Cargese - Bridging Classical and Superfluid Turbulence - July 2023

Intermittency and Lagrangian dynamics of velocity gradients in fluid turbulence

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Intermittency and Lagrangian dynamics of velocity gradients in fluid turbulence

Charles Meneveau (JHU)

reporting on work by

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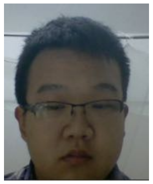
Perry

&

Yuan Luo (Beijing Univ.)



Luo

based on past contributions by:

Laurent Chevillard (now ENS Lyon)

Laurent

& **Michael Wilczek** (now U. Bayreuth)

Michael

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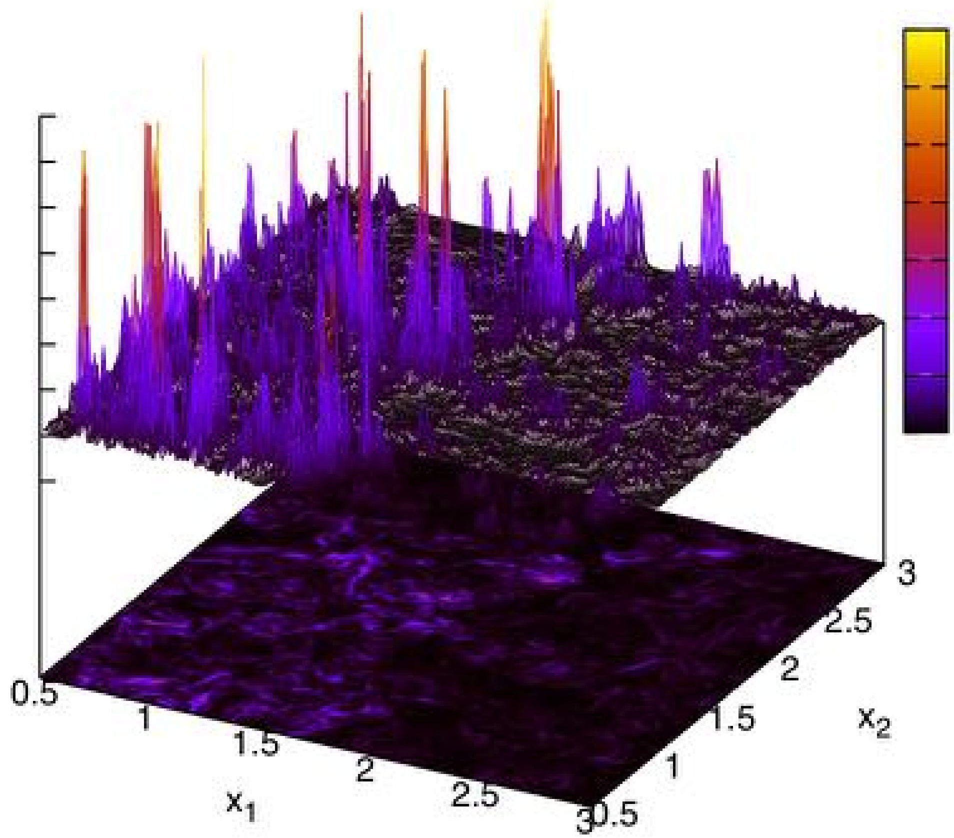
Extreme events in hydrodynamic turbulence (inner intermittency, isotropic turbulence)

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

$$\varepsilon = 2\nu S_{ij} S_{ij}$$

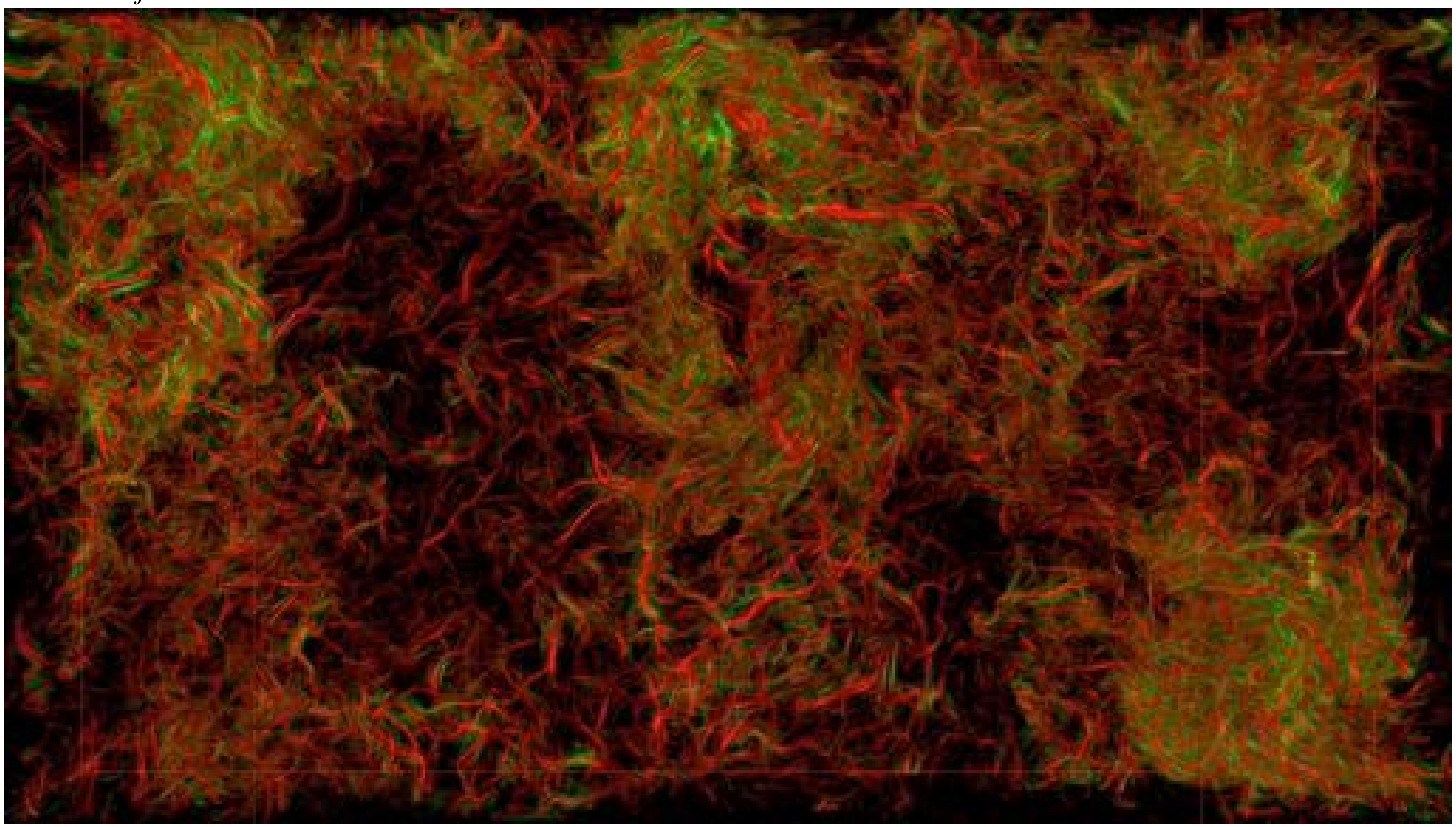
Rate of energy dissipation on a 512x512 plane in Re_λ=433, 1024^3 DNS of isotropic turbulence



Extreme events in hydrodynamic turbulence

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij} \qquad \omega^2 \propto \Omega_{ij}\Omega_{ij}$$

