Are 2D classical and quantum turbulence equivalent? Insights from velocity circulation statistics

arXiv:2306.17735 (2023)

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Bridging classical and quantum turbulence (3-15 July 2023)

PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

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Phenomenology of 2D turbulence

Absence of vortex stretching

$$\partial_t \boldsymbol{\omega} + (\mathbf{u}\cdot\nabla)\boldsymbol{\omega} = \cancel{(\boldsymbol{\omega}\cdot\nabla)\mathbf{u}} + \nu\nabla^2\boldsymbol{\omega}$$

Two conserved quadratic quantities:
Energy and enstrophy

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -2\nu\Omega = -\epsilon \qquad \frac{\mathrm{d}\Omega}{\mathrm{d}t} = -2\nu P = -\beta$$

$E = \langle u^2 \rangle / 2$ Energy

$\Omega = \langle \omega^2 \rangle / 2$ Enstrophy

$P = \langle |\nabla \times \boldsymbol{\omega}|^2 \rangle / 2$ Palinstrophy

Inverse energy cascade

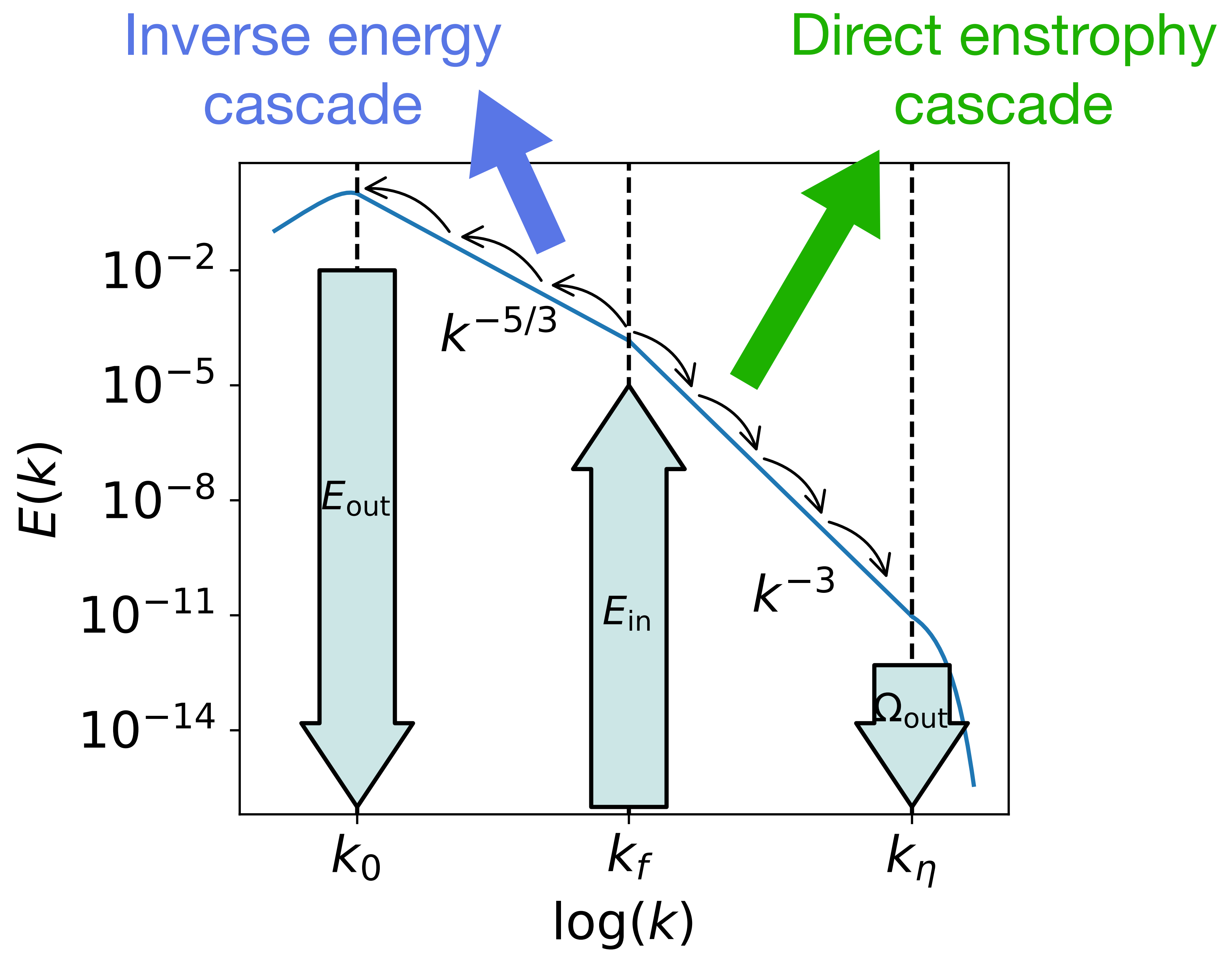
$$E(k) = C_I \epsilon^{2/3} k^{-5/3}$$

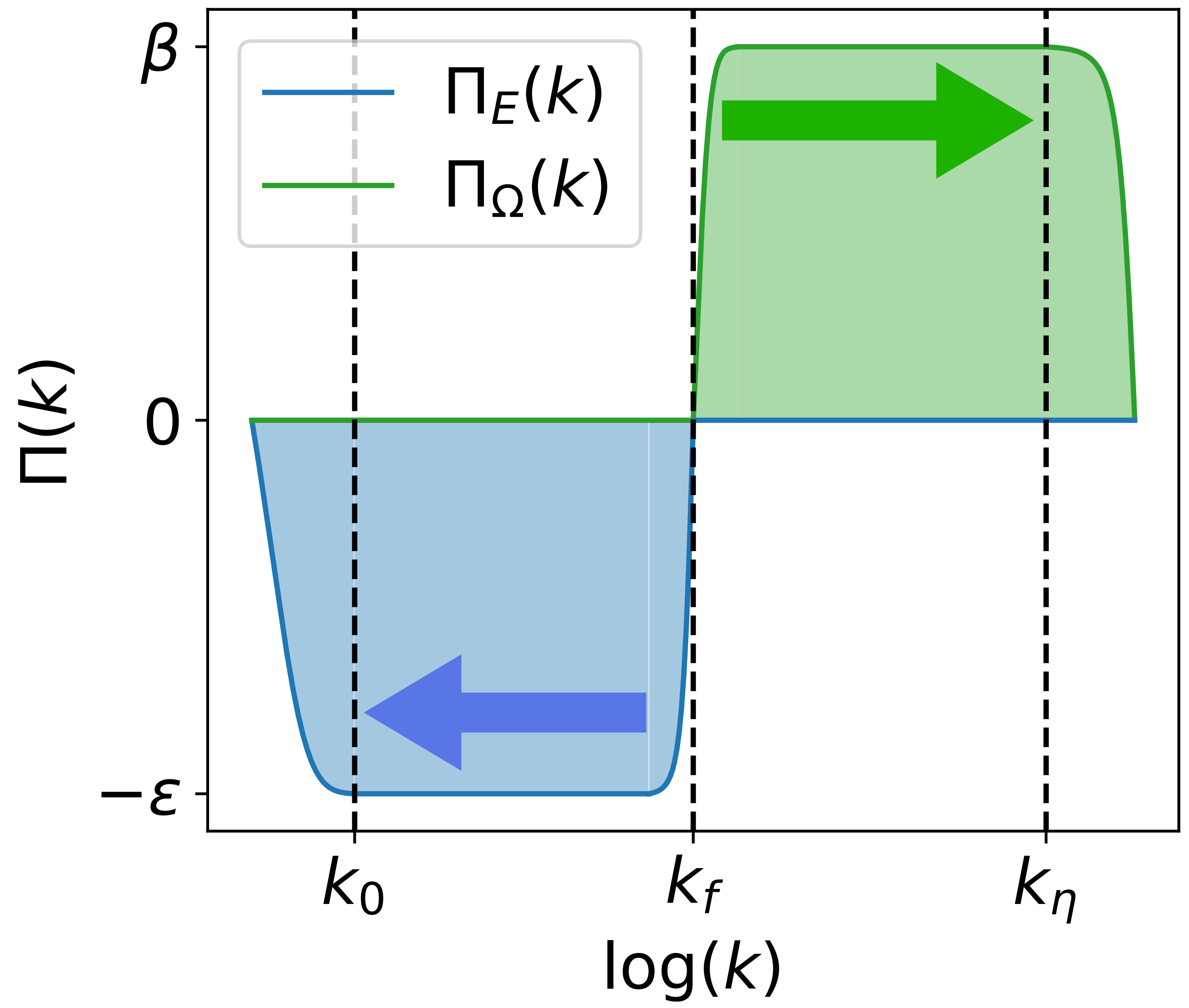
for $k_0 \ll k \ll k_f$

Direct enstrophy cascade

$$E(k) = C_D \beta^{2/3} k^{-3}$$

for $k_f \ll k \ll k_\eta$

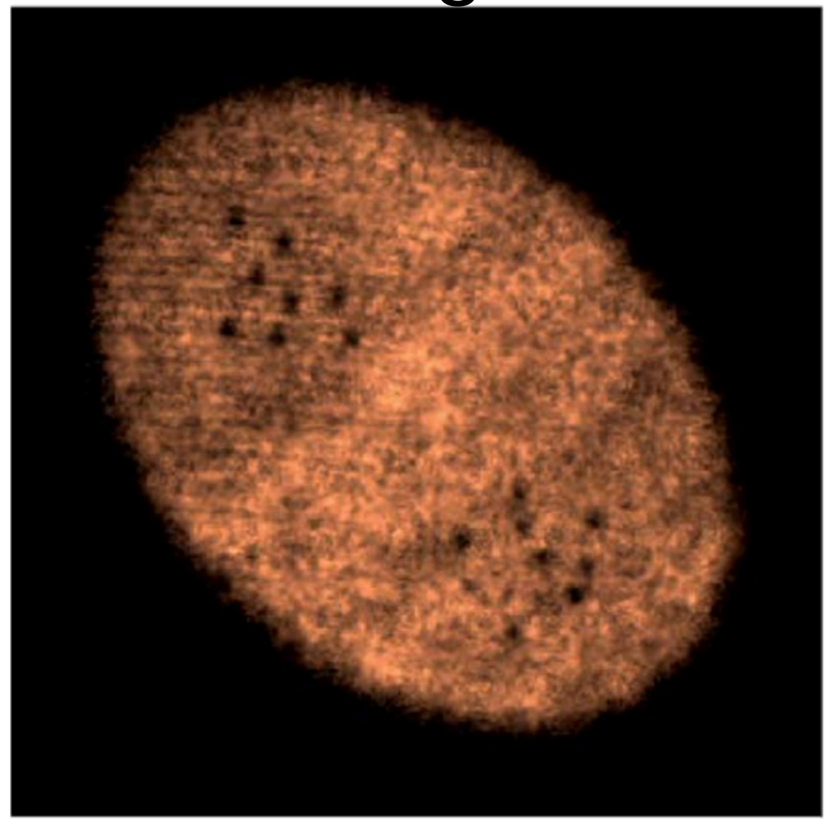




2D quantum turbulence

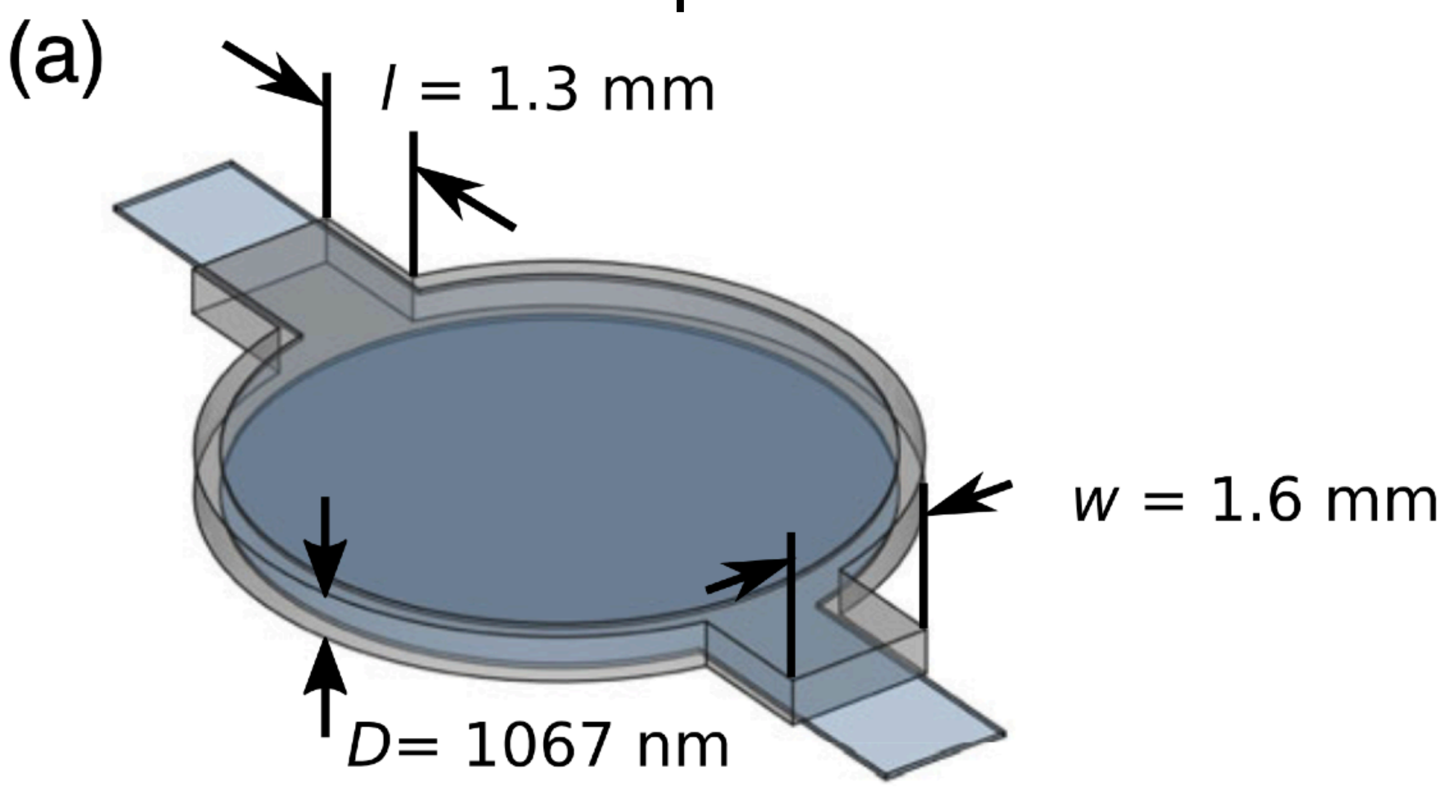
Experiments

Vortex clustering in a 2D BECs



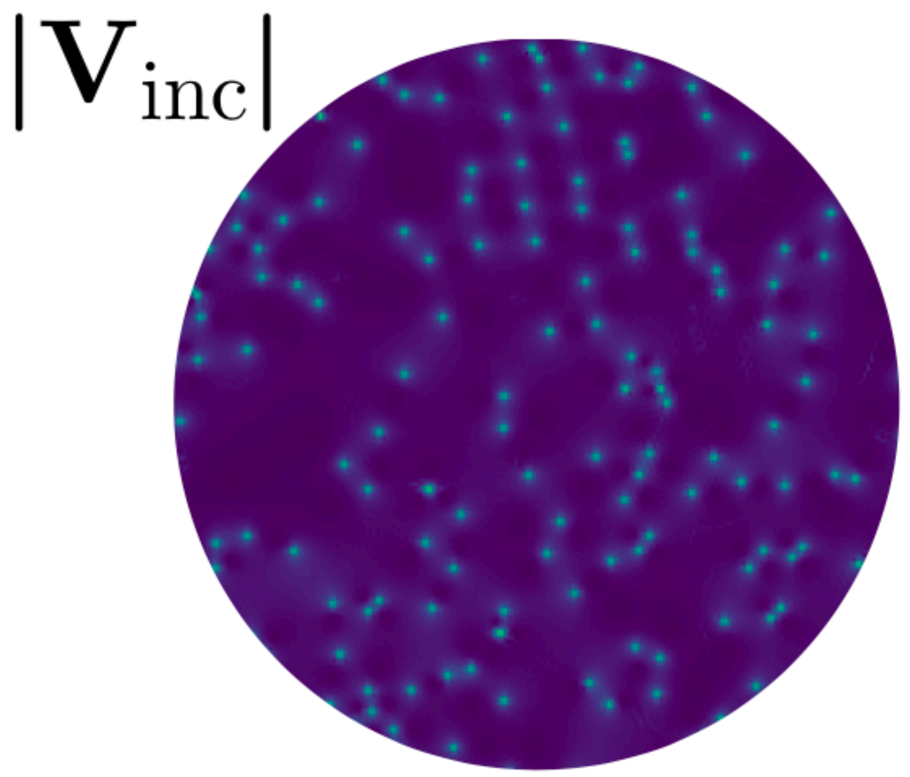
Gauthier et al. - Science (2019)

Confined superfluid ^{4}He



Varga et al. - PRL (2020)

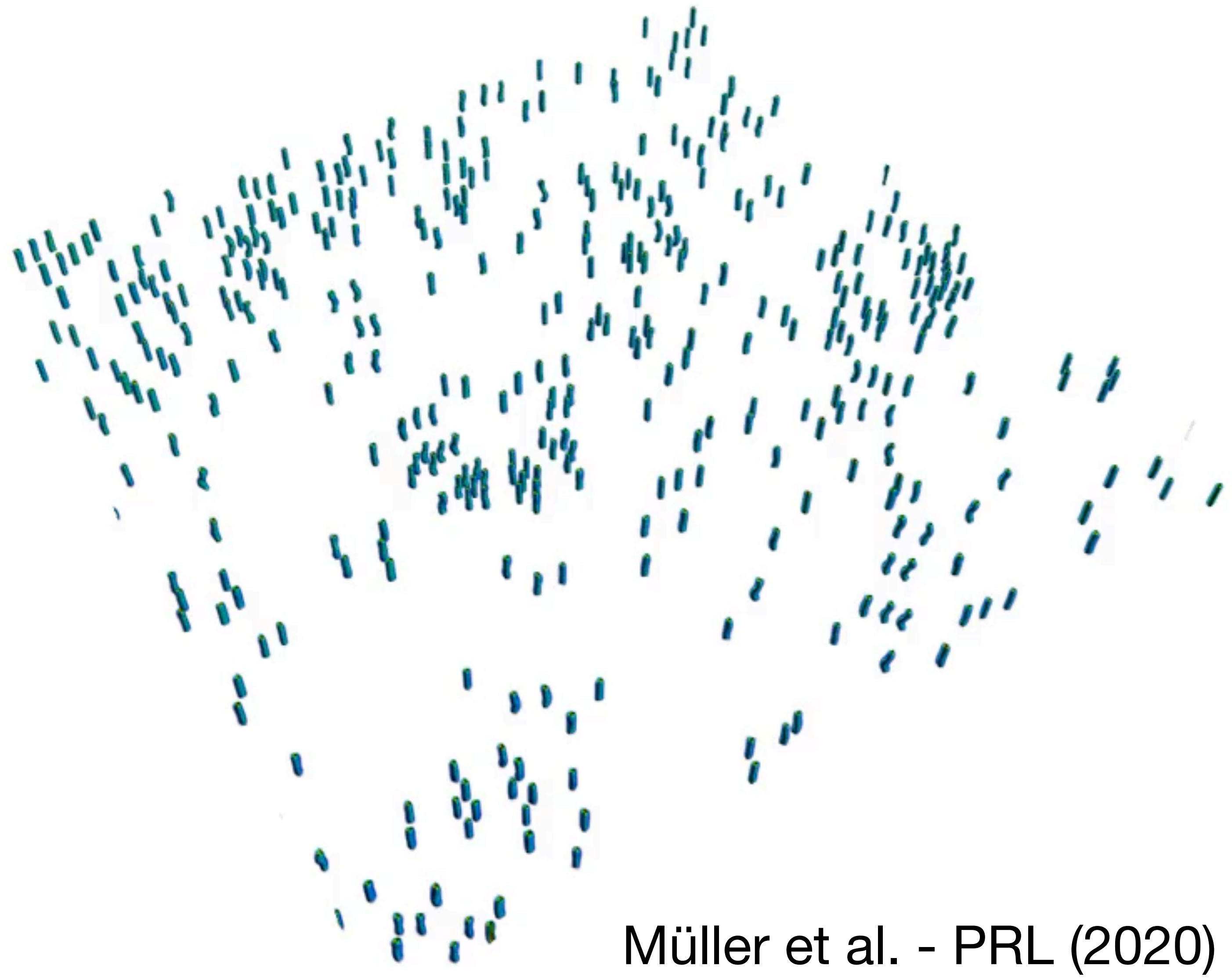
Exciton-polariton



Panico et al. - Nature Photonics (2023)

