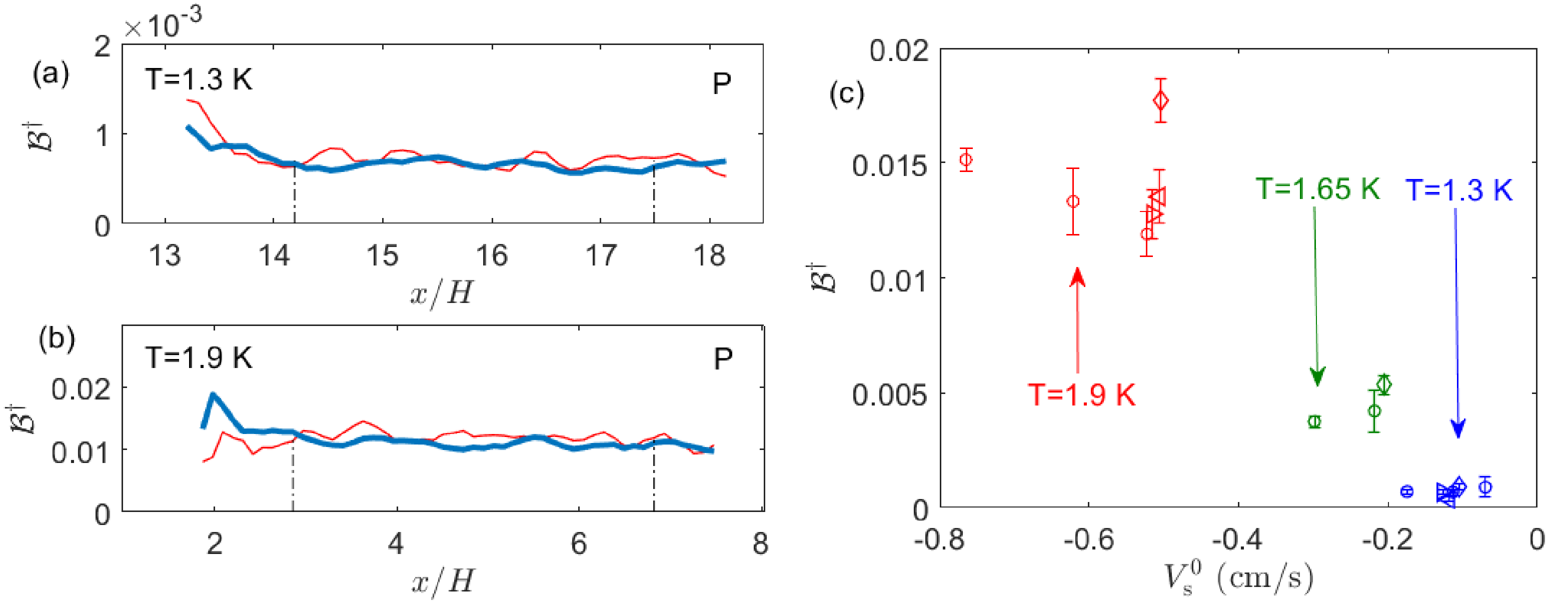
$$\tilde{F}[L(x,t)] = \mathcal{P}_1(x,t) + \mathcal{P}_2(x,t) + \mathcal{P}_3(x,t) - \frac{d\mathcal{J}_y{}^j(x,t)}{dy} - \mathcal{D}(x,t)$$

$$\mathcal{A}(x,t)\,L(x,t)$$

$$\mathcal{B}(x,t)L^2(x,t),$$
$$\mathcal{B}\,(x,t) = \alpha\,\beta\,c_2^2\,(x,t)\,\mathcal{L}_0$$

walls

core



- $\mathcal{B}$ is almost constant along the tangle
- deviate in the cold front range
- T-dependent quantity in the tangle bulk

$$\tau_{\mathrm{dec}} = \frac{1}{\mathcal{B}}$$ characteristic decay time

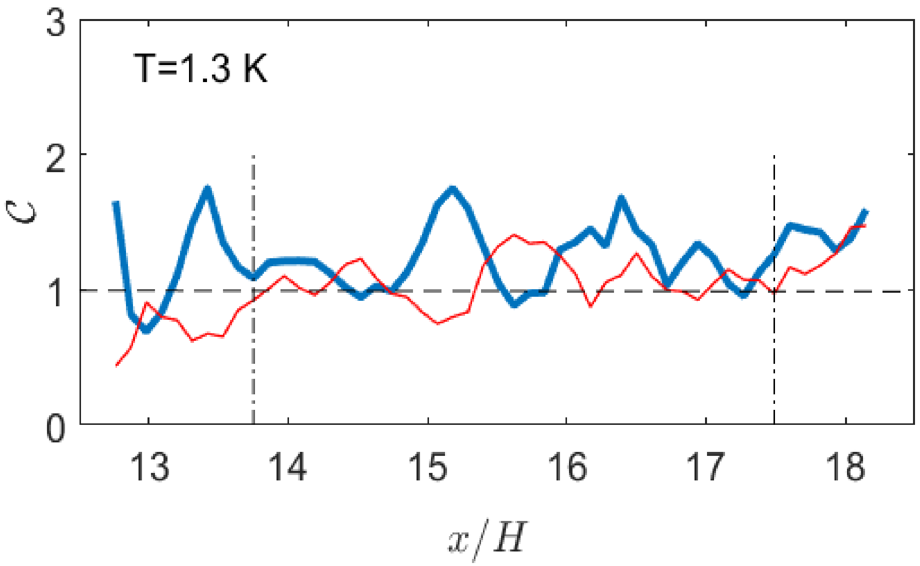
$$\tilde{F}[L(x,t)] = \mathcal{A}(x,t)L(x,t) - \mathcal{B}(x,t)L^2(x,t)$$