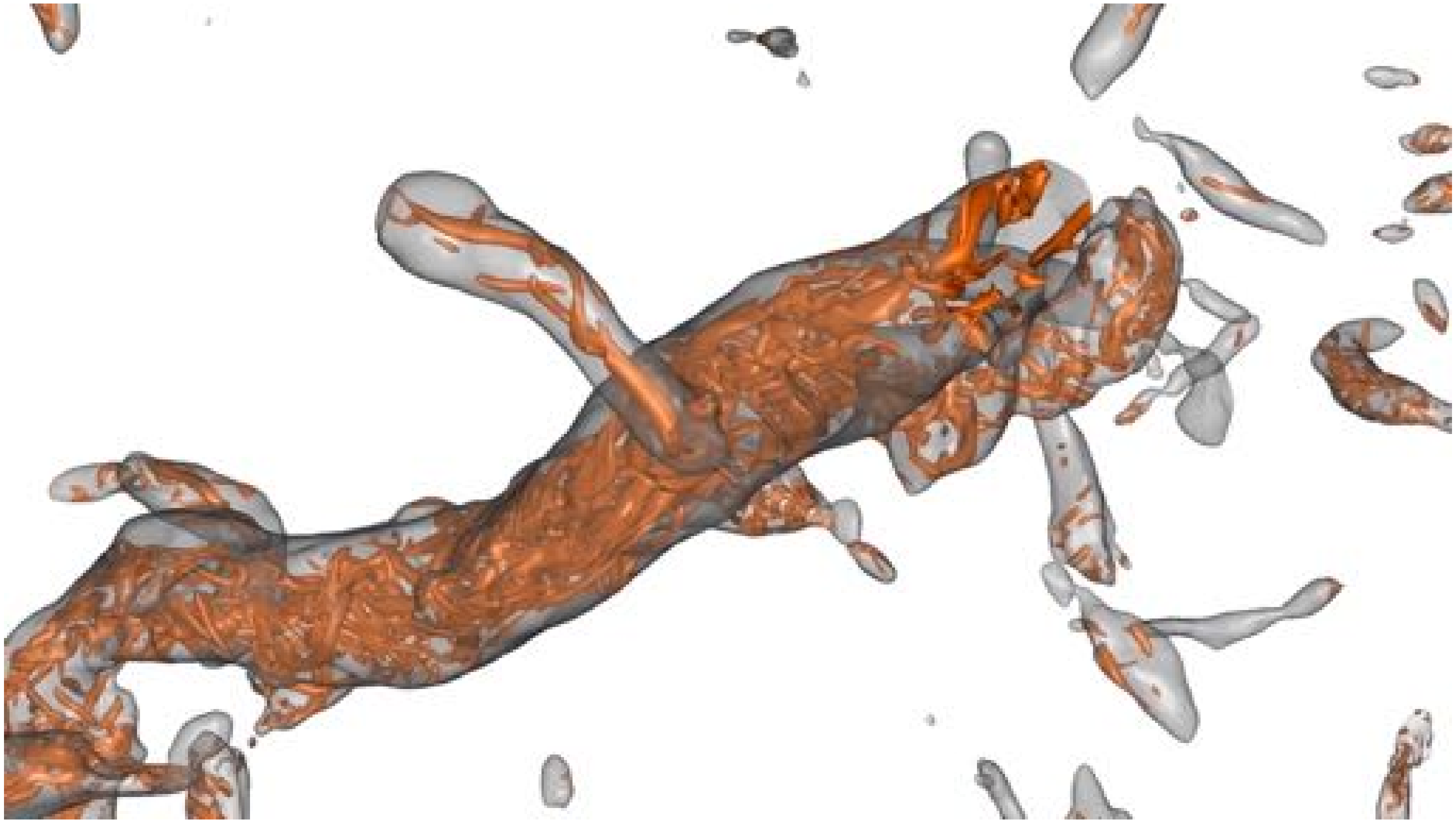
Enstrophy density plots in Re_λ=433
1024^3 DNS of isotropic turbulence



JHU database, Dr. Kai Buerger visualization

The multiscale aspect of turbulence: clustering of vortices.. vortices within vortices

2 & 3 scale v*orticity iso-contours in* Re_λ*=433, 1024*^{3} *DNS of isotropic turbulence (JHTDB)*



Bürger, K., Treib, M., Westermann, R., Werner, S., Lalescu, C.C., Szalay, A., Meneveau, C. and Eyink, G.L., 2012. Vortices within vortices: hierarchical nature of vortex tubes in turbulence. arXiv preprint arXiv:1210.3325.

K41: assuming that mean dissipation tells entire story

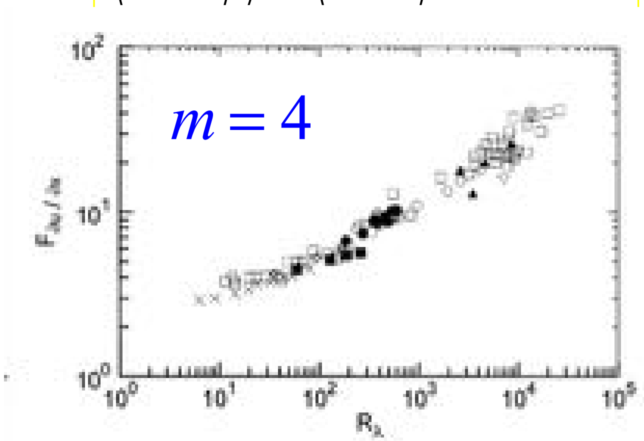
$$\langle \varepsilon \rangle \sim \frac{u'^3}{\ell} \mathrm{Re}^0 \quad \& \quad \langle \varepsilon^{m/2} \rangle \sim \left(\frac{u'^3}{\ell} \right)^{m/2} \mathrm{Re}^0 \quad \Rightarrow \quad \left\langle \left| \frac{\partial u_1}{\partial x_1} \right|^m \right\rangle \Bigg/ \left\langle \left| \frac{\partial u_1}{\partial x_1} \right|^2 \right\rangle^{m/2} \sim \mathrm{Re}^0$$

K41: assuming that mean dissipation tells entire story

$$\langle \varepsilon \rangle \sim \frac{u'^3}{\ell}\mathrm{Re}^0 \qquad \& \qquad \langle \varepsilon^{m/2} \rangle \sim \left(\frac{u'^3}{\ell}\right)^{m/2}\mathrm{Re}^0 \qquad \Rightarrow \qquad \left\langle \left|\frac{\partial u_1}{\partial x_1}\right|^m \right\rangle \Big/ \left\langle \left|\frac{\partial u_1}{\partial x_1}\right|^2 \right\rangle^{m/2} \sim \mathrm{Re}^0$$

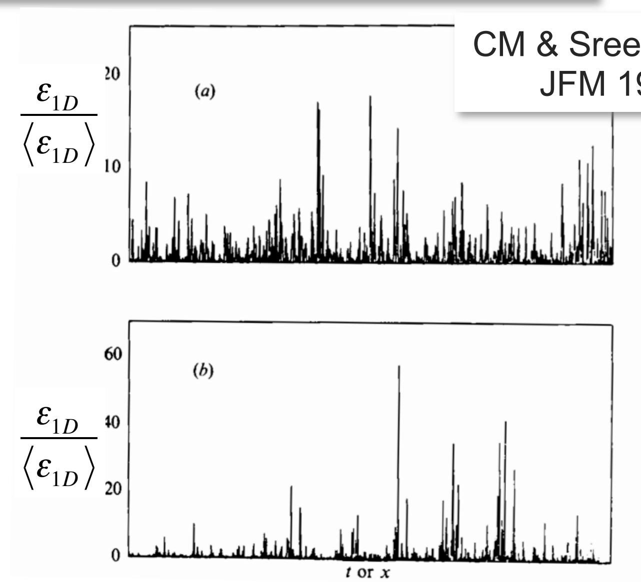
But: Intermittency in small-scale turbulence

$$\left\langle \left|\frac{\partial u_1}{\partial x_1}\right|^m \right\rangle \Big/ \left\langle \left|\frac{\partial u_1}{\partial x_1}\right|^2 \right\rangle^{m/2} \sim \mathrm{Re}^{\alpha(m)}$$



Sreenivasan & Antonia ARFM 1997

CM & Sreenivasan JFM 1991



Re ↑

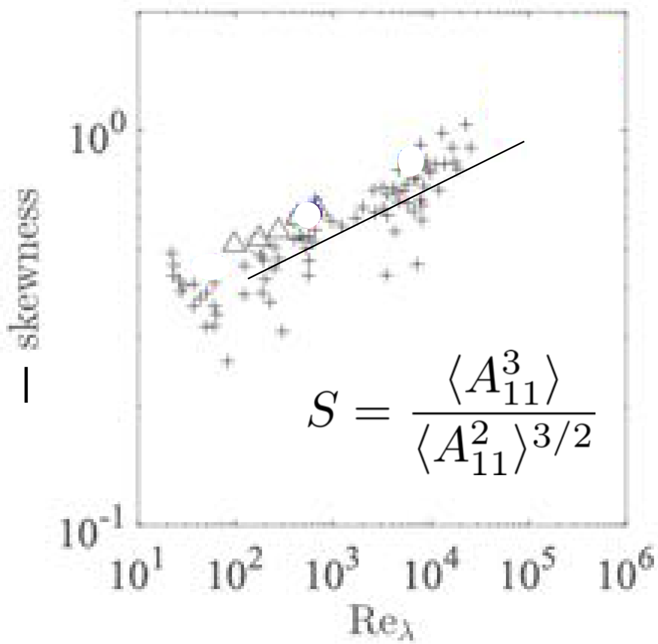
Also: anomalous scaling of structure functions in inertial range (Frisch 1995, etc..)