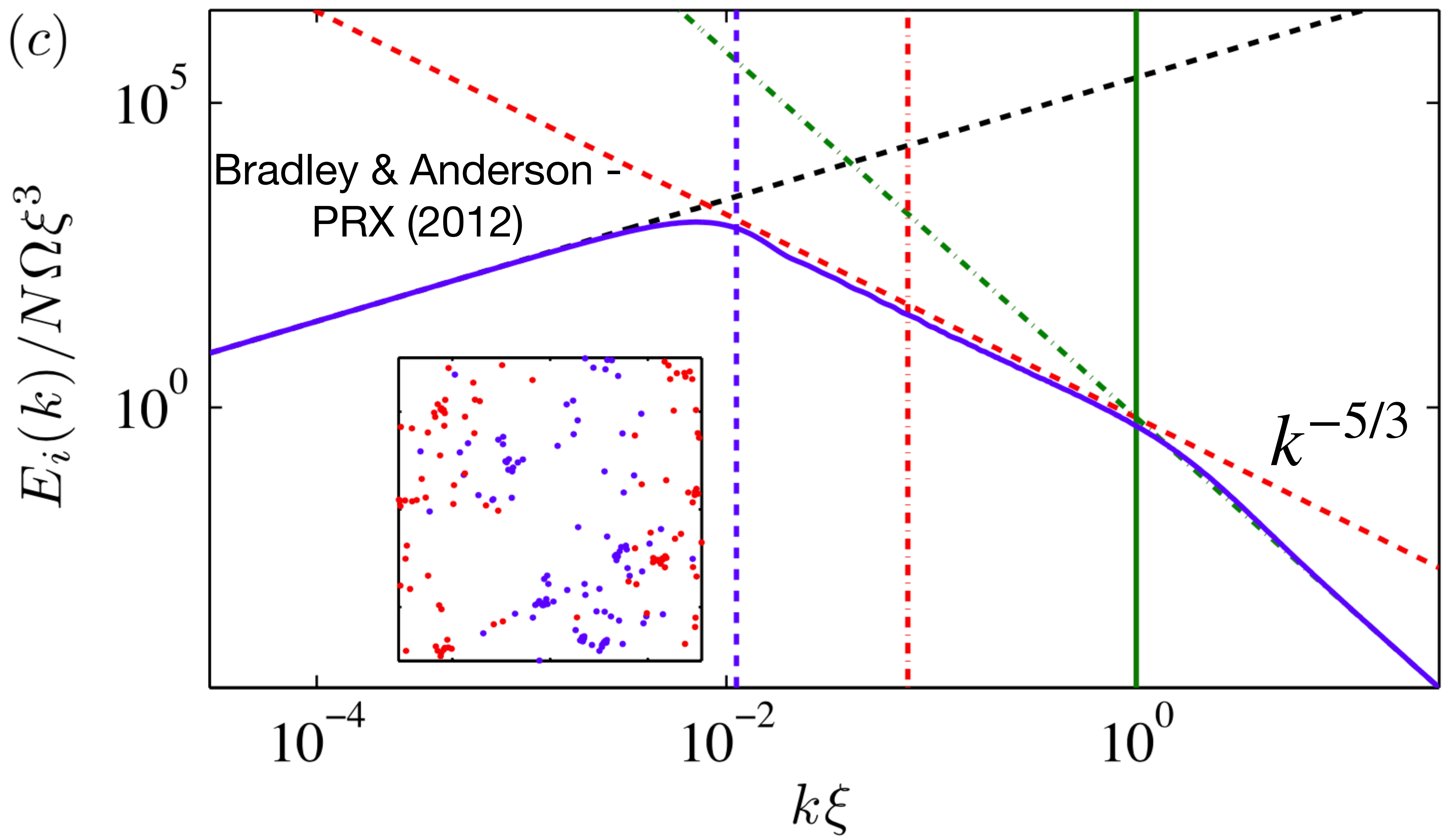
Numerical simulations

Quantum turbulence in the 3D-2D transition



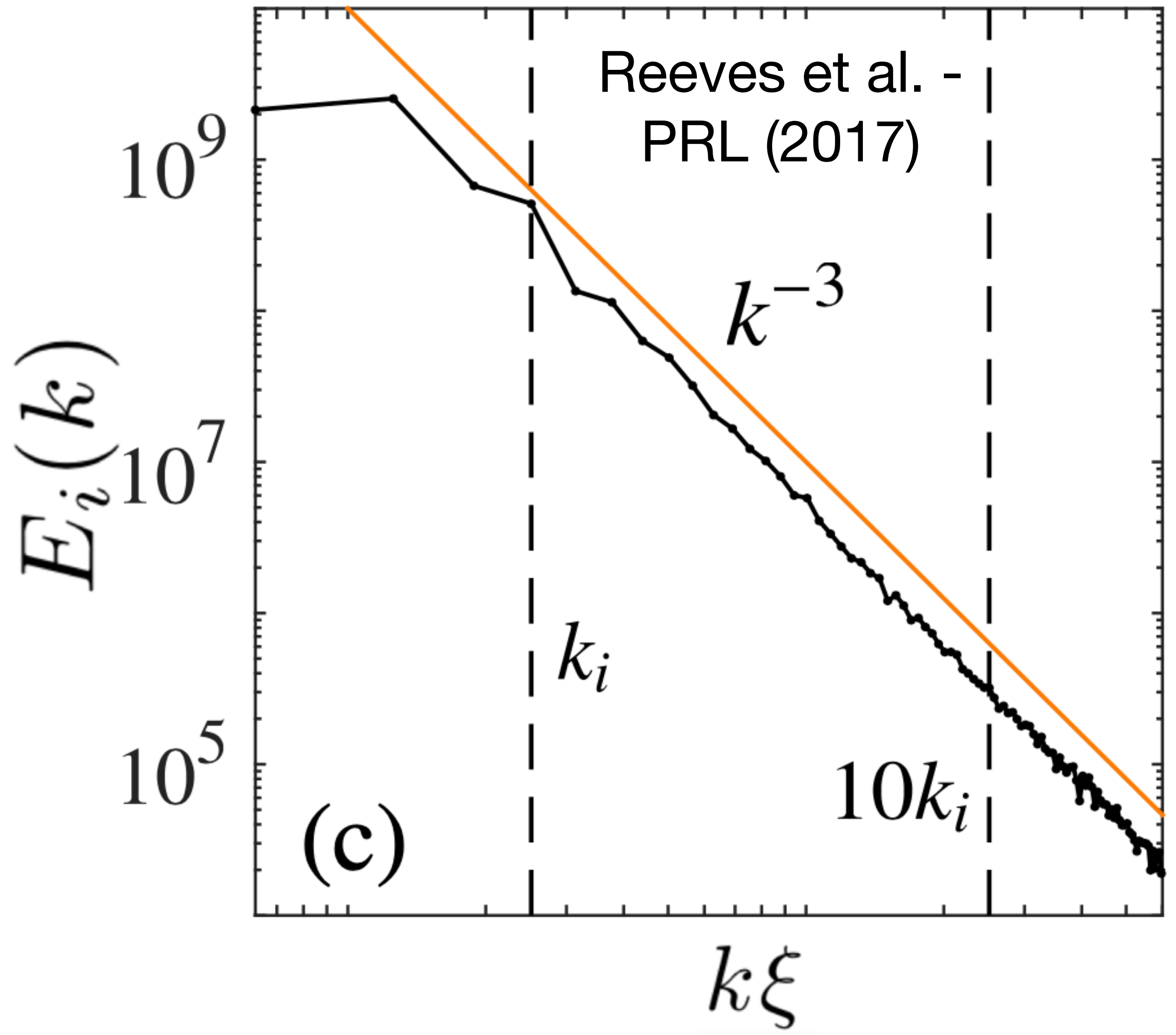
Müller et al. - PRL (2020)

Inverse cascade



Bradley & Anderson - PRX (2012)

Direct cascade



Reeves et al. - PRL (2017)

Lack of intermittency in 2D turbulence

In 3D turbulent flows, there are strong velocity fluctuations that affect high-order statistics and produce the break down of self-similarity.

Self-similar Kolmogorov 1941 theory for 3D turbulence

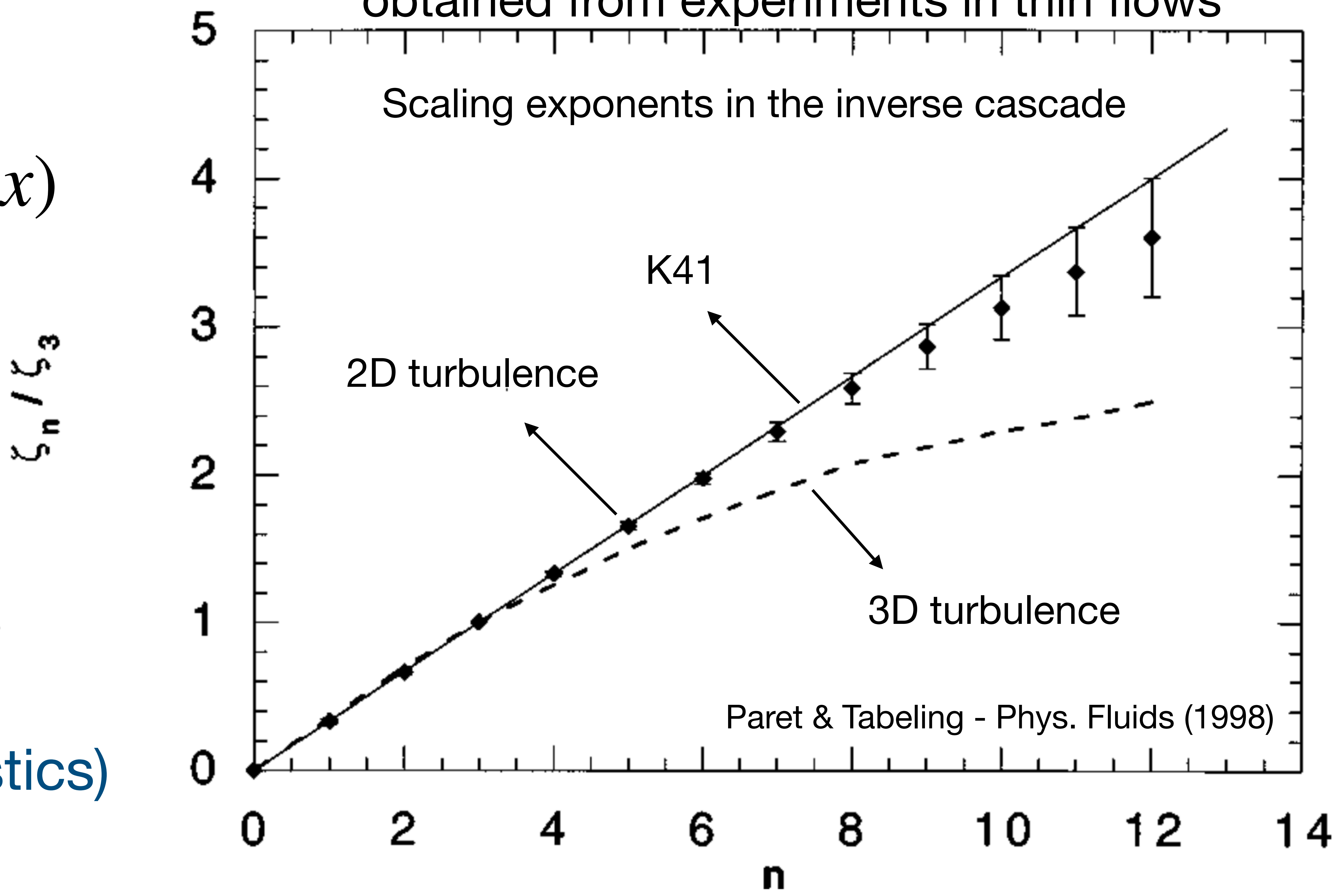
$$S_p = \langle \delta u_r^p \rangle \sim \epsilon^{p/3} r^{p/3} \qquad \delta u_r = u(x+r) - u(x)$$

$$S_p = \langle \delta u_r^p \rangle \sim r^{\zeta_p} \qquad \zeta_p^{\mathrm{K41}} = \frac{p}{3}$$

3D turbulence is intermittent (anomalous deviations)

2D turbulence is self-similar (close-to-Gaussian statistics)

Scaling exponents of structure functions obtained from experiments in thin flows



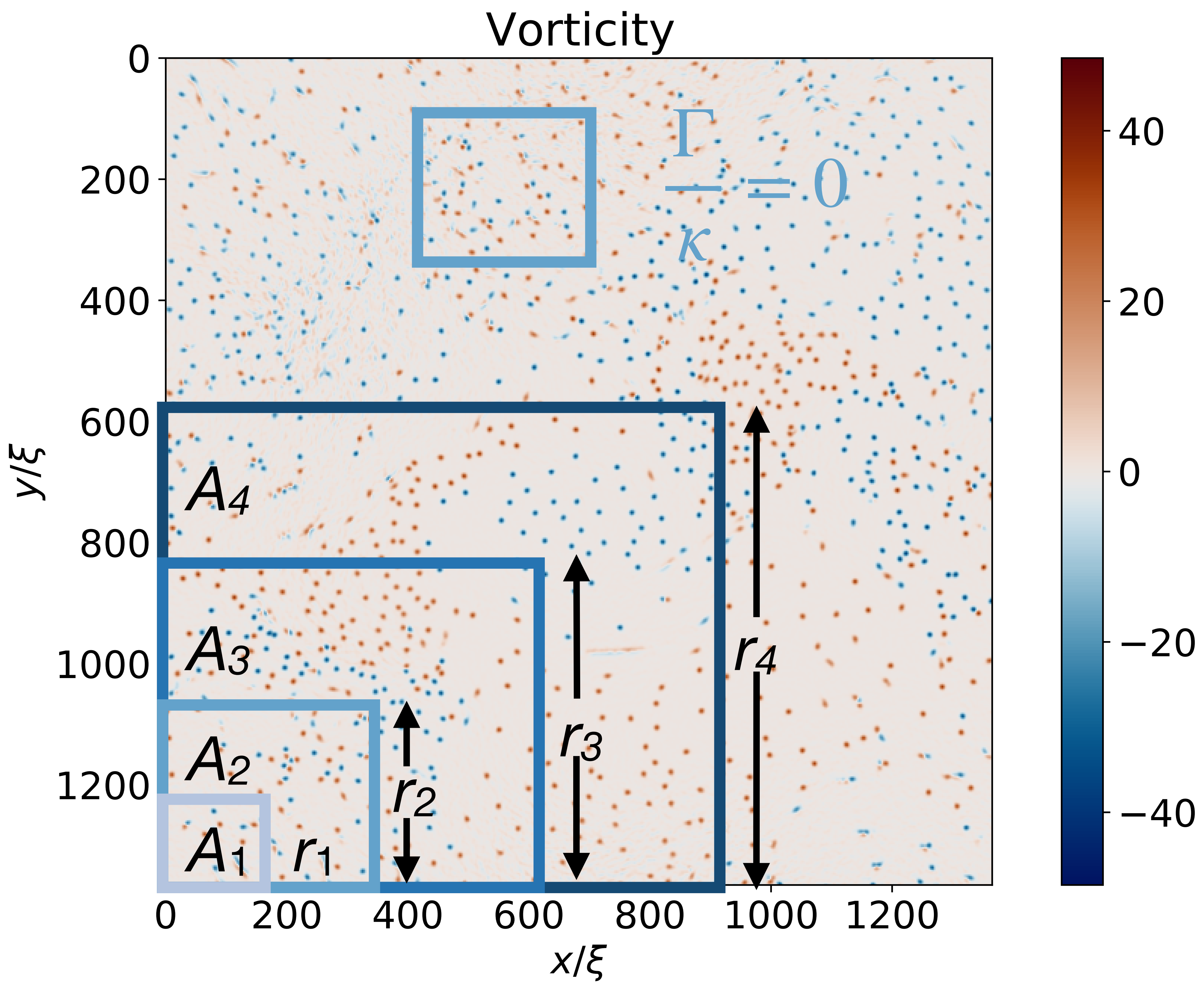
Experiments: Paret & Tabeling (PRL, 1997), Paret & Tabeling (Phys. Fluids, 1998), …
Simulations: Boffeta et al. (PRL, 2000), Boffetta & Ecke (Annu. Rev. Fluid Mech. 2012), …