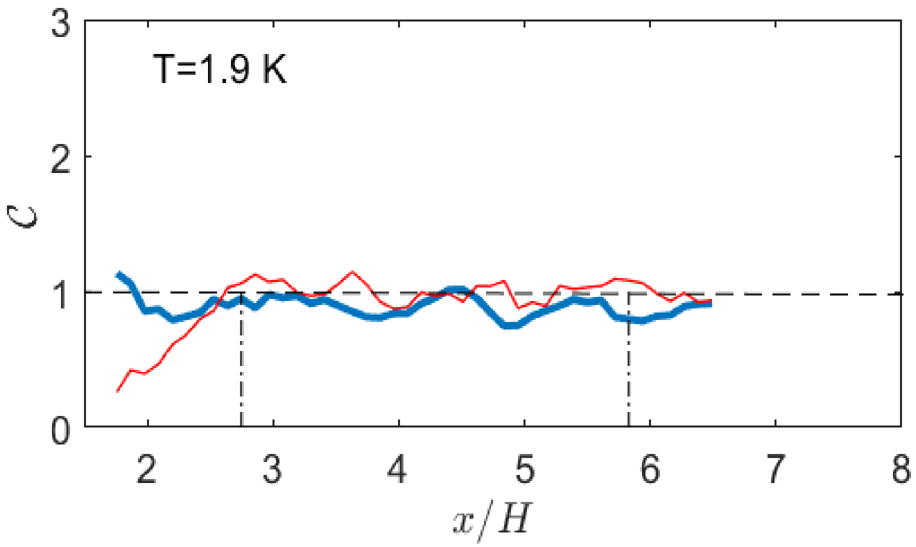
$$\tilde{F}[L] = \frac{L}{\tau}(\mathcal{C} - L), \qquad \mathcal{C} = \mathcal{A}/\mathcal{B}$$

$\tau_{\mathrm{dec}} = \frac{1}{\mathcal{B}}$ characteristic decay time

in the homogeneous tangle $\mathcal{C} = 1$

in the plug $\mathcal{C} \approx 1$





Useful review
M. Cencini, C. Lopez, D. Vergni , Lect. Notes Phys. 636 (2003)

Closed equation for each region separately (4 front regions)

$$\partial_t L^j(x,t) + V_s^x L^j(x,t) = D^j\, \partial_{x,x} L^j(x,t) + 1/\tau_{\text{dec}} L^j(x,t)[1 - Lj(x,t)\,]$$

To solve them analytically –

- change to inner variables $\tau = \frac{1}{\tau_{\text{dec}}}, l = \frac{x}{\sigma}, \sigma = \sqrt{D\tau_{\text{dec}}},\ w = V_s^x/V_{\text{diff}},\ V_{\text{diff}} = \sigma/\tau_{\text{dec}}$
- and switch to the co-moving reference frame (for each front region separately) $\zeta = c\,(l - V_f\,\tau),\quad V_f = v_f/V_{\text{diff}}$

$$[c\,v\,d_\zeta + c^2 d_{\zeta,\zeta}]L + L - L^2 = 0, \qquad v = V_f - w$$

Finally, the analytic solution in the original variables (with $\mathcal{C}$ as a constant parameter)

$$L^{j\,c}(x,t) = \frac{1}{4}\left[1 + \tanh\left(\frac{1}{\lambda^{j,c}}[x - v_f^c\, t]\right)\right]$$

$$L^{j\,h}(x,t) = \frac{1}{4}\left[1 - \tanh\left(\frac{1}{\lambda^{j,h}}[x - v_f^h t]\right)\right]$$

$\lambda = 2\sigma\sqrt{6/\mathcal{C}}$ The front width

$v_f^c = -5V_{\text{diff}}\sqrt{6\,\mathcal{C}} + V_s^x$ The front speed

$v_f^h = 5V_{\text{diff}}\sqrt{6\,\mathcal{C}} + V_s^x$

Finally, the analytic solution in the original variables (with $\mathcal{C}$ as a constant parameter)

$$L^{j\,\mathrm{c}}(x,t) = \frac{1}{4}\left[1 + \tanh\left(\frac{1}{\lambda^{j,\mathrm{c}}}\left[x - v_{\mathrm{f}}^{c}\, t\right]\right)\right]$$

$$L^{j\,\mathrm{h}}(x,t) = \frac{1}{4}\left[1 - \tanh\left(\frac{1}{\lambda^{j,\mathrm{h}}}\left[x - v_{\mathrm{f}}^{h} t\right]\right)\right]$$

$$\lambda = 2\sqrt{6D\tau_{\mathrm{dec}}/\mathcal{C}}$$ The front width

$$v_{\mathrm{f}}^{c} = -5\sqrt{6D\,\mathcal{C}/\tau_{\mathrm{dec}}} + V_s^x$$

$$v_{\mathrm{f}}^{h} = \quad 5\sqrt{6D\,\mathcal{C}/\tau_{\mathrm{dec}}} + V_s^x$$

The front speed

The front shapes and speeds are obtained by collapsing the corresponding fronts for several time snapshots, the front widths- by fitting the averaged front shapes.

$$D = \frac{\lambda^{\,2}}{24\tau_{\mathrm{dec}}} = \frac{\lambda^{\,2}\mathcal{B}}{24}$$

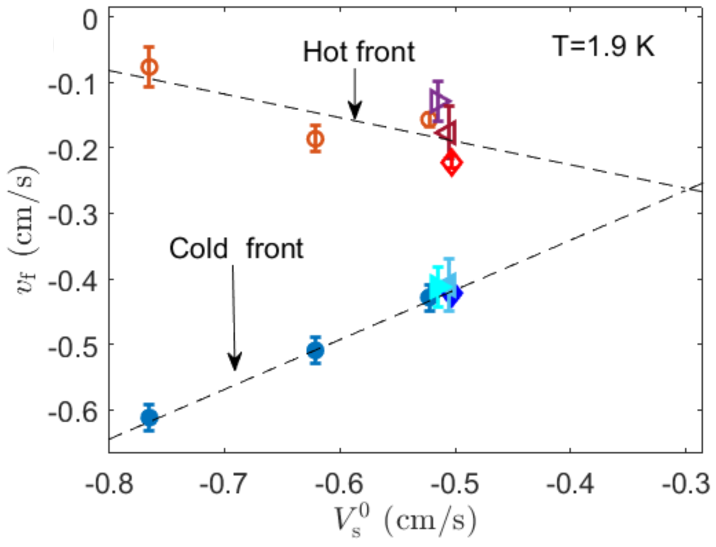
Effective diffusivity, characterizes the dynamic spread of the front

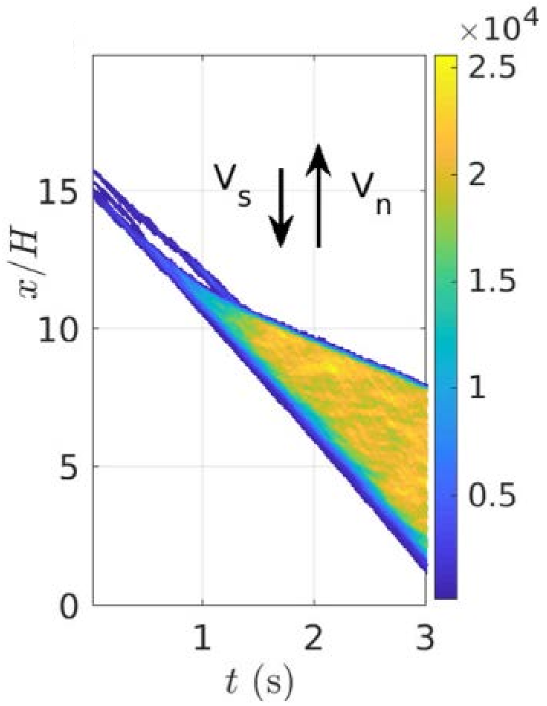
Calculated using $\mathcal{C} = 1 \pm 0.2$ and mean $\mathcal{B}$ over the bulk