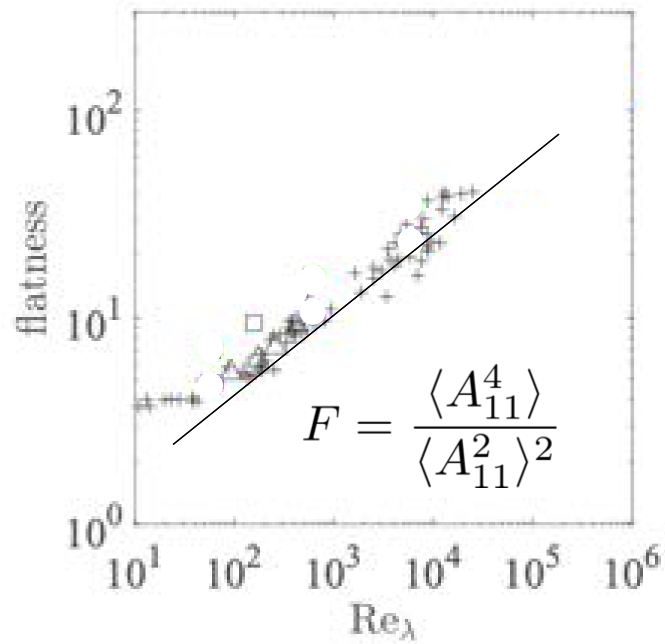
The velocity gradient tensor

Phenomenology (incompressible, NS):

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

intermittency: power-law increase of skewness and flatness with Re



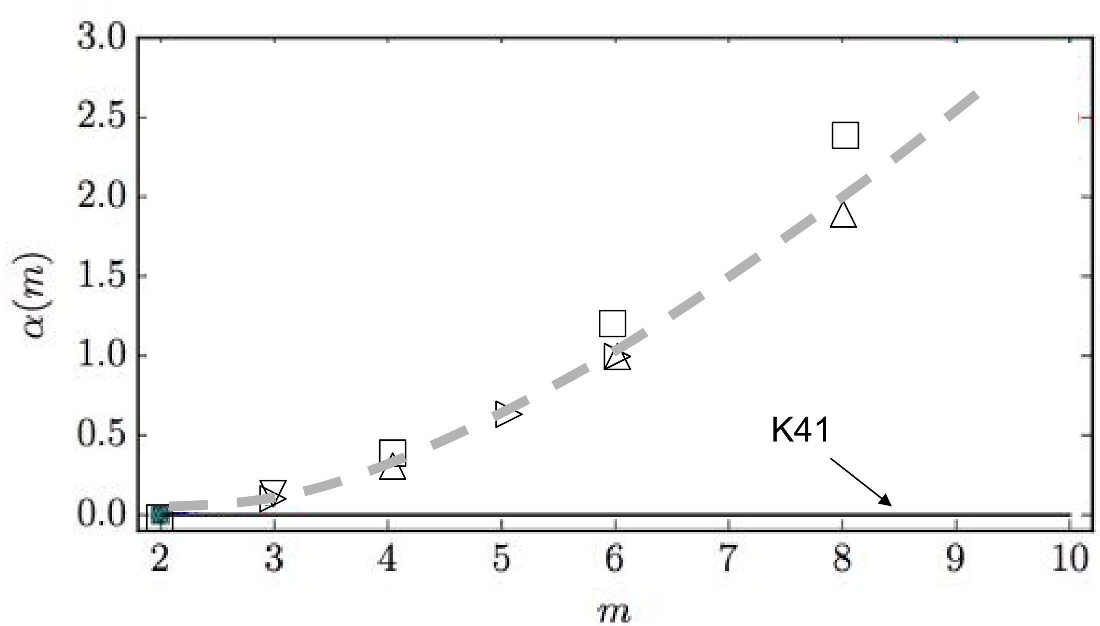


Data:

+ Sreenivasan & Antonia (Ann. Rev. Fluid Mech. 1997)
o Johnson & M (JFM, 2016)
△ Ishihara et al. (JFM, 2007)

Anomalous gradient exponents: data

$$\frac{\left\langle \left|A_{11}\right|^{m} \right\rangle}{\left\langle \left|A_{11}\right|^{2} \right\rangle^{m/2}} \sim \mathrm{Re}^{\alpha(m)}$$



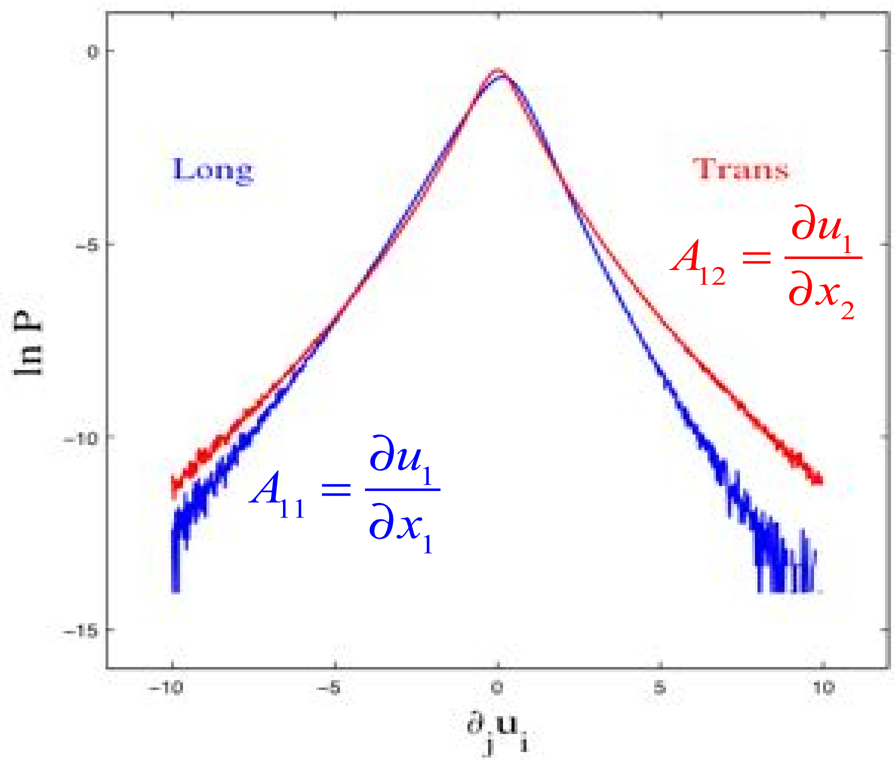
□ = DNS (Boschung 2015)   △ = DNS (Schumaker et al. 2014)
▽ = DNS (Ishihara et al. 2007)   ▷ = experiments (Antonia et al. 1981)

The velocity gradient tensor

Phenomenology (incompressible, NS):

intermittency: long tails in PDFs of gradients

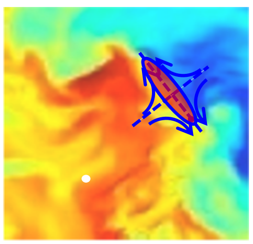
$$A_{ij} = \frac{\partial u_i}{\partial x_j}$$



The velocity gradient tensor

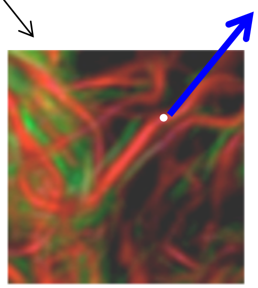
Phenomenology (incompressible, NS):