Circulation in 3D classical and quantum turbulence

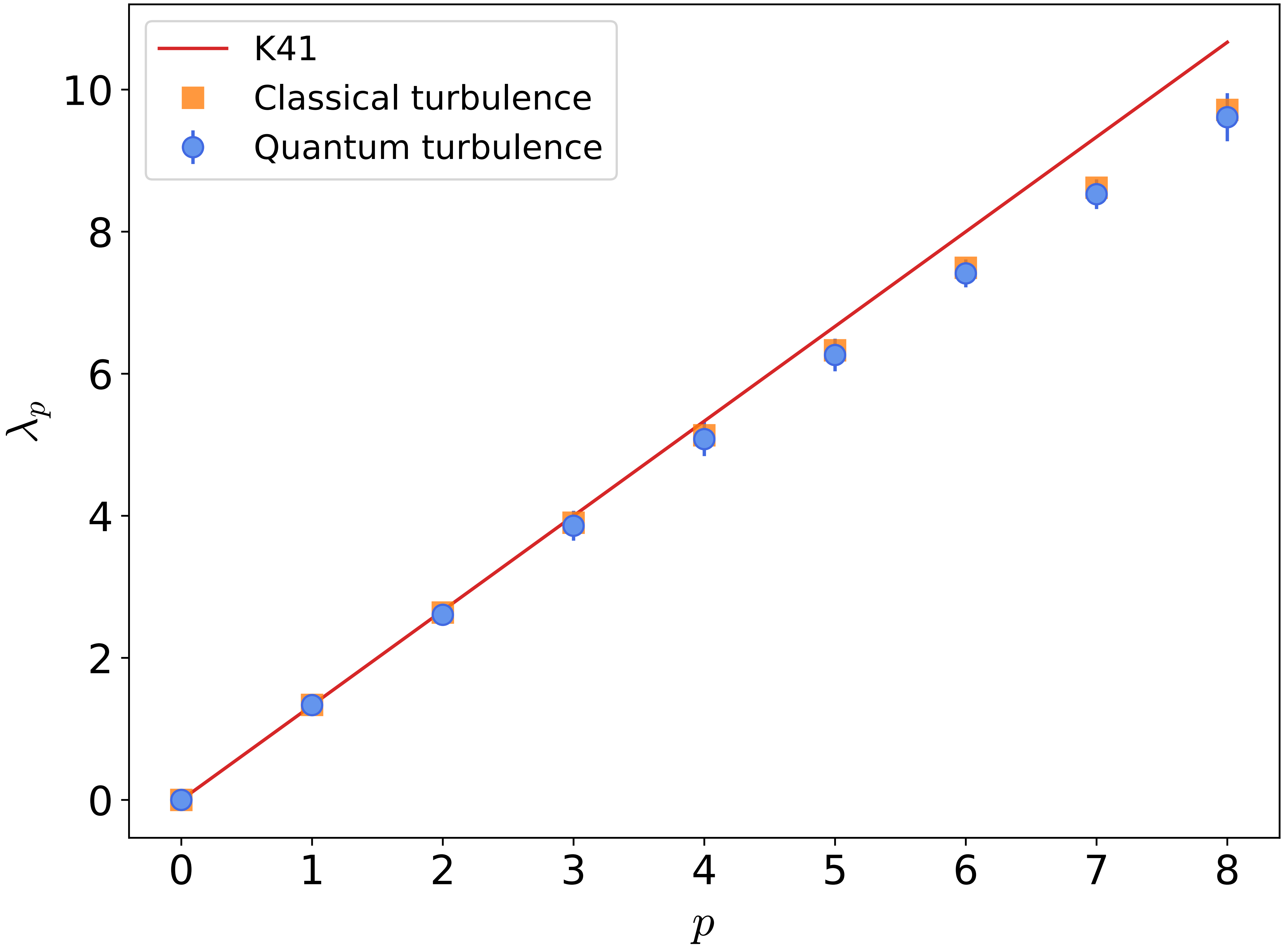
$$\Gamma = \oint_{\mathscr{C}} \mathbf{u} \cdot \mathrm{d}\mathbf{l}$$ Velocity circulation

Scaling of circulation moments using dimensional analysis $\langle \Gamma^p \rangle \sim \epsilon^{p/3} r^{4p/3}$

In general $\langle \Gamma^p \rangle \sim r^{\lambda_p}$ $\lambda_p^{\mathrm{K41}} = \frac{4p}{3}$



Same intermittent behavior in classical and quantum 3D turbulence



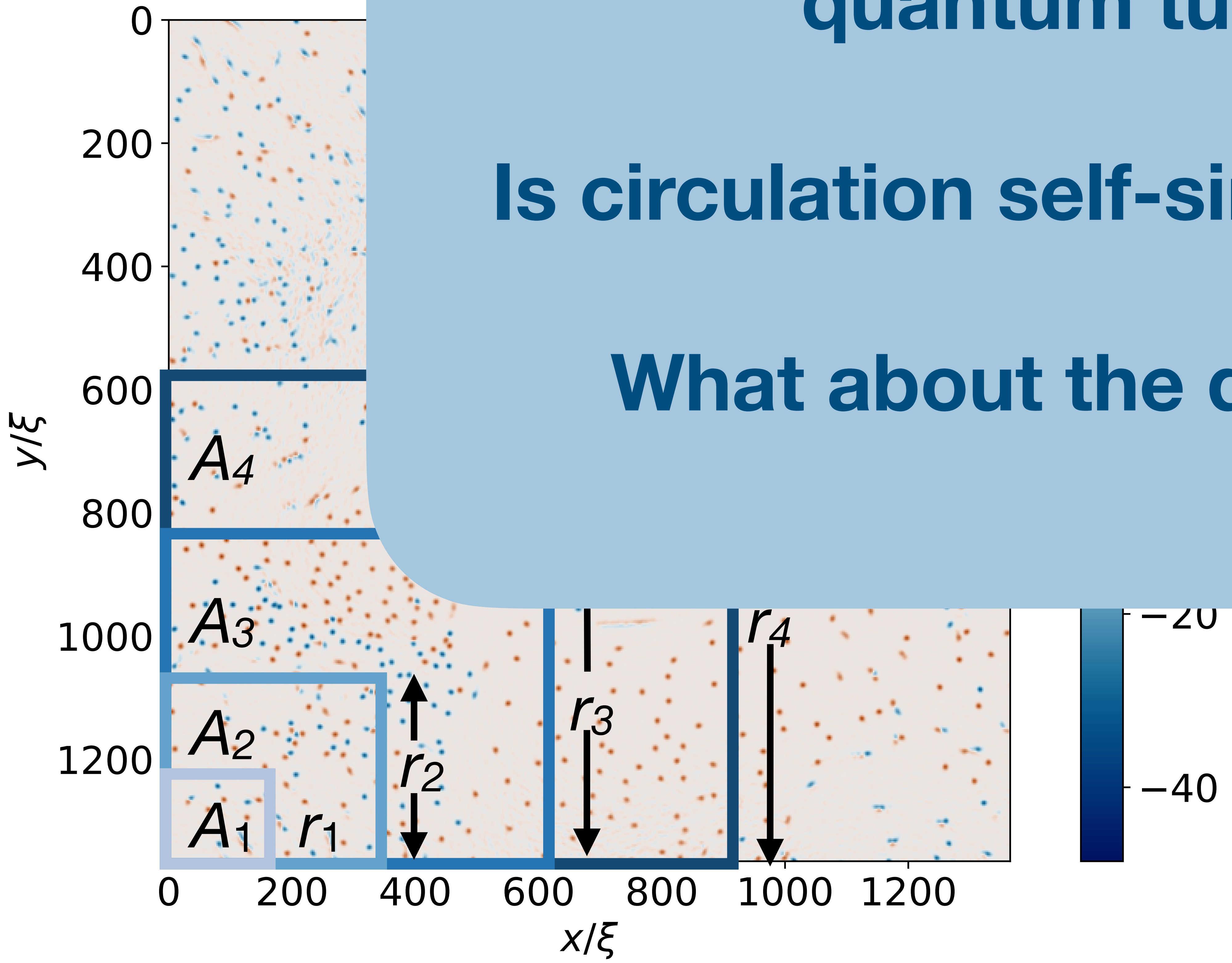
Circulation: Iyer et al. (*PRX*, 2019), Iyer et al. (*PNAS*, 2020), Müller et al. (*PRX*, 2021), Polanco et al. (*Nat. Comm.*, 2021), Müller et al. (*PRFluids*, 2022), …

Circulation in 3D classical and quantum turbulence

$$\Gamma = \oint_{\mathscr{C}} \mathbf{u} \cdot \mathrm{d}\mathbf{l}$$ Velocity circulation

Scaling of circulation moments $\langle \Gamma^p \rangle \sim \; ... \; ^{p/3} \, ^{4p/3}$
using dimensi...

In general



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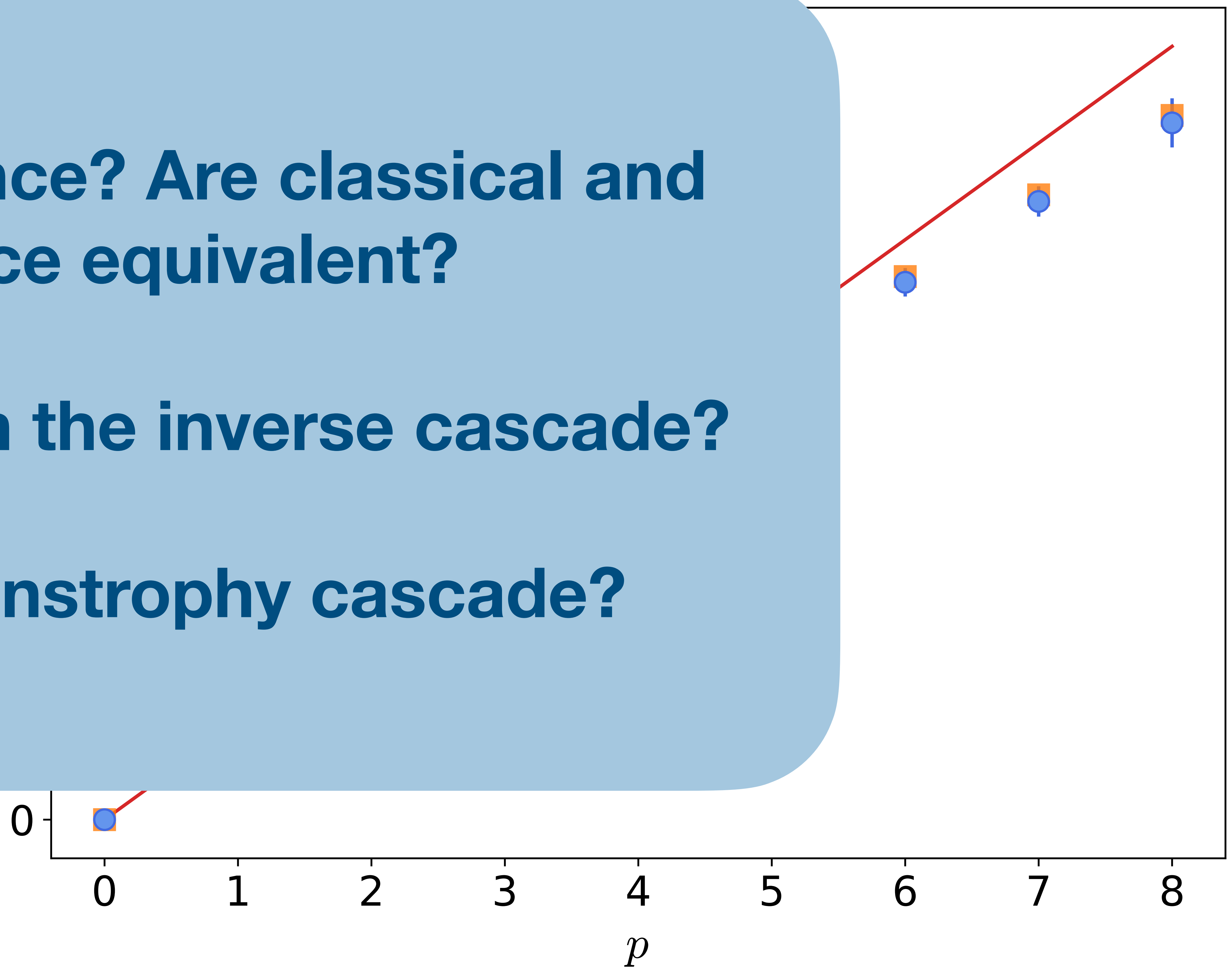


What about in 2D turbulence? Are classical and quantum turbulence equivalent?