Front shape and speed

At high T

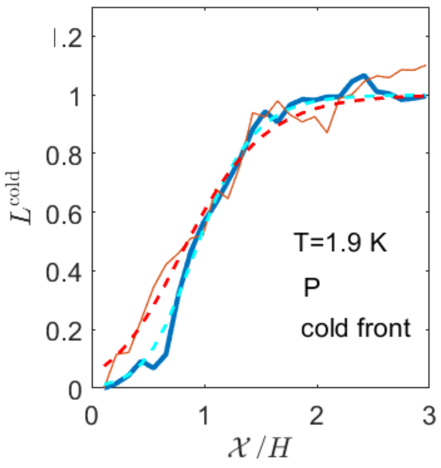
- Front speeds are linear in mean superfluid velocity for all conditions (including various H)
- The cold front is wider and faster
- The hot front is narrow and slow
- VLD peak in the core for the parabolic V_n
- Transverse flux play important role in the VLD dynamics





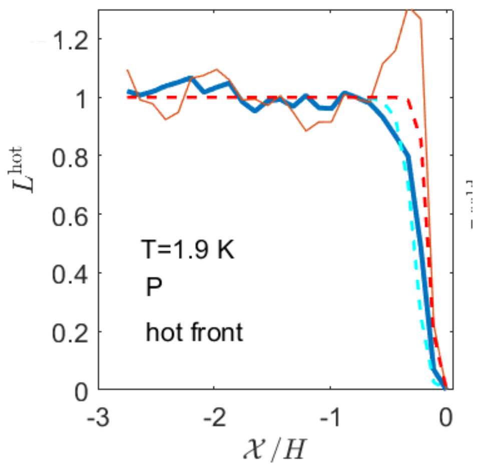
$D/\kappa = 1.3 \pm 0.4$

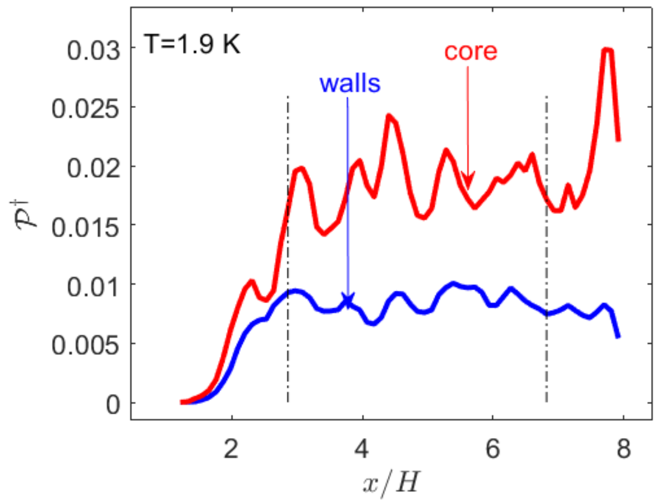
1.3 ± 0.3

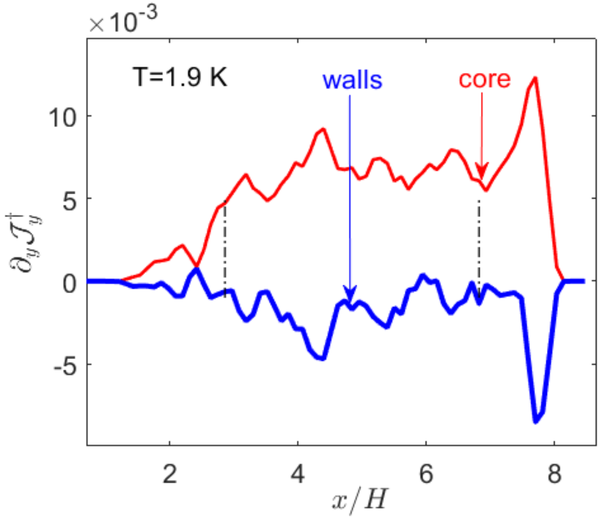


$D/\kappa = 0.04 \pm 0.4$

0.2 ± 0.05

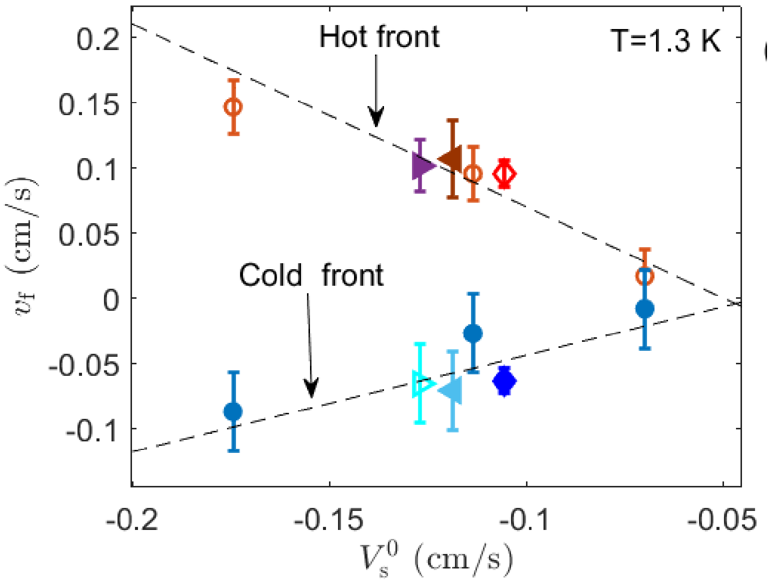


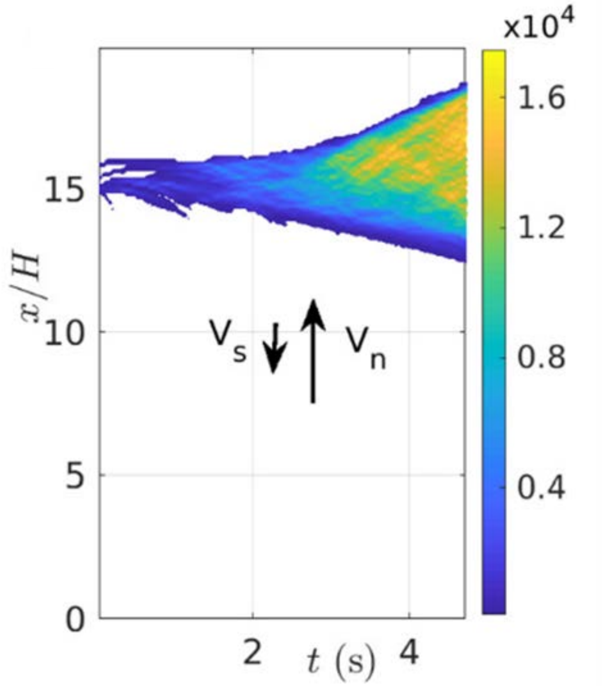




At low T

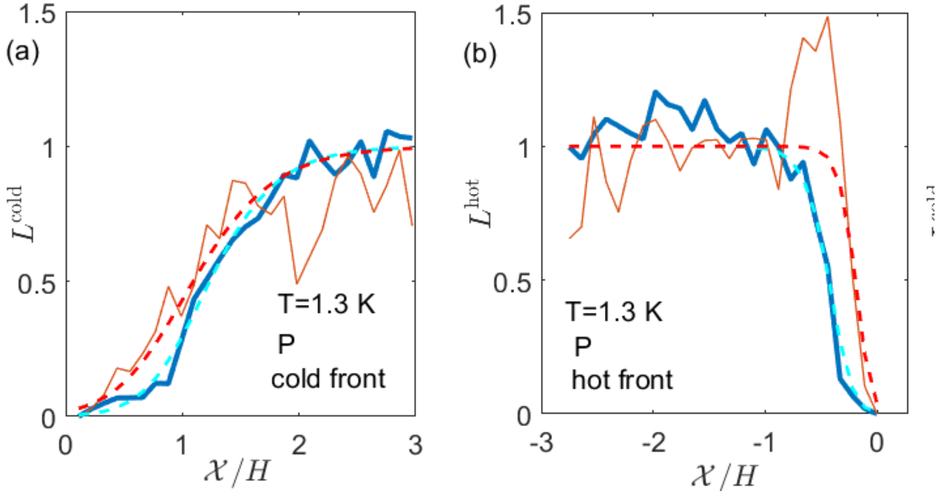
- Front speeds are linear in mean superfluid velocity for all conditions (including various H)
- The cold front is wider but slower
- The hot front is narrow and faster
- VLD peak in the core for the parabolic V_n
- No peak for flattened profile