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

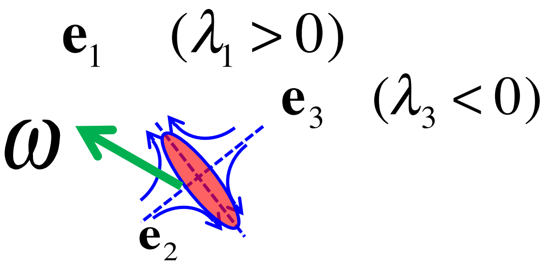


Strain-rate tensor: eigen-values, eigen-vectors

$$\omega = \nabla \times \mathbf{u}$$

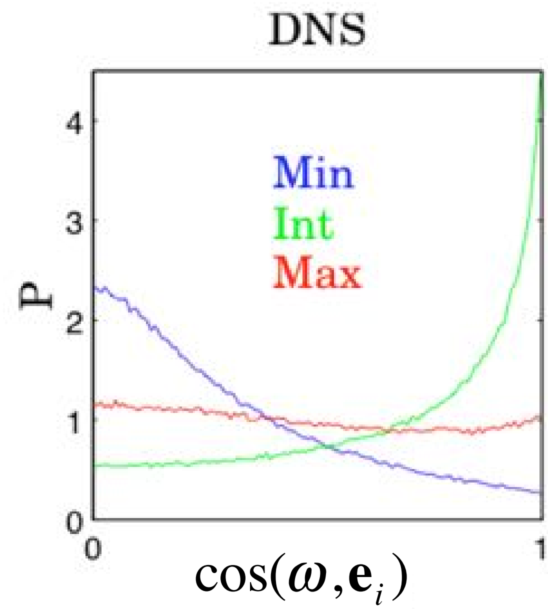


Rotation tensor Vorticity vector



Geometric aspects:

E.g. preferential alignment of vorticity with intermediate strain-rate eigenvector *(Ashurst et al. 1987, Kerr 1988, etc.)*:



The velocity gradient tensor

Phenomenology (incompressible, NS):

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

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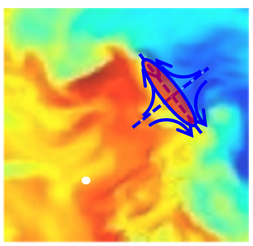
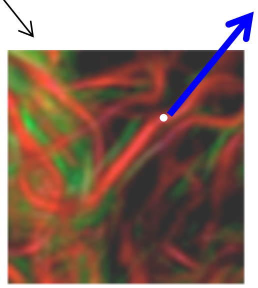


Strain-rate tensor: eigen-values, eigen-vectors



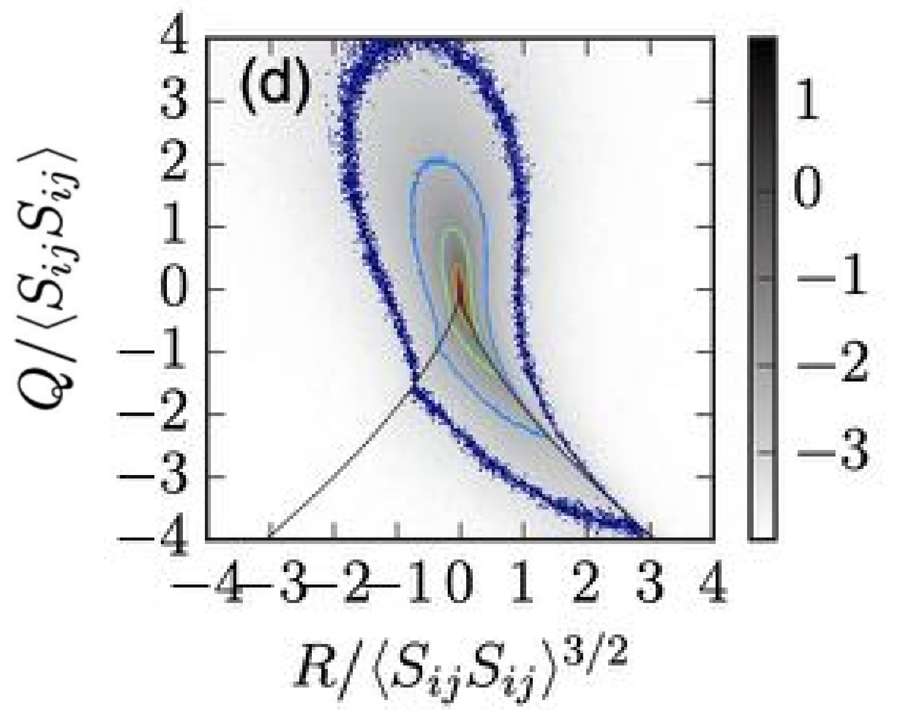
Rotation tensor Vorticity vector

$$Q = -\frac{1}{2} A_{ij} A_{ji} \qquad R = -\frac{1}{3} A_{ij} A_{jk} A_{ki}$$

Viellefosse 1982, Cantwell 1992

Geometric aspects:

Joint PDF of invariants: tear-drop shape



Also, many practical engineering motivations:

Small scale intermittency of turbulence generates extreme events, difficult to characterize based on local averages

Small scale intermittency affects:

- droplet and bubble deformations in turbulence
- flame structure (quenching) in combustion
- micro-organism motility and nutrient uptake
- polymer stretching–relaxation dynamics

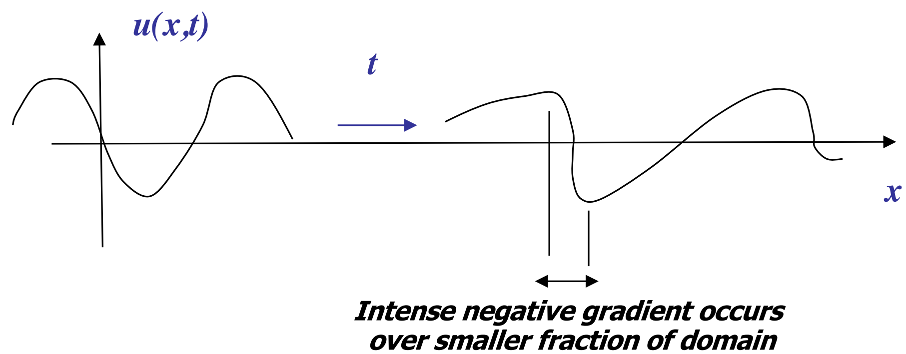
The velocity gradient tensor

in its Lagrangian evolution

Trivial case: 1-D inv Burgers equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

$$A = \frac{\partial u}{\partial x}$$



The velocity gradient tensor

in its Lagrangian evolution

Trivial case: 1-D inv Burgers equation

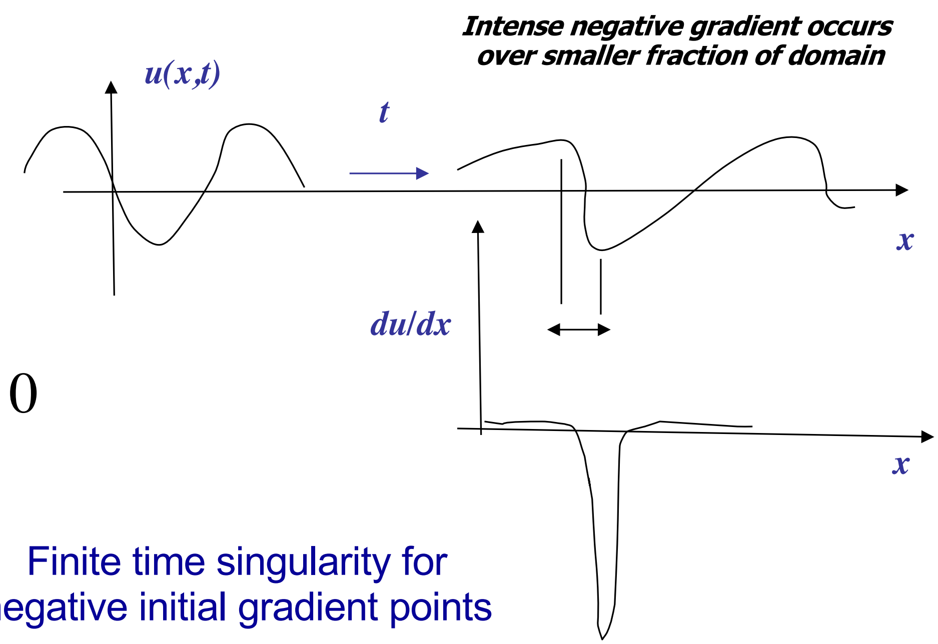
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

$$A = \frac{\partial u}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = \frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + AA = 0$$

$$\frac{dA}{dt} = -A^2 \quad \Rightarrow A = \frac{1}{t-(-A_0^{-1})}$$

Finite time singularity for negative initial gradient points



The velocity gradient tensor: 3D

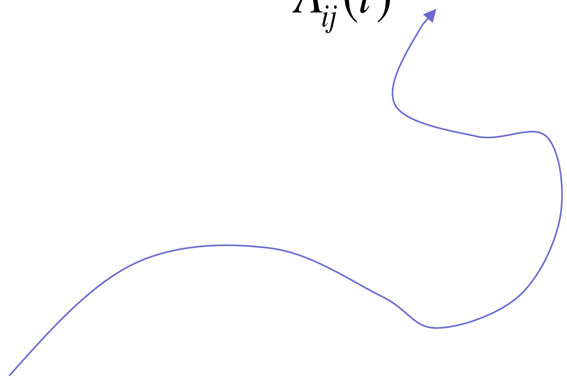
in its Lagrangian evolution

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

$$\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial t} + \frac{\partial u_k u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i + g_i\right)$$