Is circulation self-similar in the inverse cascade?

What about the direct enstrophy cascade?

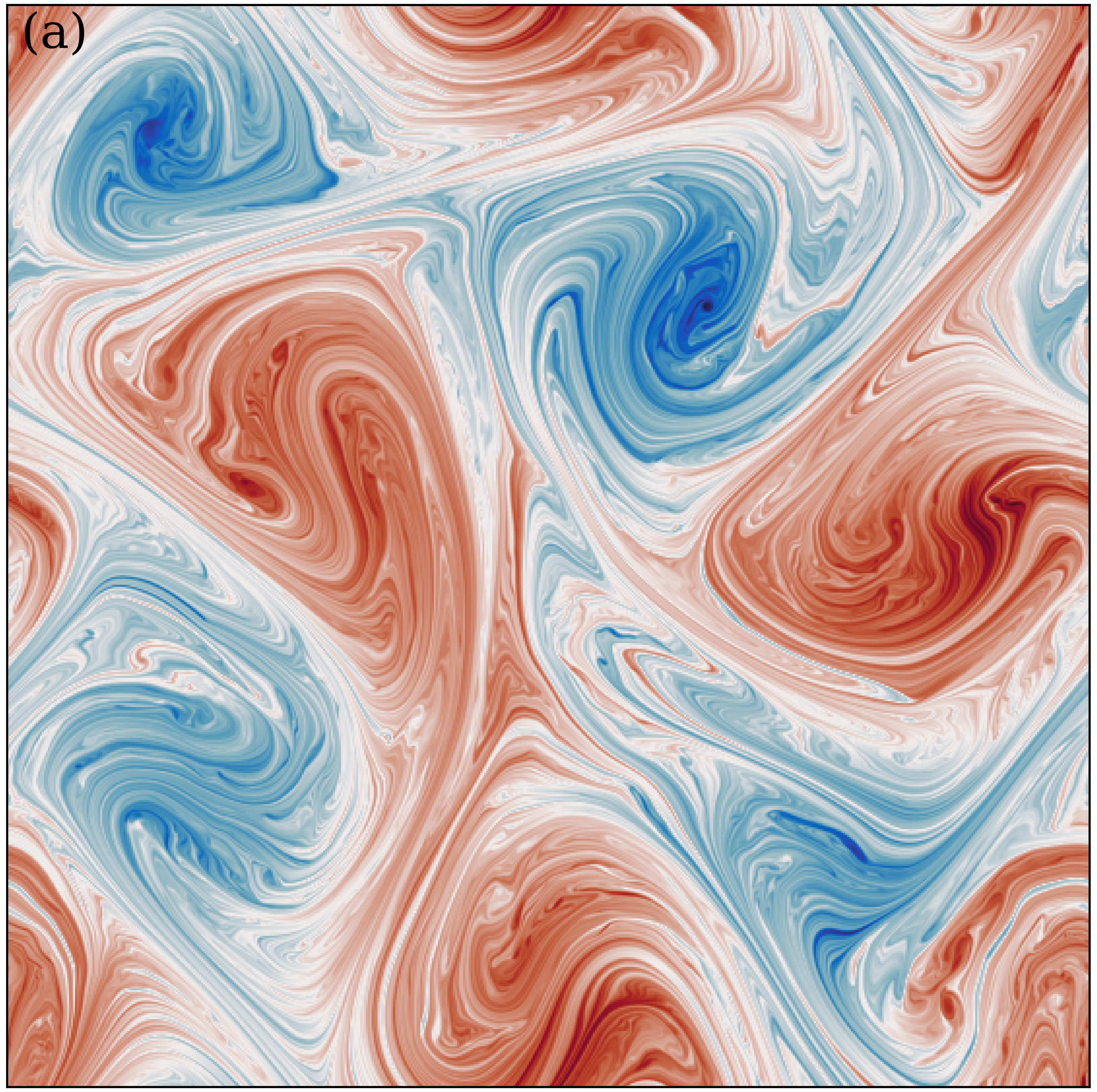
Circulation: Iyer et al. (*PRX*, 2019), Iyer et al. (*PNAS*, 2020), Müller et al. (*PRX*, 2021), Polanco et al. (*Nat. Comm.*, 2021), Müller et al. (*PRFluids*, 2022), …

2D turbulence: Results

Incompressible Navier—Stokes equation

$$\partial_t \omega + (\mathbf{u} \cdot \nabla)\omega = \nu \nabla^2 \omega - \alpha\omega + f$$

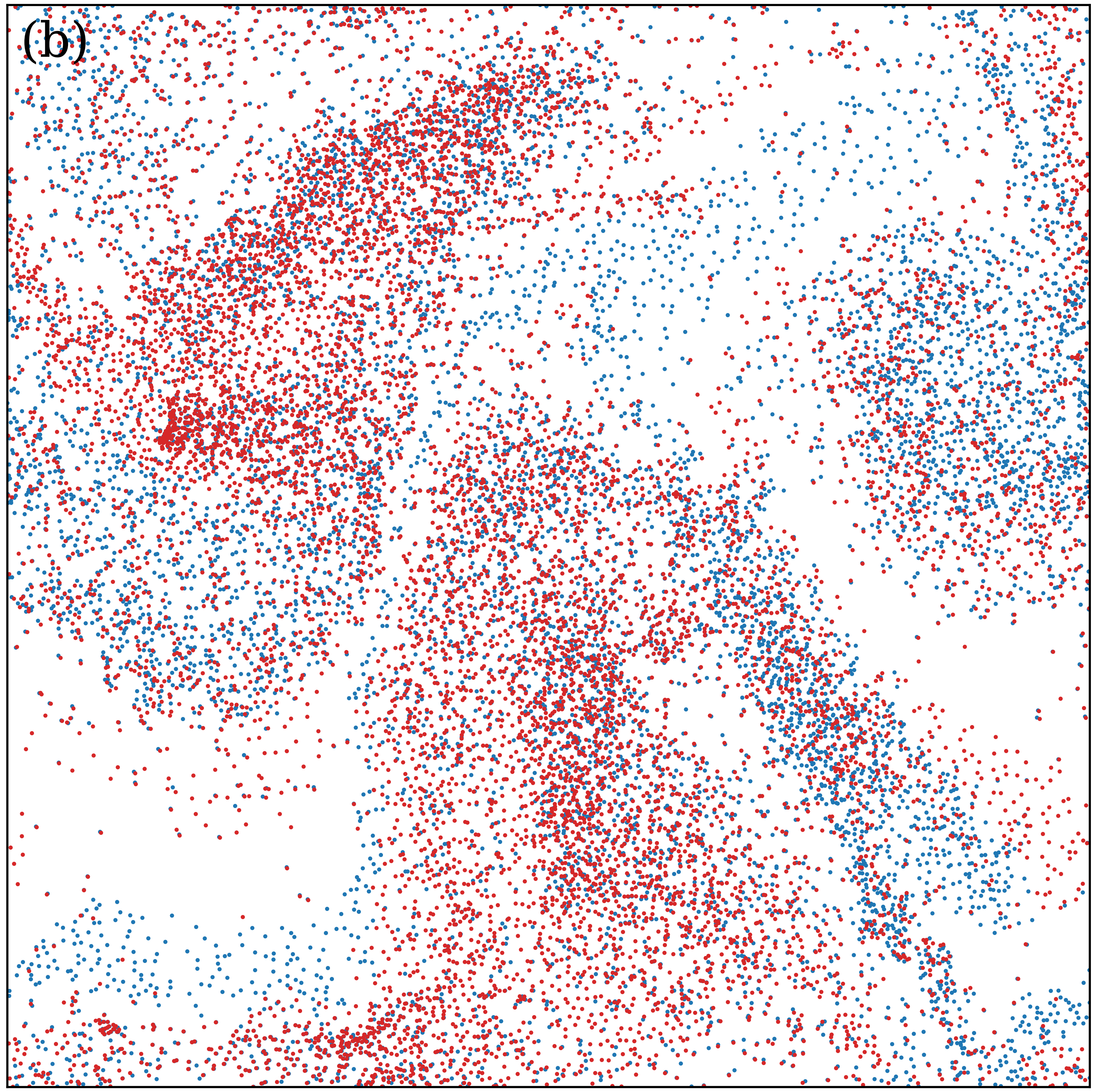
$N_c^2 = 6144^2$ collocation points



Gross—Pitaevskii equation

$$i\hbar\partial_t \psi = -\frac{\hbar^2}{2m}\nabla^2\psi + g\,|\psi|^2\psi - \mu\psi$$