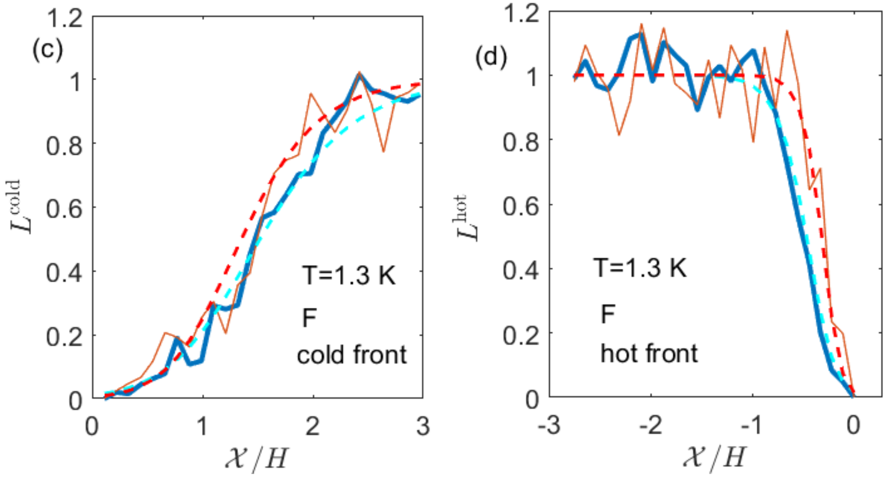


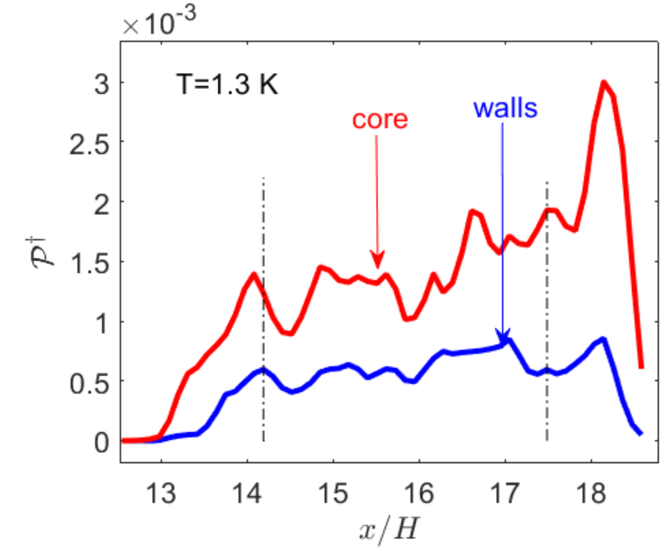
$D/\kappa = 0.5 \pm 0.2$
0.4 ± 0.1

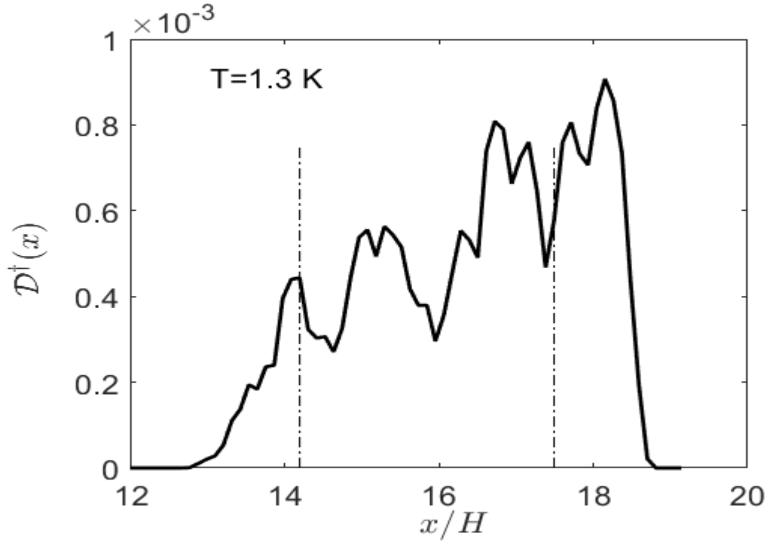
$D/\kappa = 0.01 \pm 0.4$
0.03 ± 0.01

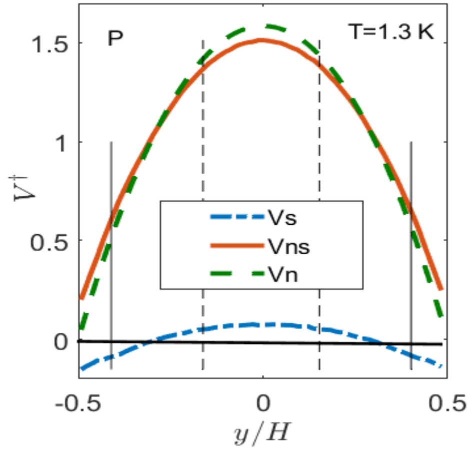


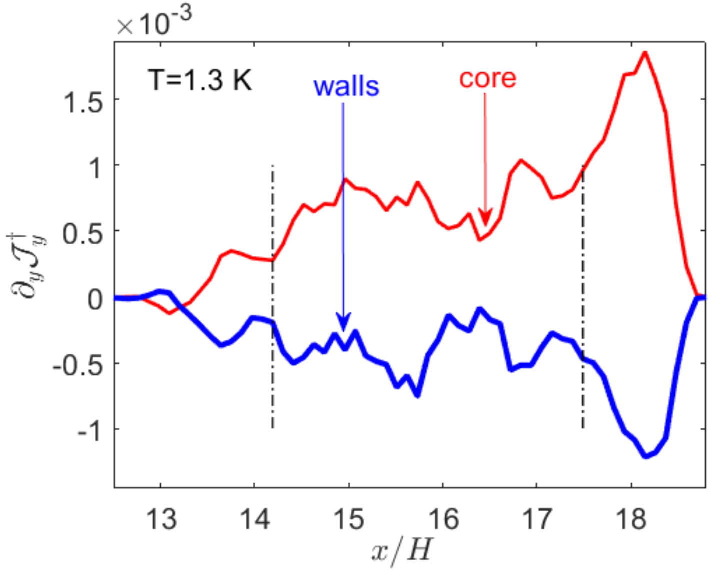
Flattened V_n

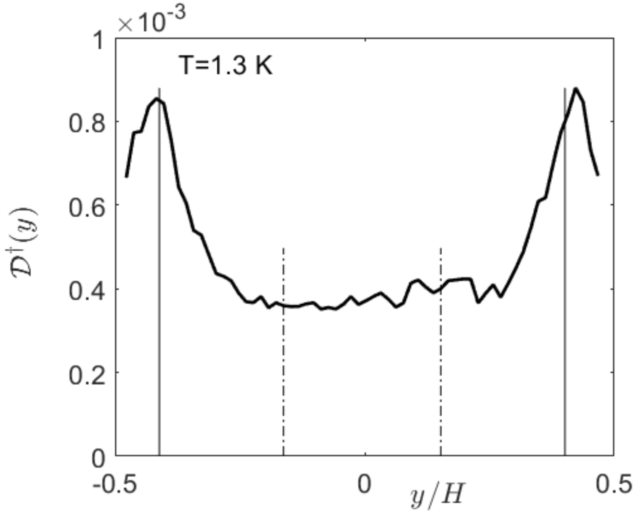