System of 9 (8) ODEs (not closed)
if viewed in Lagrangian frame
(dependent on non-local variables)

$A_{ij}(t)$



$$\frac{dA_{11}}{dt} = \ldots$$

$$\frac{dA_{12}}{dt} = \ldots$$

$$\frac{dA_{13}}{dt} = \ldots$$

$\ldots$

$\ldots$

$$\frac{dA_{33}}{dt} = \ldots$$

The velocity gradient tensor: 3D

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij} \qquad \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial t} + \frac{\partial u_k u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i + g_i \right)$$

$$\frac{dA_{ij}}{dt} = -\underbrace{\left(A_{iq}A_{qj} - \frac{1}{3}A_{mn}A_{nm}\delta_{ij} \right)}_{\text{Self-stretching}} \underbrace{- \left(\frac{\partial^2 p}{\partial x_i \partial x_j} - \frac{1}{3}\nabla^2 p \delta_{ij} \right) + \nu \frac{\partial^2 A_{ij}}{\partial x_q \partial x_q}}_{\text{unclosed}} + \underbrace{W_{ij}}_{\text{forcing}}$$

" $\frac{dA}{dt} = -A^2$ " in 3D

The velocity gradient tensor: 3D

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij} \qquad \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial t} + \frac{\partial u_k u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i + g_i \right)$$

$$\frac{dA_{ij}}{dt} = -\underbrace{\left(A_{iq}A_{qj} - \frac{1}{3}A_{mn}A_{nm}\delta_{ij} \right)}_{\text{Self-stretching}} \underbrace{- \left(\frac{\partial^2 p}{\partial x_i \partial x_j} - \frac{1}{3}\nabla^2 p \delta_{ij} \right) + \nu \frac{\partial^2 A_{ij}}{\partial x_q \partial x_q}}_{\text{unclosed}} + \underbrace{W_{ij}}_{\text{forcing}}$$

Restricted Euler Equation
Viellefosse 1982, Cantwell 1992:

$$\frac{dA_{ij}}{dt} = -\left(A_{iq}A_{qj} - \frac{1}{3}A_{mn}A_{nm}\delta_{ij} \right) \qquad A_{ii} = \frac{\partial u_i}{\partial x_i} = 0$$

System of 8 independent ODEs
if viewed in Lagrangian frame

The velocity gradient tensor: 3D

Restricted Euler Equation

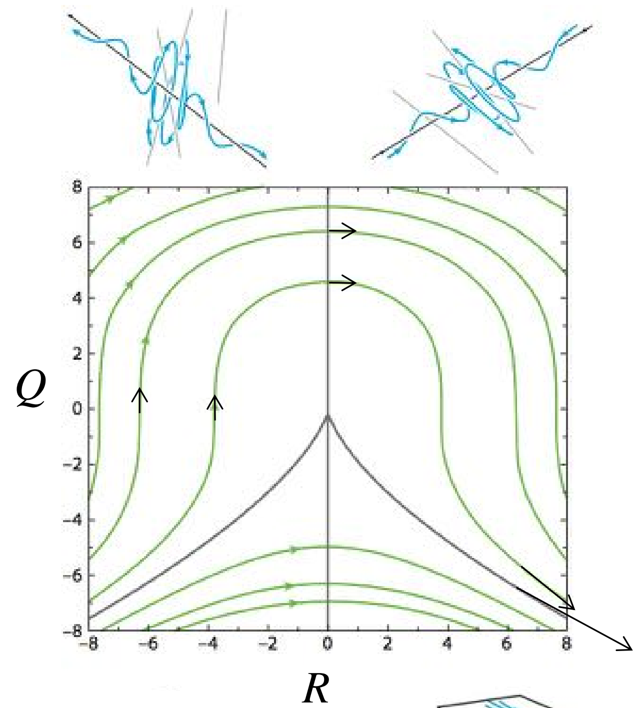
Viellefosse 1982, Cantwell 1992:

$$\frac{dA_{ij}}{dt} = -\left(A_{iq}A_{qj} - \frac{1}{3}A_{mn}A_{nm}\delta_{ij} \right)$$

$$Q = -\frac{1}{2}A_{mn}A_{nm} \qquad R = -\frac{1}{3}A_{mn}A_{np}A_{pm}$$

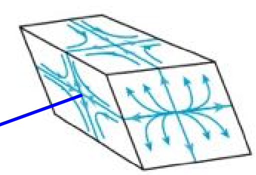
Two very “special” invariants, *R,Q*:

$$\frac{dQ}{dt} = -3R, \quad \frac{dR}{dt} = \frac{2}{3}Q^2,$$



En-route to singylarity, RE reproduces several very realistic trends, e.g.

vorticity becomes aligned with intermediate eigenvector



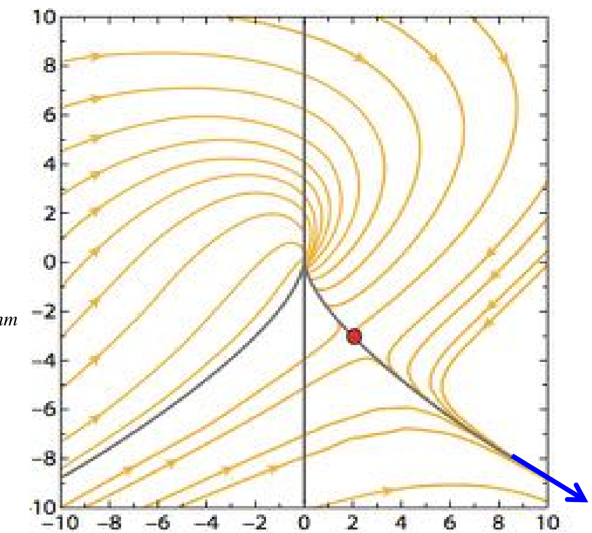
The velocity gradient tensor: how to damp RE?

Restricted Euler Equation with **linear damping: not enough!**

Martin et al. Phys Fluids 1998:

$$\frac{dA_{ij}}{dt} = -\left(A_{iq}A_{qj} - \frac{1}{3}A_{mn}A_{nm}\delta_{ij} \right) - \frac{1}{T}A_{ij}$$

$$A_{ii} = \frac{\partial u_i}{\partial x_i} = 0$$



Still, most initial conditions
have finite-time **divergence** to infinity

Goal: develop **Lagrangian** model for missing physics keeping simplicity of 8 ODEs (or SDE if with stochastic ingredients)

Families of models (review: Annu Rev Fluid Mech, 2011):

$$\frac{dA_{ij}}{dt} = -\left(A_{iq}A_{qj} - \frac{1}{3}A_{mn}A_{nm}\delta_{ij}\right) - \left(\frac{\partial^2 p}{\partial x_i \partial x_j} - \frac{1}{3}\nabla^2 p\delta_{ij}\right) + \nu\frac{\partial^2 A_{ij}}{\partial x_q \partial x_q} + W_{ij}$$

- Restricted Euler Equation (1982 Viellefosse,1992 Cantwell..)
- Linear relaxation (Martin et al.. 1998)
- Specified lognormal and stochastic (Chen, Pope, Girimaji)
- Tetrads.. (Chertkov, Pumir)
- Recent Fluid Deformation closure (Chevillard & CM, 2006)
- Linear combination, with a shell-model approach (Biferale et al.)
- Gaussian and “enhanced” Gaussian (Wilczek & CM, JFM 2014)
- Deformed Gaussian Fields Model (Johnson & CM, JFM 2017)

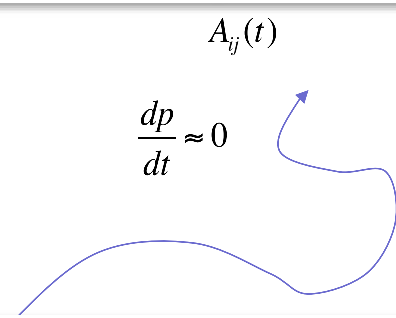
Pressure Hessian model: assume pressure is slowly varying along Lagrangian trajectories over short time-period τ

$$\frac{dp}{dt} \approx 0 \rightarrow$$

$$\nabla\nabla\left(\frac{dp}{dt}\right) = \frac{d\nabla\nabla p}{dt} + \nabla\mathbf{u}\nabla\nabla p + \nabla\mathbf{u}^T\nabla\nabla p$$

$$\rightarrow \frac{d\nabla\nabla p}{dt} \approx -\nabla\mathbf{u}\ \nabla\nabla p - \nabla\mathbf{u}^T\ \nabla\nabla p$$

$$\rightarrow \nabla\nabla p(t) \approx e^{-\tau\nabla\mathbf{u}} \cdot \nabla\nabla p(0) \cdot e^{-\tau\nabla\mathbf{u}^T}$$