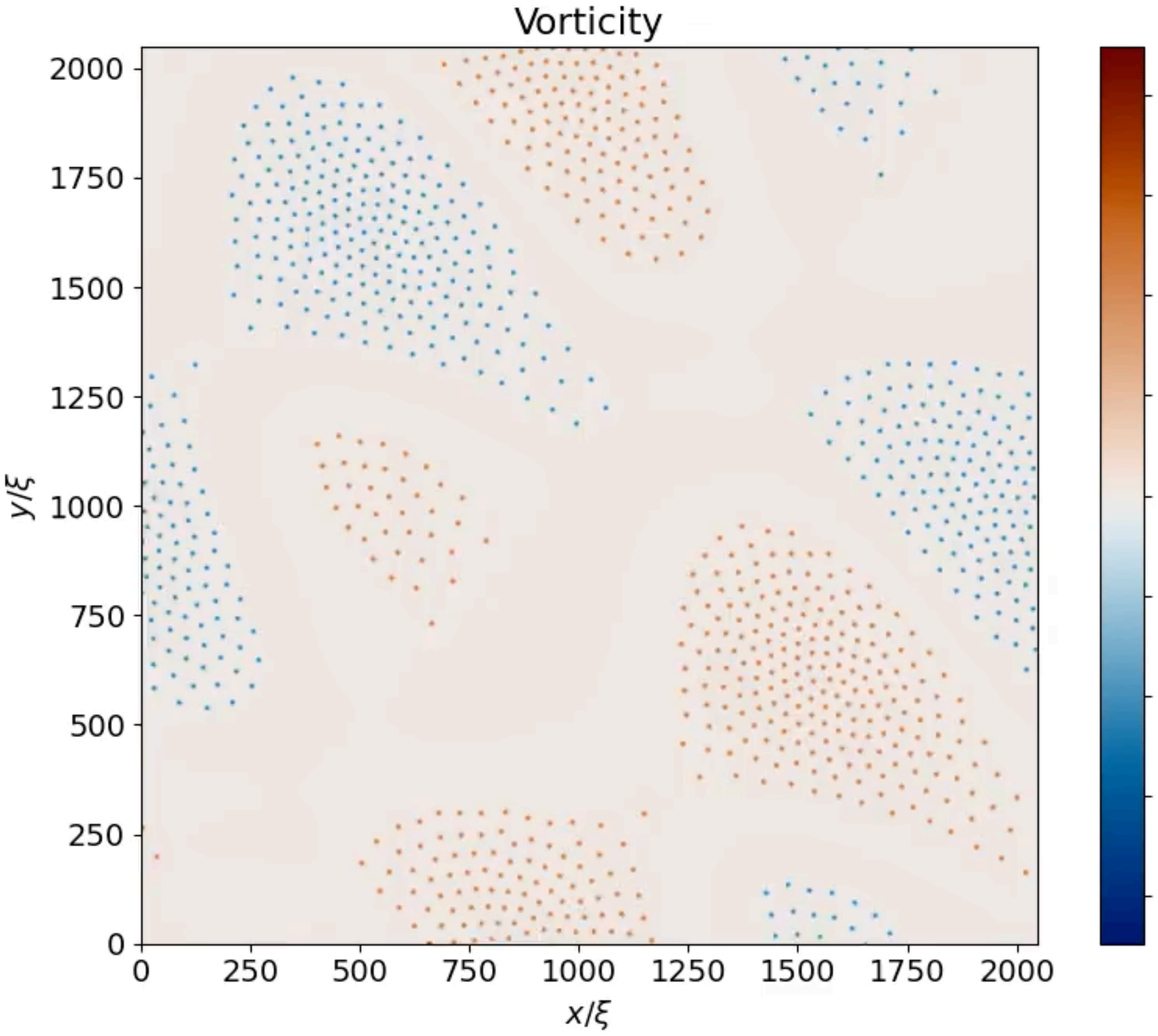
$N_c^2 = 8192^2$ collocation points



Numerical simulations of GP

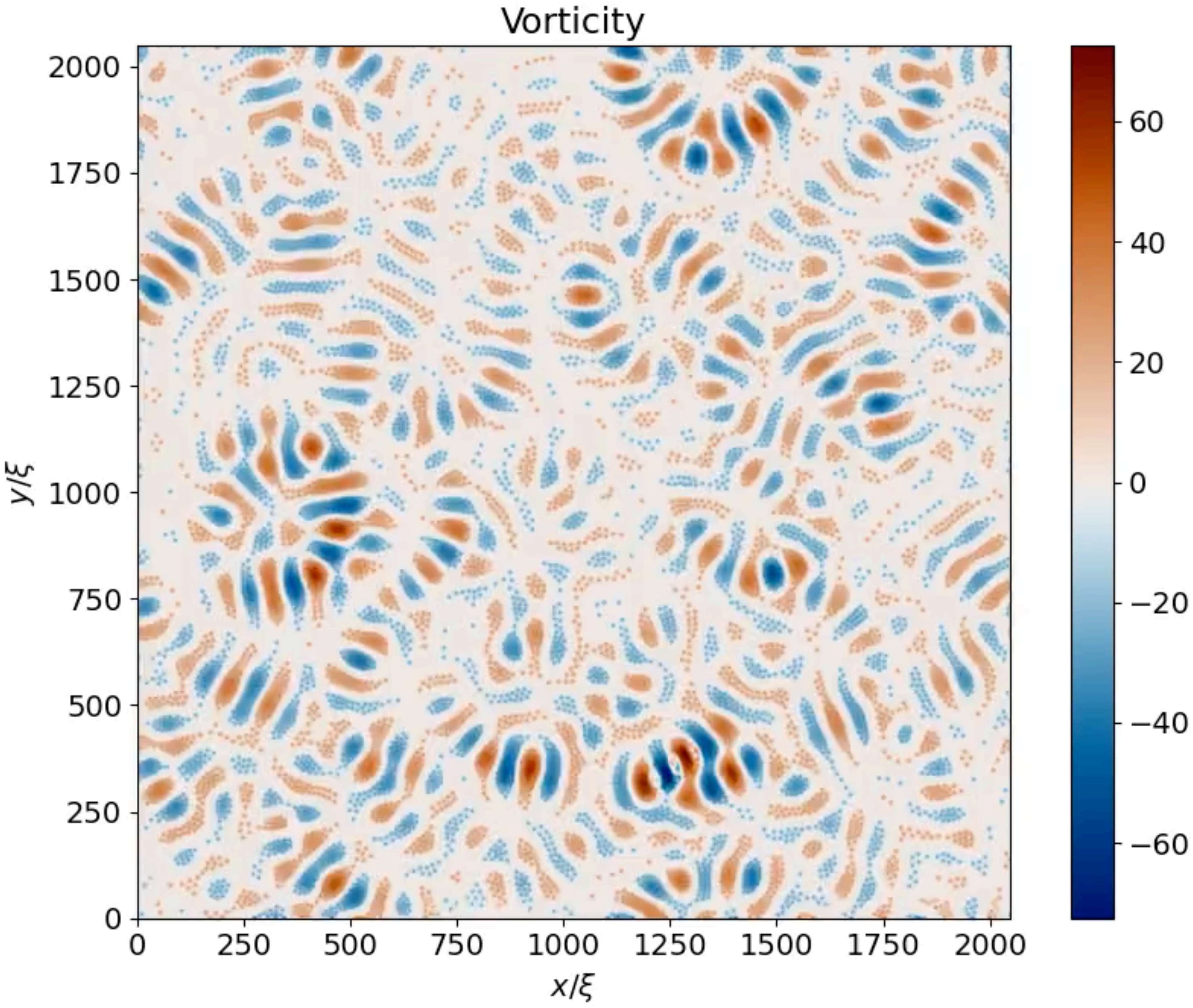
Direct enstrophy cascade

Initial condition at k=1



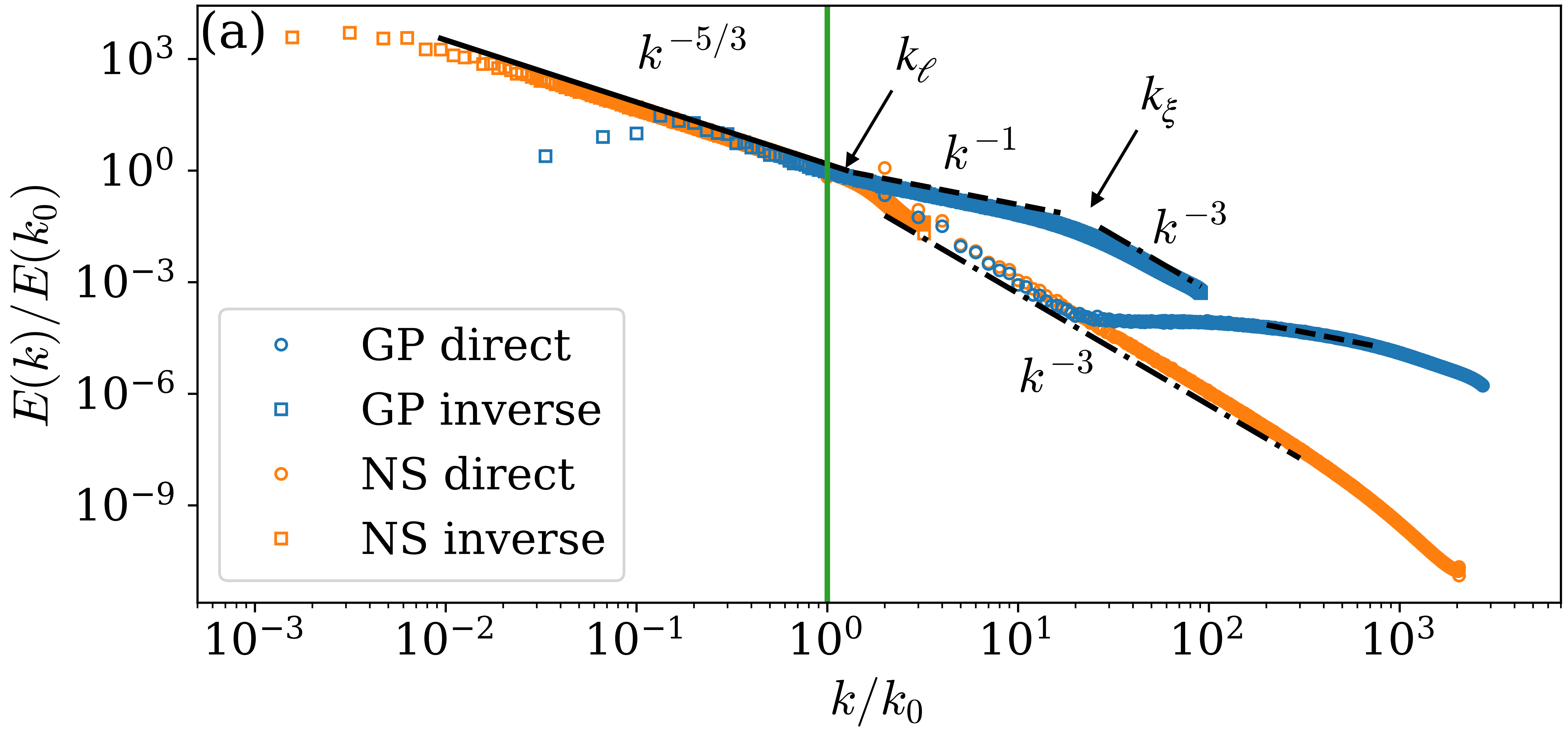
Inverse energy cascade

Initial condition at k=20



2D turbulence: Results

We perform 4 numerical simulations: 2 for NS, and 2 for GP. In each case, one for the inverse and another for the direct cascades.



Inverse energy cascade

$$E(k) = C_I \epsilon^{2/3} k^{-5/3} \quad \text{for} \quad k_I \ll k \ll k_0$$

Direct enstrophy cascade

$$E(k) = C_D \beta^{2/3} k^{-3} \quad \text{for} \quad k_f \ll k \ll k_\eta \,;\, k_0 \ll k \ll k_\ell$$

Other scalings in quantum turbulence

$$E(k) \sim k^{-1} \quad \text{for} \quad k_\ell \ll k \ll k_\xi$$

$$E(k) \sim k^{-3} \quad \text{for} \quad k \gg k_\xi$$

2D turbulence: Results

We perform 4 numerical simulations: 2 for NS, and 2 for GP. In each case, one for the inverse and another for the direct cascades.

Inverse energy cascade

$$E(k) = C_I \epsilon^{2/3} k^{-5/3} \quad \text{for} \quad k_I \ll k \ll k_0$$

$$\langle \Gamma_r^2 \rangle \sim r^{8/3} \quad \text{for} \quad L_0 \ll r \ll L_I$$

Direct enstrophy cascade

$$E(k) = C_D \beta^{2/3} k^{-3} \quad \text{for} \quad k_f \ll k \ll k_\eta \,;\, k_0 \ll k \ll k_\ell$$

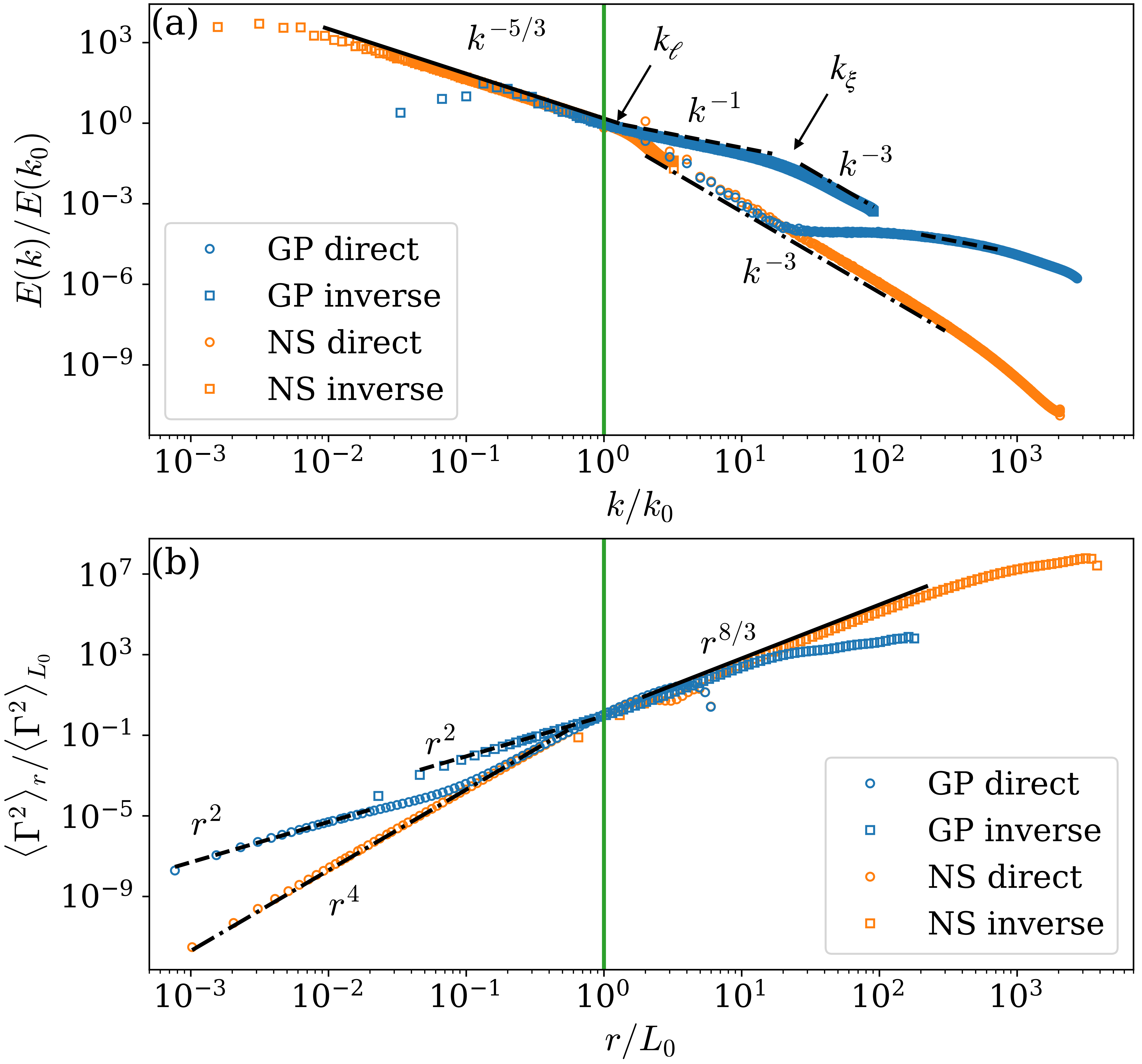
$$\langle \Gamma_r^2 \rangle \sim r^4 \quad \text{for} \quad r \ll L_0 \,;\, \ell \ll r \ll L_0$$