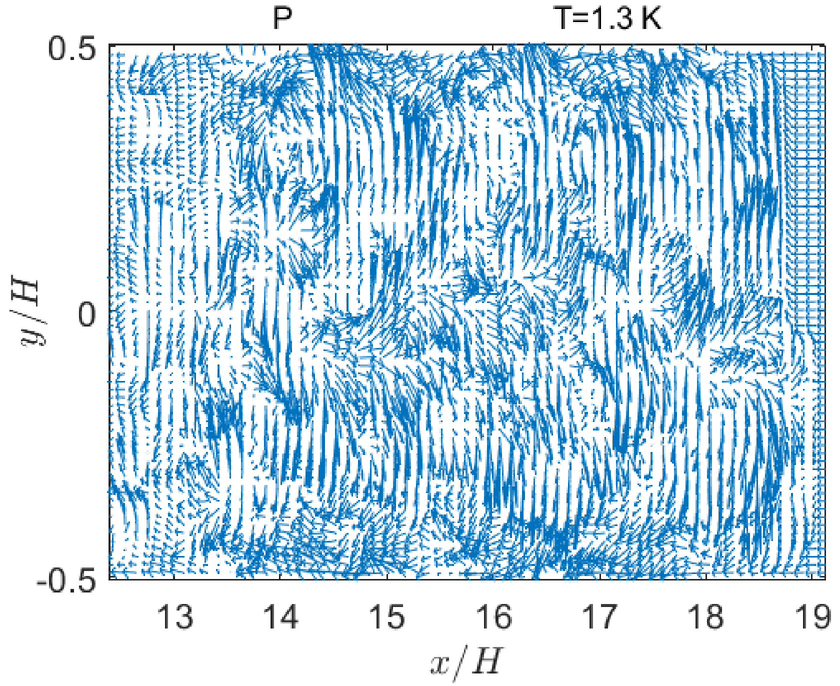

×10-3
T=1.3 K
core
walls
$\mathcal{P}^\dagger$
x/H


×10-3
T=1.3 K
$\mathcal{D}^\dagger(x)$
x/H


P
T=1.3 K
Vs
Vns
Vn
$V^\dagger$
y/H


×10-3
T=1.3 K
walls
core
$\partial_y \mathcal{J}_y^\dagger$
x/H

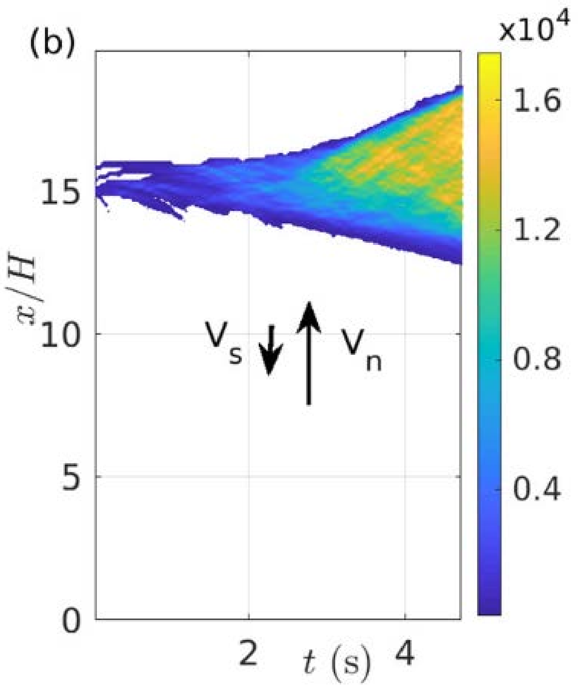

×10-3
T=1.3 K
$\mathcal{D}^\dagger(y)$
y/H


P
T=1.3 K
y/H
x/H

Are slugs and plugs similar?