Olroyd (upper convective) derivative = zero

Chevillard &CM (2006): Recent Fluid Deformation Map

Also assumed I.C. is an isotropic tensor

(Lagrangian restricted Euler)

$$\langle \nabla\nabla p(0) | \mathbf{A} \rangle = \langle \frac{\partial^2 p(0)}{\partial x_i \partial x_j} | \mathbf{A} \rangle \approx C\,\delta_{ij}$$

Another view: Gaussian closure for conditional pressure Hessian

(Wilczek & CM, JFM, 2015)

$$\langle \nabla\nabla p(0)|\mathbf{A}\rangle = \langle \frac{\partial^2 p(0)}{\partial x_i \partial x_j}|\mathbf{A}\rangle \approx \alpha\left(\boldsymbol{\mathcal{S}}_1^2 - \tfrac{1}{3}\mathrm{Tr}\left(\boldsymbol{\mathcal{S}}_1^2\right)\boldsymbol{I}\right) + \beta\left(\boldsymbol{\mathcal{W}}_1^2 - \tfrac{1}{3}\mathrm{Tr}\left(\boldsymbol{\mathcal{W}}_1^2\right)\boldsymbol{I}\right) + \gamma\left(\boldsymbol{\mathcal{S}}_1\boldsymbol{\mathcal{W}}_1 - \boldsymbol{\mathcal{W}}_1\boldsymbol{\mathcal{S}}_1\right)$$

with

$$\alpha = -\frac{4}{105 f_u''(0)^2}\int \mathrm{d}r\left(8\frac{f_u'^2}{r^3} - 4\frac{f_u' f_u''}{r^2} - 4\frac{f_u' f_u'''}{r} - 4\frac{f_u''^2}{r} + f_u'' f_u'''\right)$$

$$\beta = -\frac{4}{125 f_u''(0)^2}\int \mathrm{d}r\left(16\frac{f_u'^2}{r^3} - 12\frac{f_u' f_u''}{r^2} - 4\frac{f_u' f_u'''}{r} - 4\frac{f_u''^2}{r} - f_u'' f_u'''\right)$$

$$\gamma = \frac{4}{75 f_u''(0)^2}\int \mathrm{d}r\left(4\frac{f_u' f_u''}{r^2} - 4\frac{f_u''^2}{r} - f_u'' f_u'''\right).$$

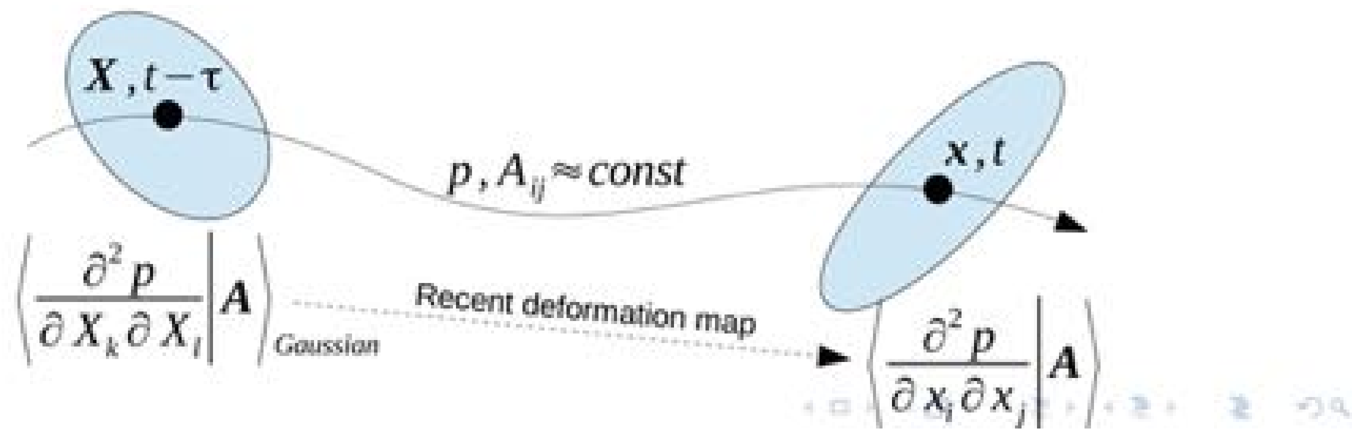
These terms can be significantly simplified by partial integration and identifying product rules, which then leads to

$$\alpha = -\tfrac{2}{7}$$

$$\beta = -\tfrac{2}{5}$$

$$\gamma = \frac{6}{25} + \frac{16}{75 f_u''(0)^2}\int \mathrm{d}r \frac{f_u' f_u'''}{r}.$$

The Recent Deformation of Gaussian Fields (RDGF) closure
(P. Johnson & CM, JFM 2016)



$$\langle \nabla\nabla p(t) | \mathbf{A} \rangle = \langle \frac{\partial^2 p(t)}{\partial x_i \partial x_j} | \mathbf{A} \rangle \approx \exp(-\boldsymbol{\tau}\mathbf{A}) \cdot \langle \nabla\nabla p(0) | \mathbf{A} \rangle \cdot \exp(-\boldsymbol{\tau}\mathbf{A}^{\mathrm{T}})$$

$$\langle \nabla\nabla p(0) | \mathbf{A} \rangle \approx \alpha \left(\boldsymbol{\mathcal{S}}_1^2 - \tfrac{1}{3}\mathrm{Tr}\left(\boldsymbol{\mathcal{S}}_1^2\right) \boldsymbol{I} \right) + \beta \left(\boldsymbol{\mathcal{W}}_1^2 - \tfrac{1}{3}\mathrm{Tr}\left(\boldsymbol{\mathcal{W}}_1^2\right) \boldsymbol{I} \right) + \gamma \left(\boldsymbol{\mathcal{S}}_1 \boldsymbol{\mathcal{W}}_1 - \boldsymbol{\mathcal{W}}_1 \boldsymbol{\mathcal{S}}_1 \right)$$

The Recently-Deformed Gaussian Fields (RDGF) Model

- Langevin equation, $dA_{ij} =$

$$\left[-\left((\mathbf{A}^2)_{ij} - \frac{C_{ij}^{-1}}{C_{kk}^{-1}}\mathrm{tr}\,(\mathbf{A}^2)\right) - \left(G_{ij} - \frac{C_{ij}^{-1}}{C_{kk}^{-1}}\mathrm{tr}\,(\mathbf{G})\right) + V_{ij}\right] dt + dF_{ij}$$

where

$$G_{ij} = D_{mi}^{-1}\left[-\tfrac{2}{7}(\mathbf{S}^2)_{ij}^{(d)} - \tfrac{2}{5}(\mathbf{\Omega}^2)_{ij}^{(d)} + \tfrac{86}{1365}\left(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}\right)\right] D_{nj}^{-1},$$

$$V_{ij} = \frac{-7}{10\sqrt{15}}\frac{C_{kk}}{3}\tau_K^{-1}\left(T_{ij}C_{kk}^{-1} + 2T_{ik}B_{kj}^{-1} - \frac{4}{21}B_{ik}^{-1}S_{kj} - \frac{2}{21}B_{k\ell}^{-1}S_{k\ell}\delta_{ij}\right)$$

$$S_{ij} = \frac{(A_{ij} + A_{ji})}{2}, \qquad \Omega_{ij} = \frac{(A_{ij} - A_{ji})}{2}, \qquad T_{ij} = \frac{23}{105}A_{ij} + \frac{2}{105}A_{ji}.$$