Other scalings in quantum turbulence

$$E(k) \sim k^{-1} \quad \text{for} \quad k_\ell \ll k \ll k_\xi$$

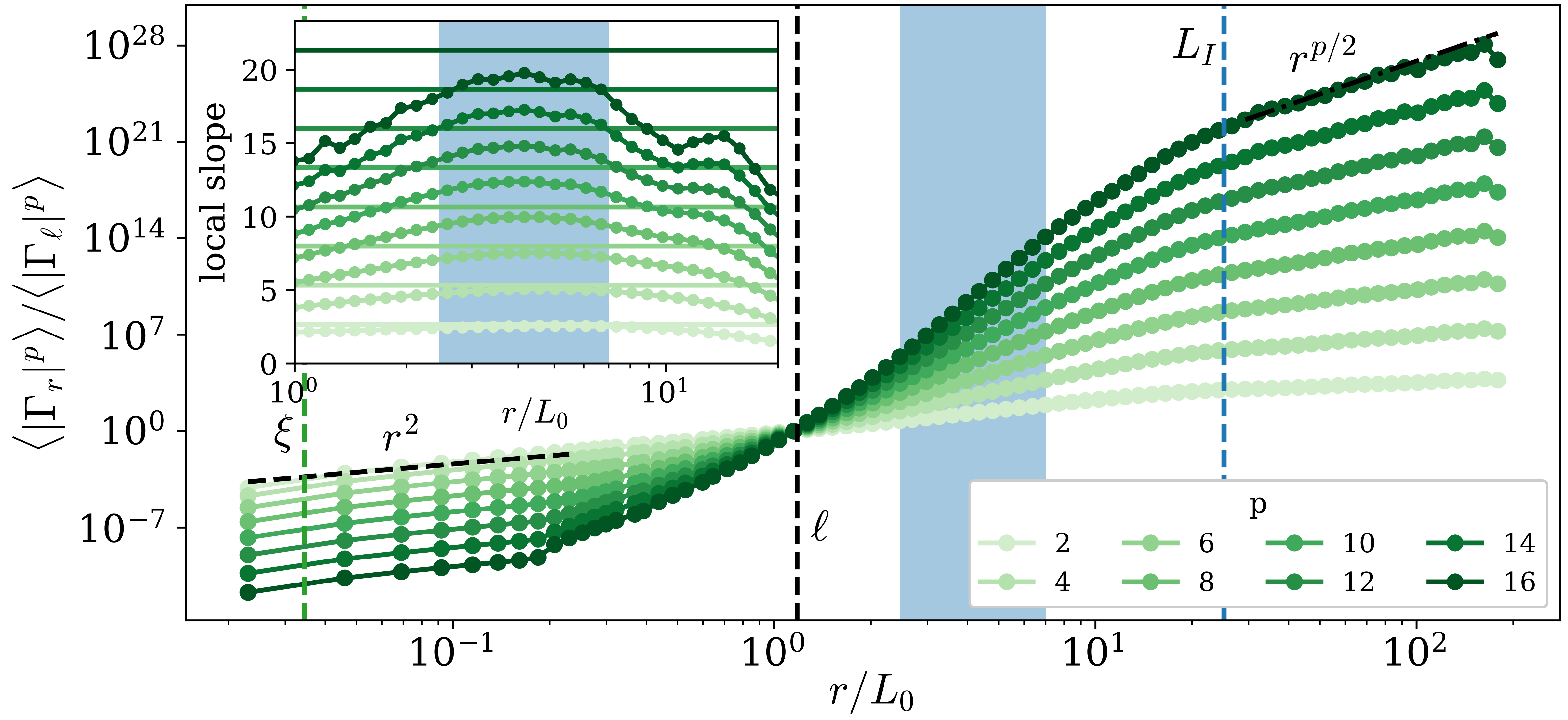
$$E(k) \sim k^{-3} \quad \text{for} \quad k \gg k_\xi$$