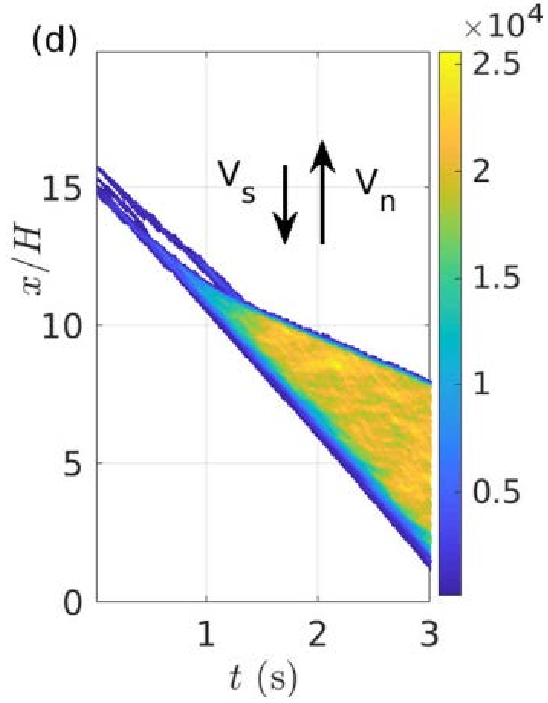
- The advection direction is defined by the mean superfluid velocity ✓
- The hot front is steep and has VLD peak = upstream front,
  Independent on the actual direction of propagation ✓
- The cold front is wide and always faster than
  the mean advection velocity= downstream front ✓
- At high *T*, the hot front is slower than the mean advection velocity ✓
- Temperature and applied heat flux (= V_n) decide the direction of the hot front ✗
- At low *T*, the hot front not only lags, it moves in the opposite direction ✗
- The speeds of both fronts are linear in mean advection velocity,
  for all conditions, various *H* and V_n profiles ✗
- Transverse VLD flux plays very important role in the VLD dynamics ✗
- The two fronts are driven by different types of nonlinearity ✗

+ many more …



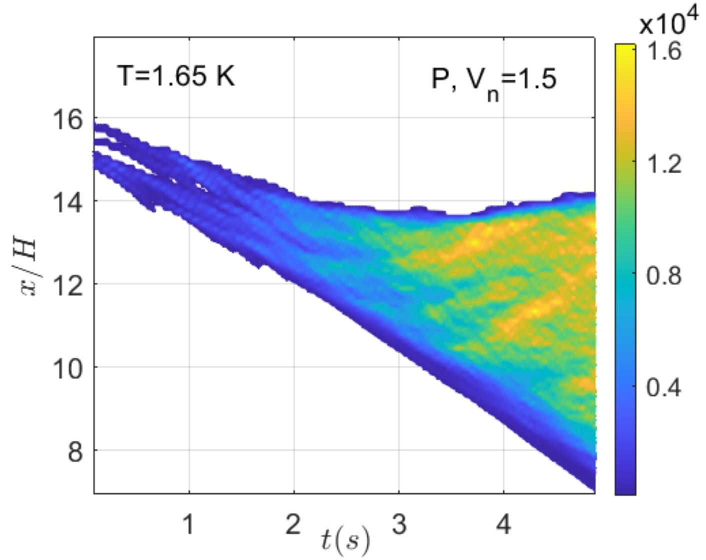
$T = 1.3\ K$

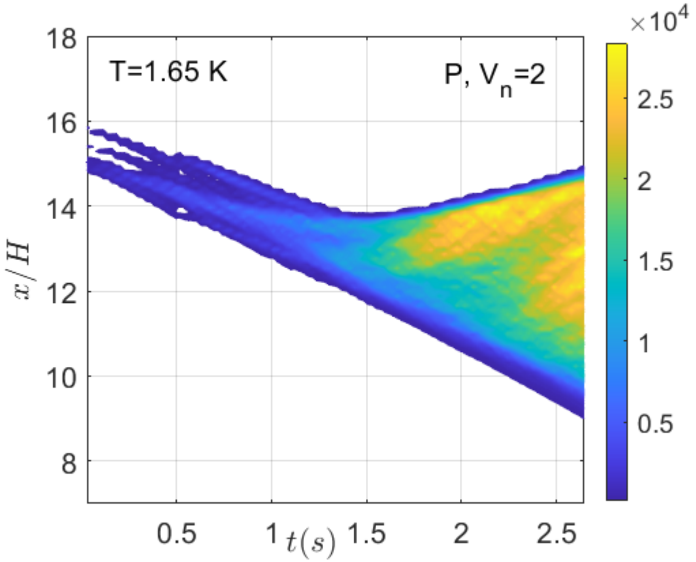
$T = 1.9\ K$

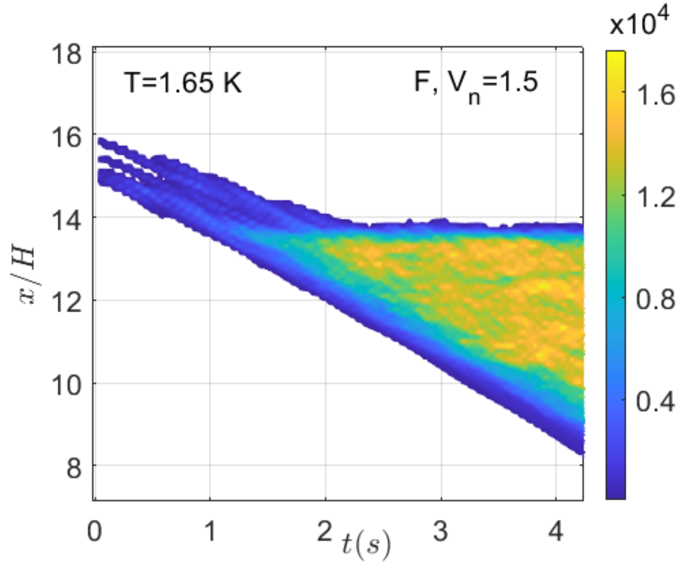


The cold front is settled first at all conditions
The hot front is formed only after a fully 3D tangle appears
Puff-like localized structures are early-time transient