- Recent deformation tensor:

$$D_{ij}^{-1} = \exp\left(-A_{ij}\tau\right), \qquad C_{ij}^{-1} = D_{ki}^{-1}D_{kj}^{-1}, \qquad B_{ij}^{-1} = D_{ik}^{-1}D_{jk}^{-1}.$$

$$\tau = 0.13\ \tau_K$$

0.13 chosen such that $\left\langle 2S_{ij}S_{ij}\right\rangle^{-1/2} = \tau_K$

2 more parameters for **forcing** strength D_a, D_s: fixed from 2 required Betchov symmetries:

$$\langle Q\rangle = 0$$

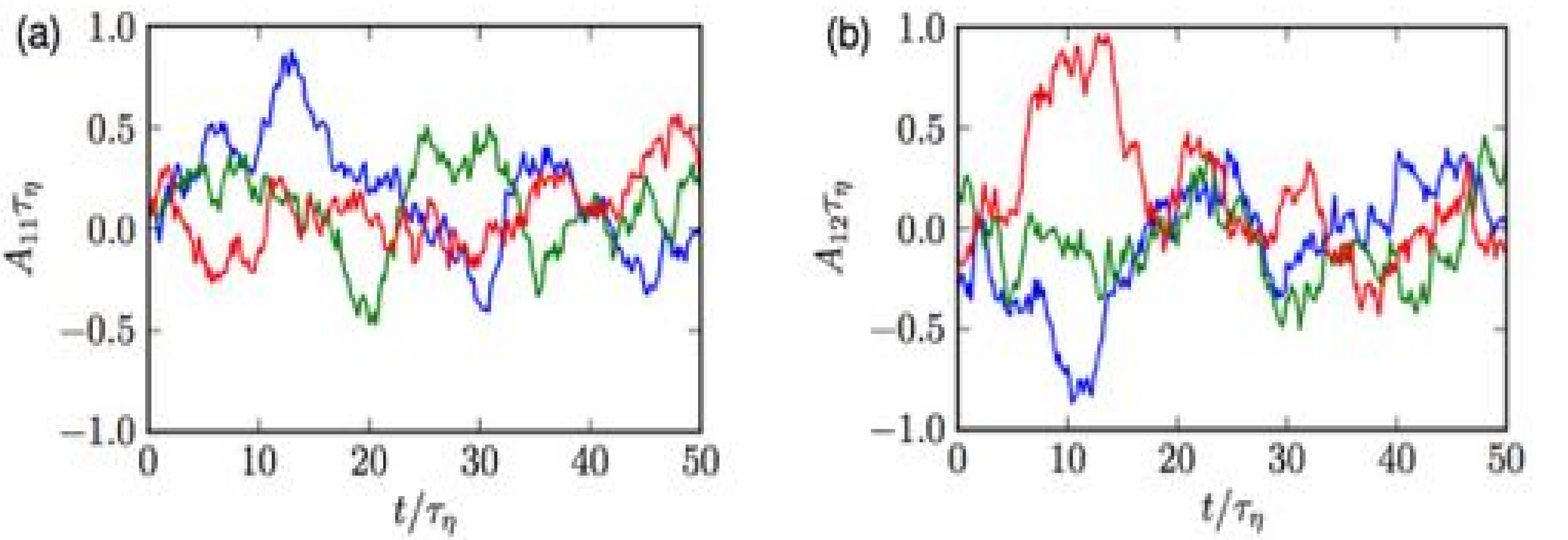
$$\langle R\rangle = 0$$

The Recently-Deformed Gaussian Fields (RDGF) Model

$$dA_{ij} = \left[-\left(A_{ik}A_{kj} - \frac{1}{3} A_{pq}A_{pq}\delta_{ij} \right) + h_{ij}(\mathbf{A}, \tau_K) \right] dt + dF_{ij}(\tau_K)$$

- 9 x SDE solver: 2nd order predictor-corrector method,
- dt/τ_K = 0.04, 0.02, 0.01 for comparisons & checks
- 2^{16} trajectories advanced for 1000 τ_K.

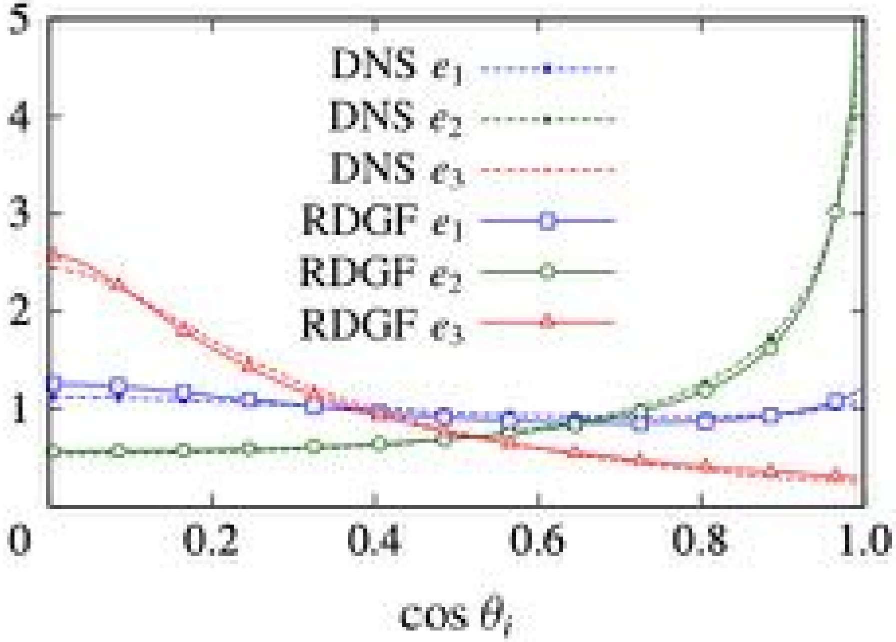
✓ Lagrangian time series (3 sample realizations)

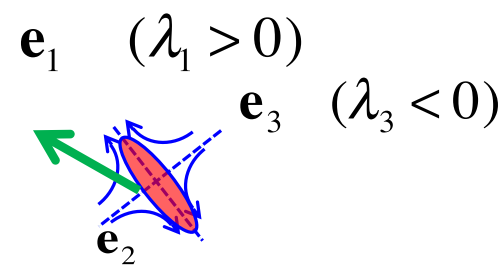


The Recently-Deformed Gaussian Fields (RDGF) Model

$$dA_{ij} = \left[-\left(A_{ik}A_{kj} - \frac{1}{3} A_{pq}A_{pq}\delta_{ij} \right) + h_{ij}(\mathbf{A}, \tau_K) \right] dt + dF_{ij}(\tau_K)$$

✓ Alignments

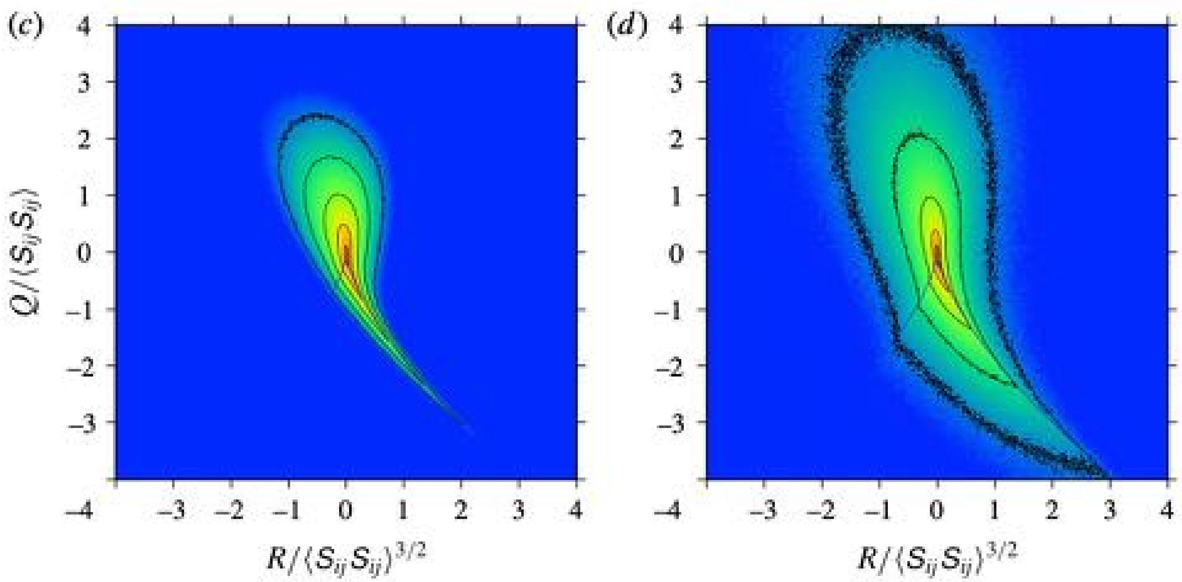




The Recently-Deformed Gaussian Fields (RDGF) Model

$$dA_{ij} = \left[-\left(A_{ik}A_{kj} - \frac{1}{3}A_{pq}A_{pq}\delta_{ij}\right) + h_{ij}(\mathbf{A},\tau_K)\right]dt + dF_{ij}(\tau_K)$$