$$\langle \Gamma_r^2 \rangle \sim r^2 \quad \text{for} \quad r \ll \ell$$

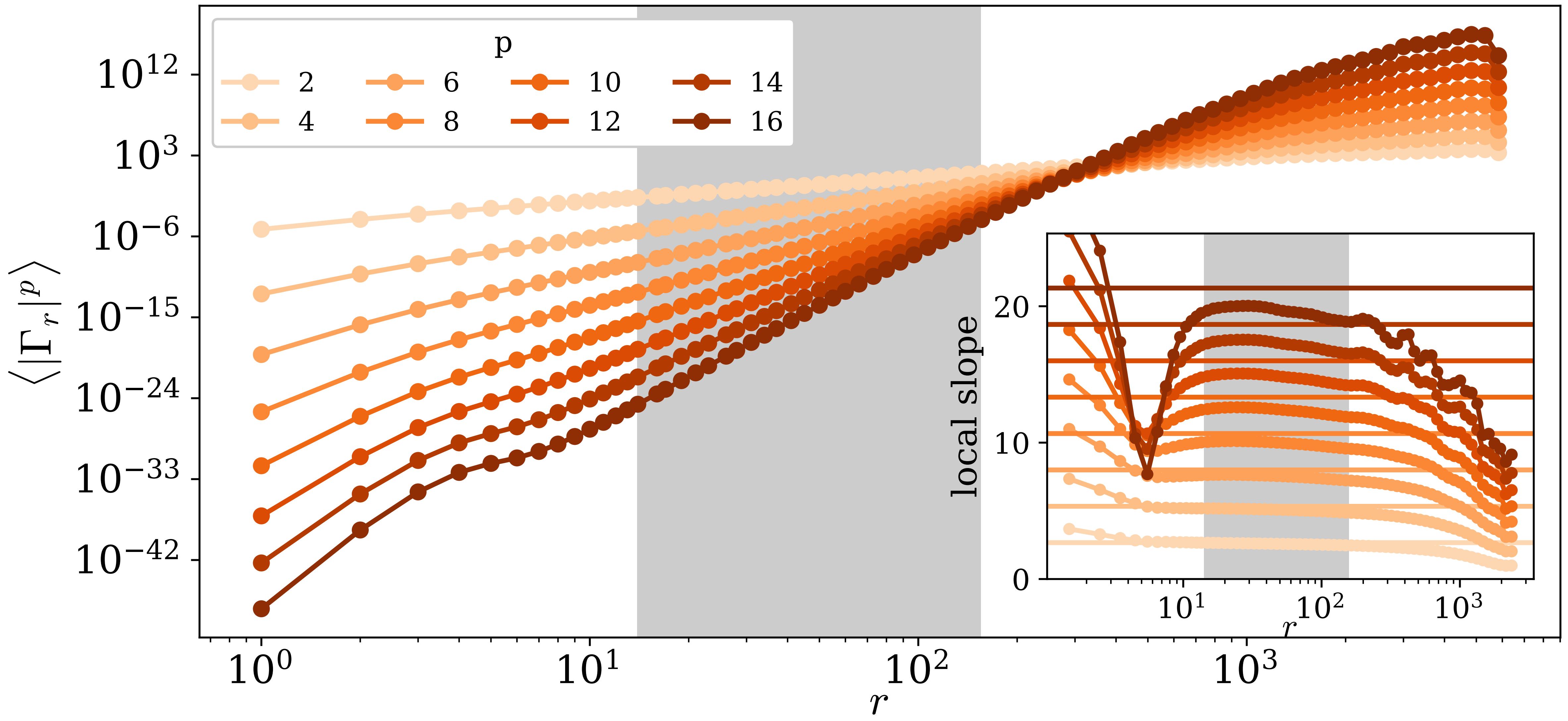


Circulation moments for the inverse energy cascade

Quantum turbulence

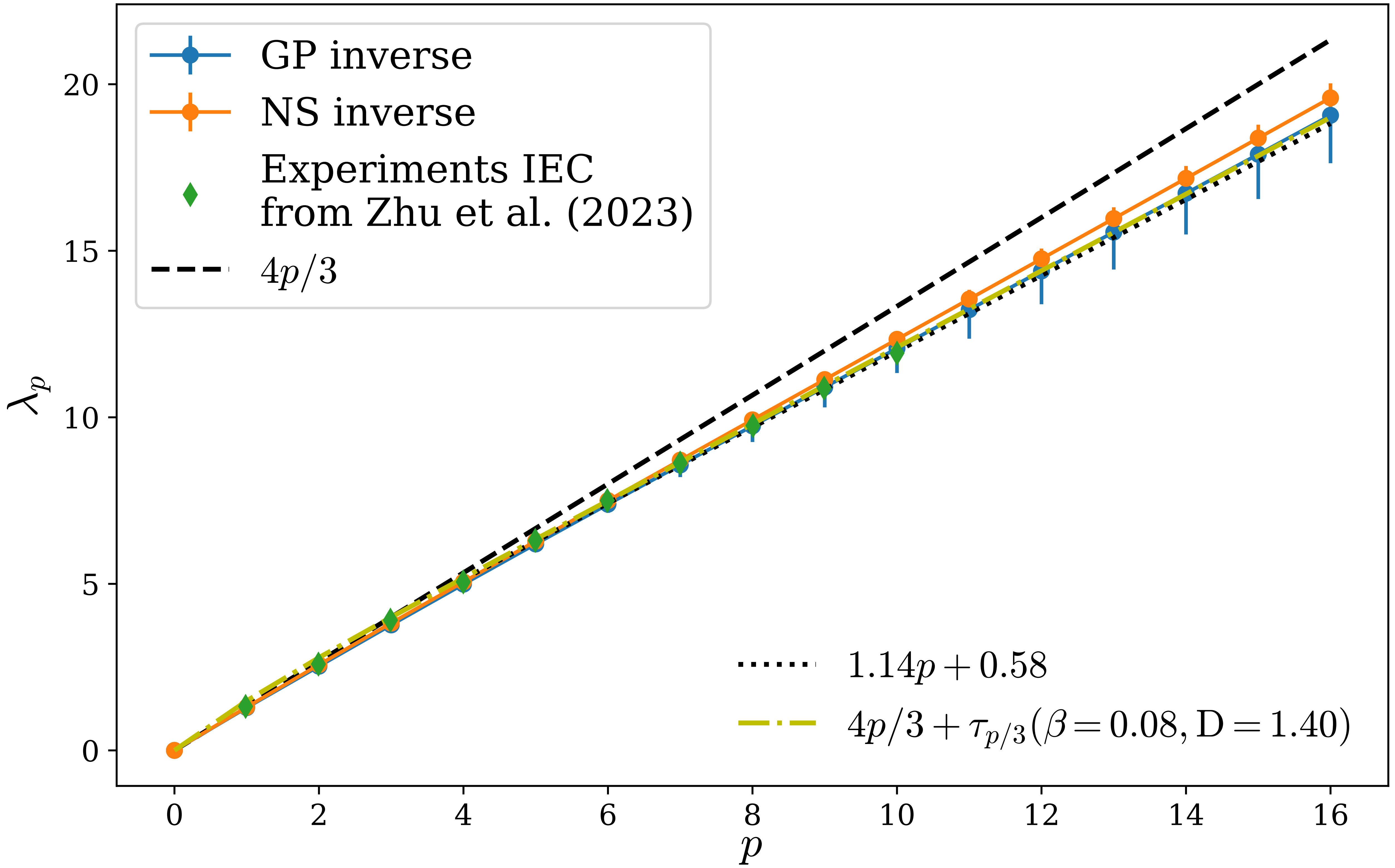


Classical turbulence



Self-similar scaling

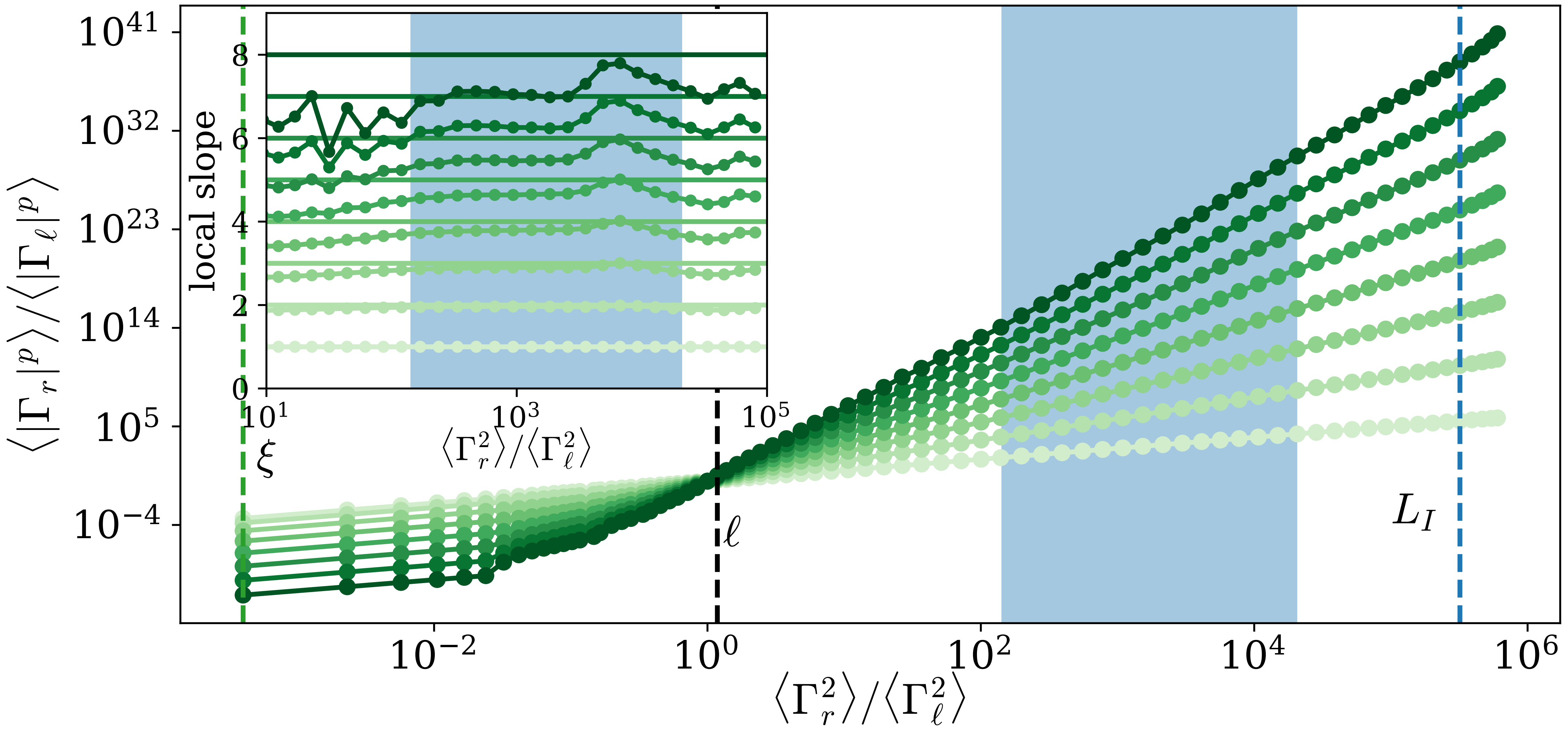
$$\langle|\Gamma|^p\rangle \sim r^{\lambda_p} \qquad \lambda_p^{\mathrm{K41}} = 4p/3$$



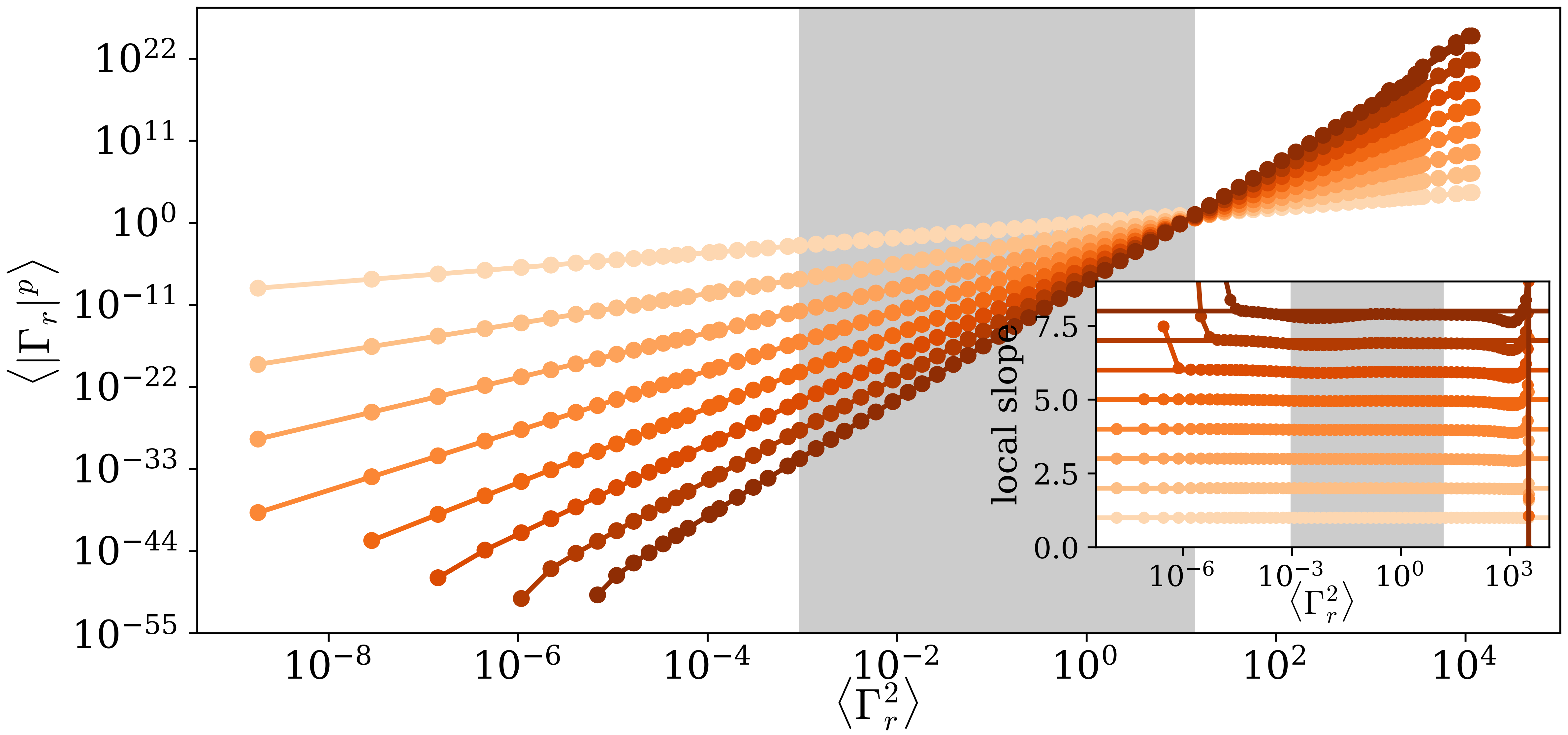
- **Similar** intermittent behavior in classical and quantum turbulence
- Consistent with recent experiments from Zhu et al. (*PRL*, 2023)
- Reminiscent to the equivalence observed in 3D turbulence

Circulation moments for the direct enstrophy cascade

Quantum turbulence

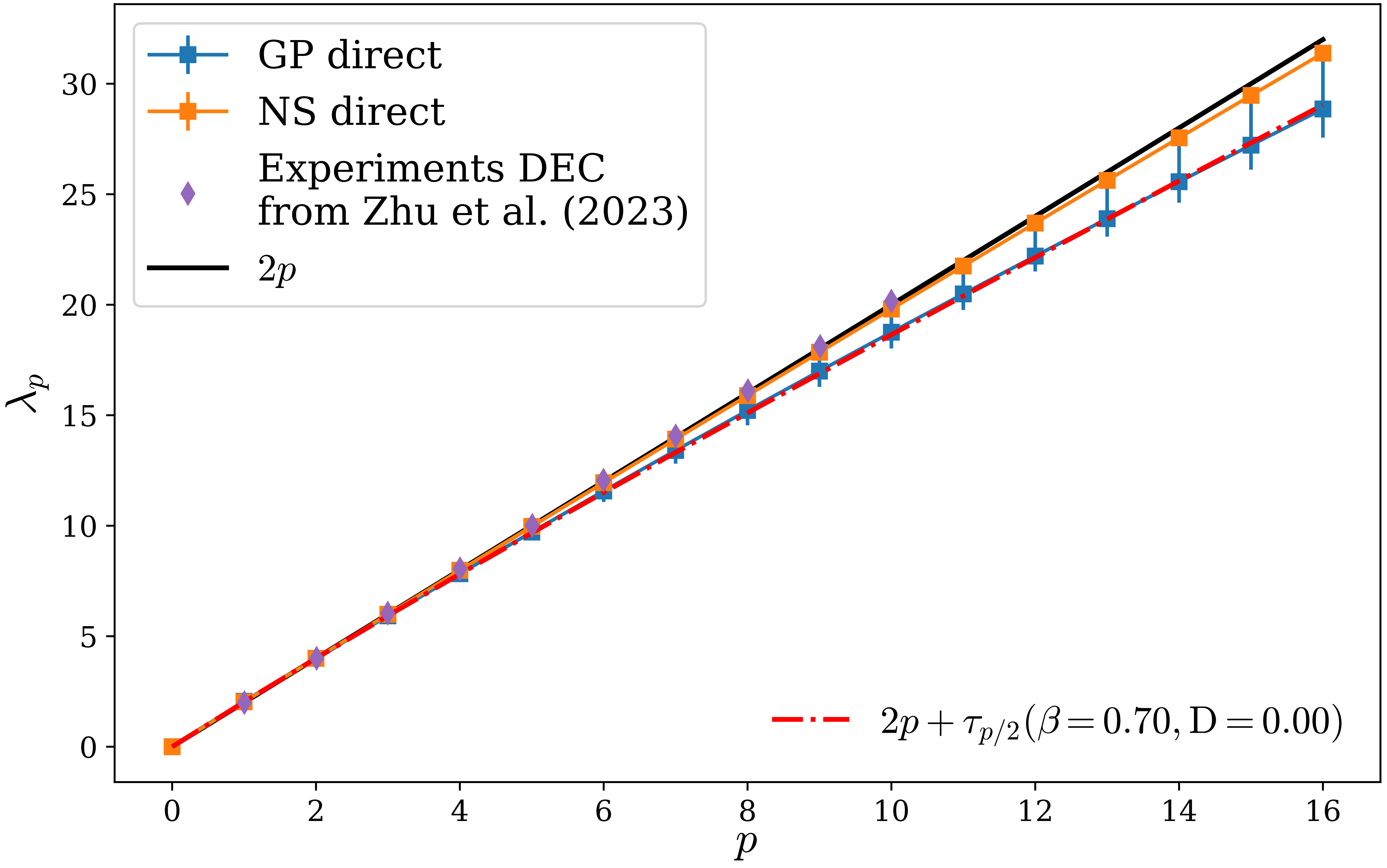


Classical turbulence



Scaling of a smooth field

$$\langle |\Gamma|^p \rangle \sim r^{\lambda_p} \qquad \lambda_p = 2p$$



- **Different** intermittent behavior in classical and quantum turbulence
- Quantum vortices play a crucial role for high-order statistics

Conclusions

We characterised the limits of the equivalence between 2D classical and quantum turbulence

High-order statistics of velocity circulation in 2D turbulence

Inverse energy cascade:

- Quantum turbulence exhibits an intermittent behavior
- Consistent with numerical simulations in classical turbulence
- Consistent with recent experimental measurements of Zhu et al. (*PRL*, 2023)
- Strong difference with velocity increments structure functions

Direct enstrophy cascade:

- Simulations and experiments in classical turbulence show no intermittency
- Quantum turbulence is intermittent
- Quantum vortices introduce strong fluctuations in the enstrophy cascade