$T = 1.65\ K$







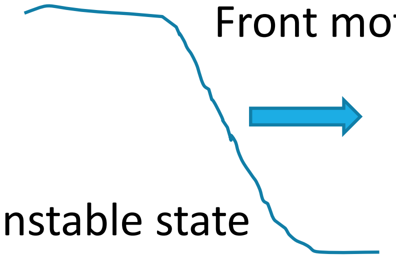
Advection-Reaction-Diffusion Dynamics

$$\frac{d\theta}{dt} + \nabla[u(r,t)\cdot\theta] = D\,\nabla^2\theta + F(\theta)$$

$\theta \in [0,1]$ Reaction concentration,

for laminar $u(r,t)$ ignore back reaction of θ on $u(r,t)$

θ=1 –stable state



Front motion

θ=0 –unstable state

Chemical reaction fronts in liquids
Population dynamics
Combustion

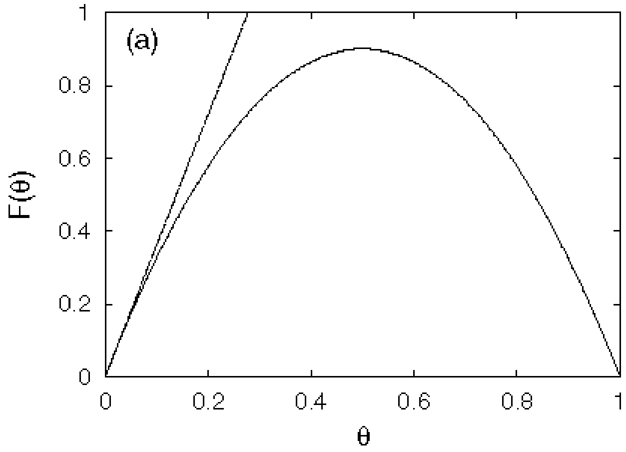
$F(\theta) = f(\theta)/\tau$ Reaction term, typically nonlinear

$F(0) = F(1) = 0$,
$F(\theta)$ >0 for $0 < \theta < 1$

$F(\theta)/\,\theta$ is the measure of the growth rate

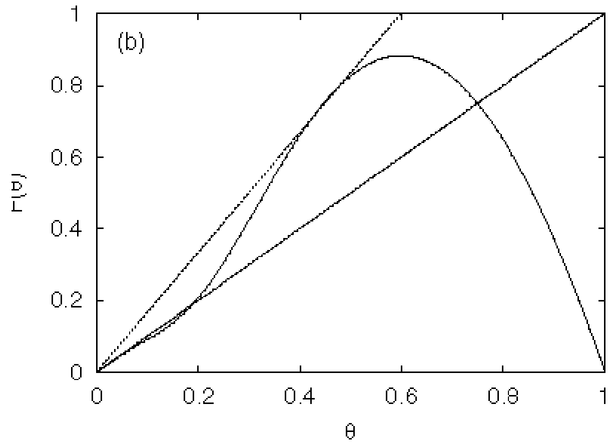
M. Cencini, C. Lopez, D. Vergni ,
The Kolmogorov Legacy in Physics, 636 (2003)

$\sup_\theta[F(\theta)/\,\theta]$ at θ=0



Pulled front

$\sup_\theta[F(\theta)/\,\theta]$ at θ>0



Pushed front

Minimum front speed is bounded

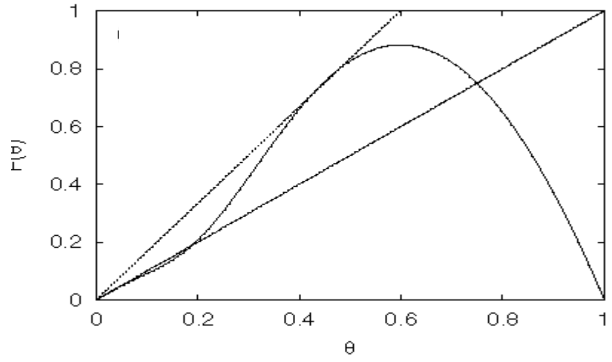
$$2\sqrt{D\,F'(0)} \le v_{\min} \le \sqrt{D\,\sup_\theta[F(\theta)/\,\theta]}$$

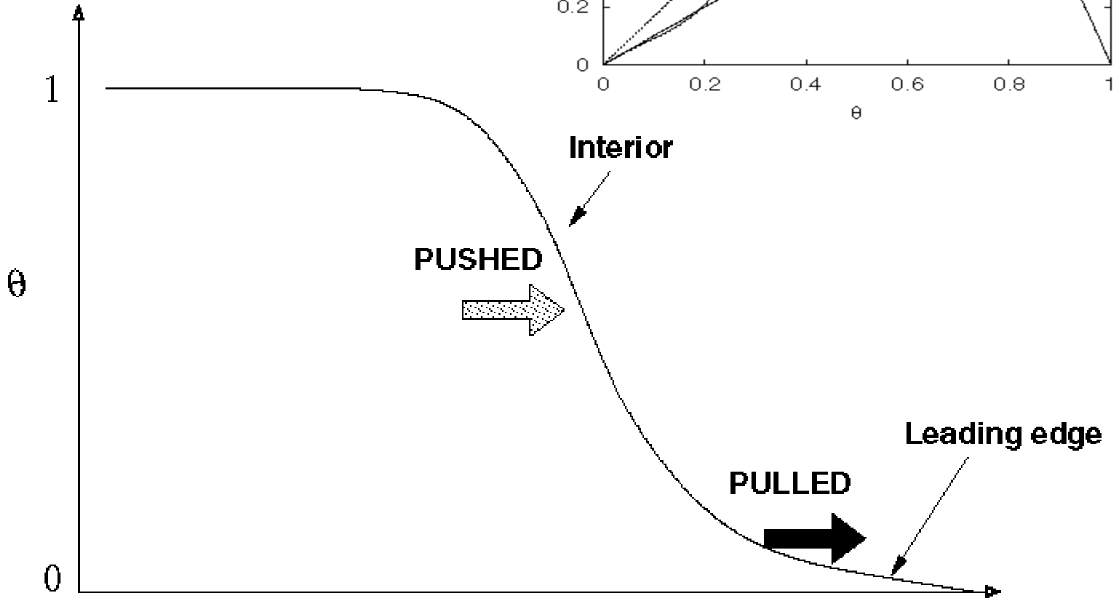
R.A. Fischer, Proc. Annu. Symp. Eugen. Soc. 7, 355 (1937).

A.N. Kolmogorov, I. Petrovskii and N. Piskunov,
Bull. Univ. Moscow, Ser. Int. A, 1, 1 (1937).

Y.B. Zeldovich and D.A. Frank-Kamenetskii,
Acta Physicochimica U.R.S.S., Vol. XVII, 1-2 42 (1938).

$\sup_\theta[F(\theta)/\,\theta]$ at $\theta>0$

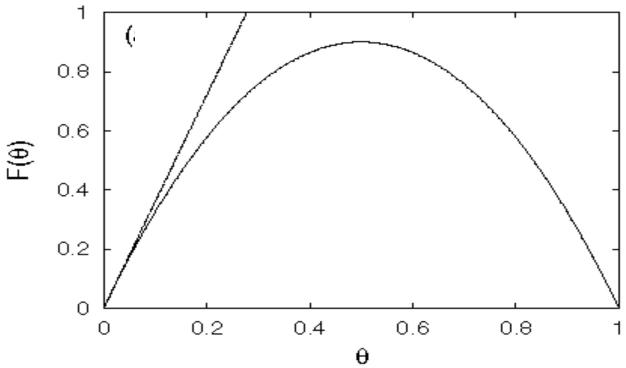


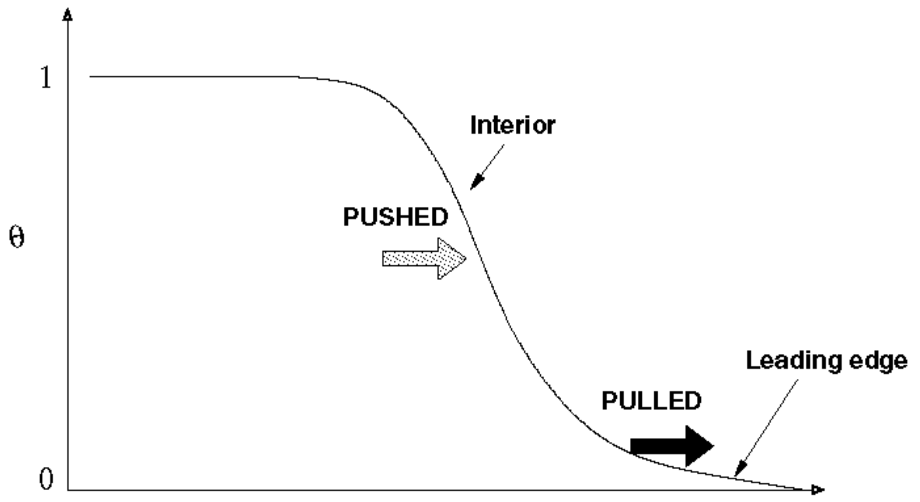


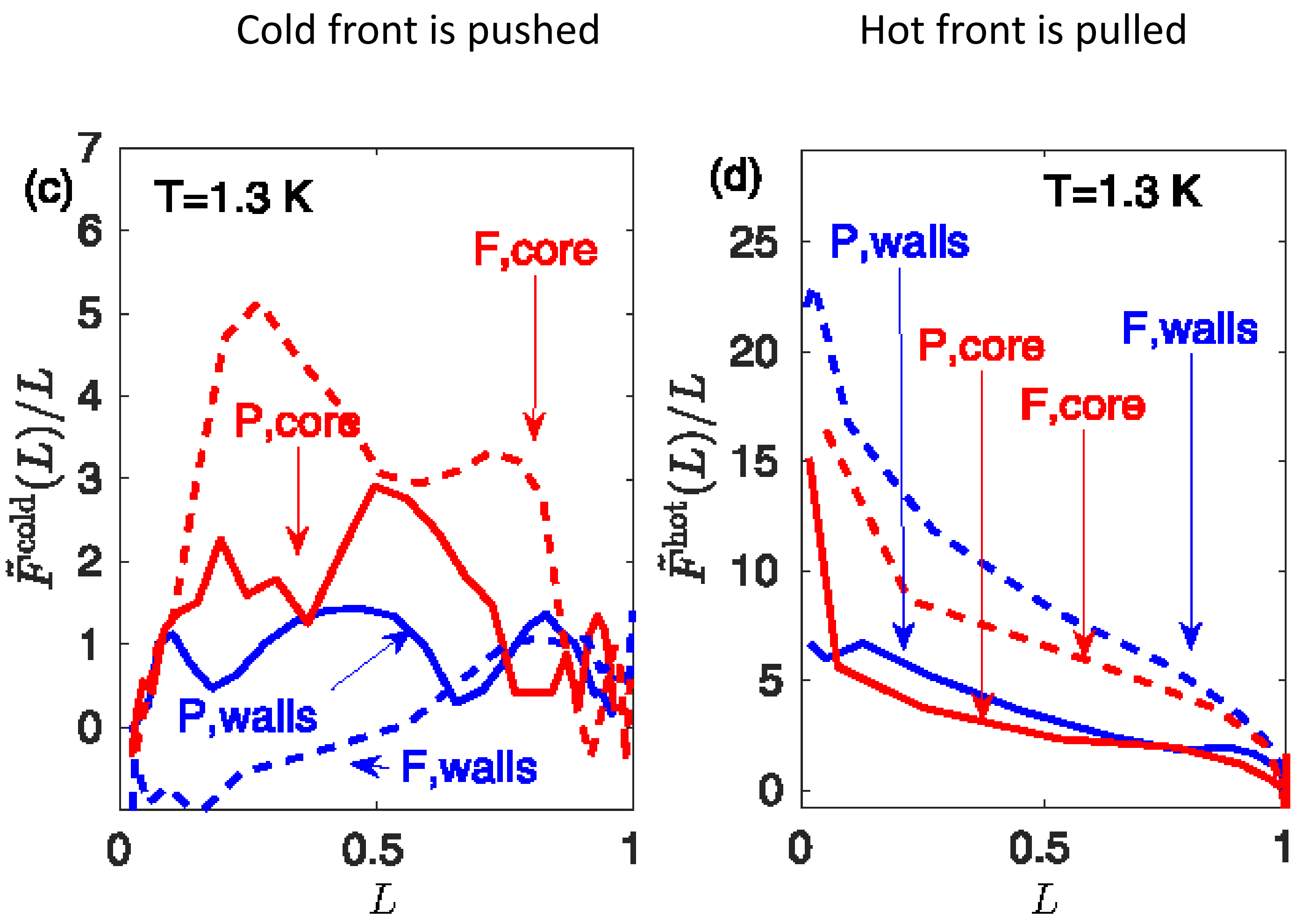
Fischer-Kolmogorov-Petrovskii-Peskunov (FKPP)
nonlinearity $F(\theta)=\theta(1-\theta)$ or any convex function $F''(\theta)<0$

$\sup_\theta[F(\theta)/\,\theta]$ at $\theta=0$

$v_{\min}=2\sqrt{D\,F'(0)}$







$\sup_\theta[F(\theta)/\theta]$ at θ>0

$\sup_\theta[F(\theta)/\theta]$ at θ=0