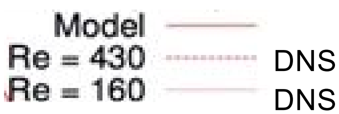
✓ tear-drop joint PDF

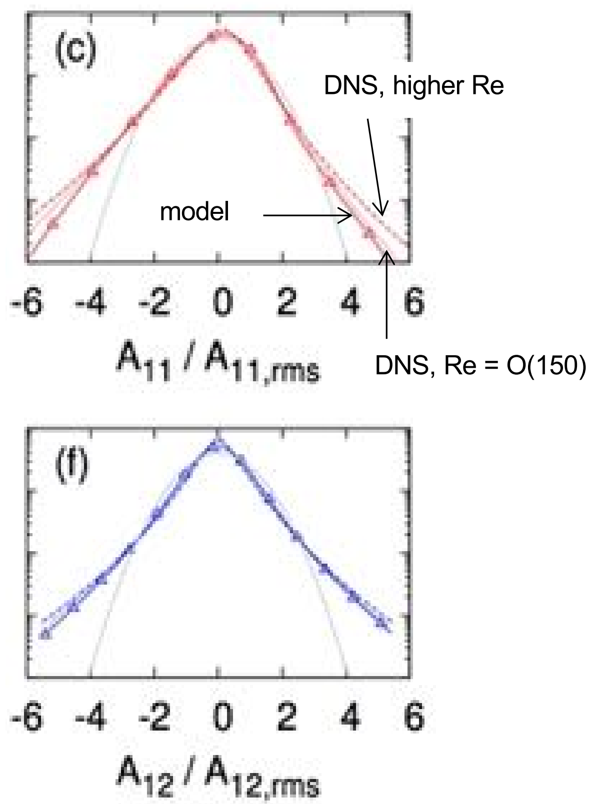


The Recently-Deformed Gaussian Fields (RDGF) Model

$$dA_{ij} = \left[-\left(A_{ik}A_{kj} - \frac{1}{3} A_{pq}A_{pq}\delta_{ij} \right) + h_{ij}(\mathbf{A}, \tau_K) \right] dt + dF_{ij}(\tau_K)$$

✓ gradient PDFs





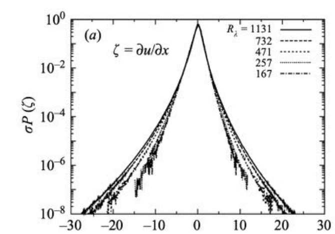
Summary:

✓ Skewness
✓ Alignments
✓ Joint PDF (Q,R)

However:

➢ Behavior like at a **fixed Re** ($Re_\lambda \sim 60$ -120).

➢ Can't describe continued "widening" of tails in PDF at large Re



A multiple time-scale model

(Johnson & CM, Phys Rev Fluids, 2017, Luo, Shi & CM, Phys. Rev. Fluids 7, 2022):

$$dA_{ij} = \left[-\left(A_{ik}A_{kj} - \frac{1}{3}A_{pq}A_{pq}\delta_{ij} \right) + h_{ij}(\mathbf{A},\tau_K) \right] dt + dF_{ij}(\tau_K)$$

Instead of constant τ_K (constant $\nu/\langle\varepsilon\rangle$), "variable" background $\nu/\varepsilon(t)$:

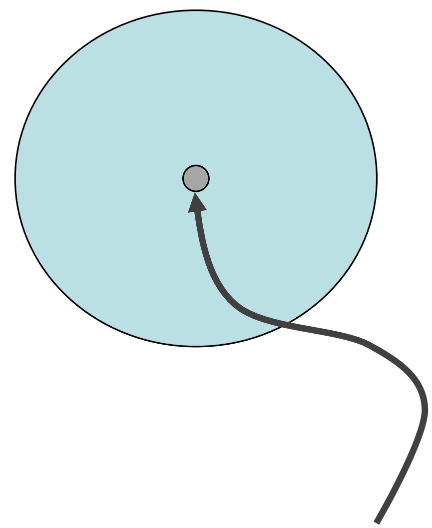
$$dA_{ij}^{(n)} = \left[-\underbrace{\left(A_{ik}^{(n)}A_{kj}^{(n)} - \frac{1}{3}A_{pq}^{(n)}A_{pq}^{(n)}\delta_{ij} \right)}_{\text{local-in-scale interactions: from N-S}} + \underbrace{h_{ij}^{n}(\mathbf{A},\tau_n) - \frac{\dot{\tau}_n}{\tau_n}A_{ij} \Big] dt + dF_{ij}(\tau_n)}_{\text{non-local interactions}}$$

local-in-scale interactions: from N-S

non-local interactions

$$\tau_1 = \beta^{N-1}\tau_K$$

$$\tau_n(t) = \frac{1}{\beta}\left(2S_{ij}^{(n-1)}S_{ij}^{(n-1)}\right)^{-1/2}, \quad n = 2,3,\ldots\ N$$

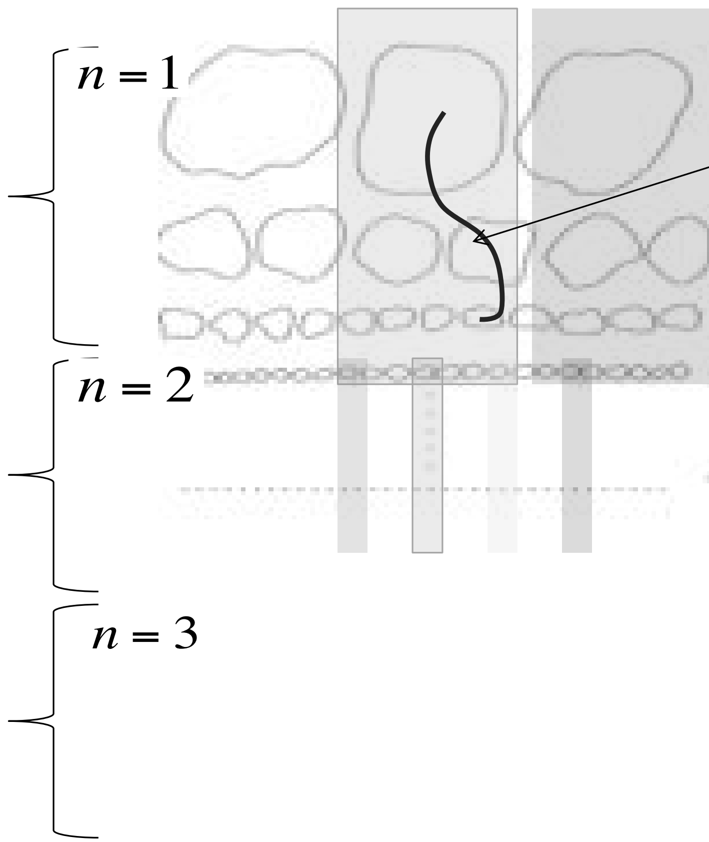


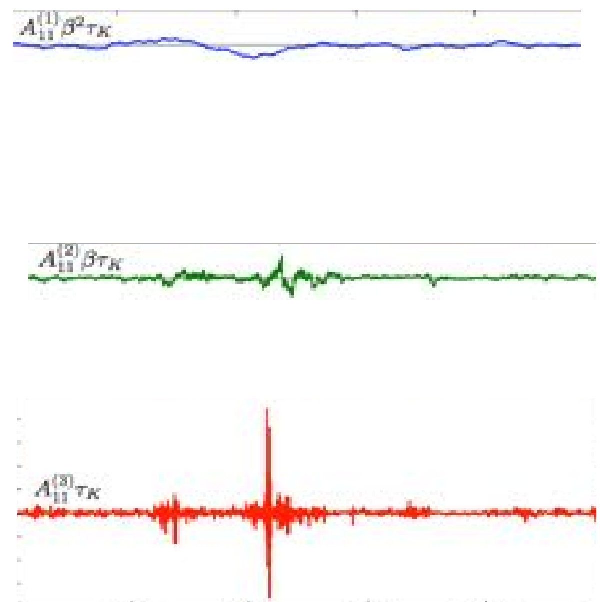
- Free parameter – timescale ratio (fitted) $\beta=10$
- levels: scale-separation needed in model
- not a "shell cascade model" (scale ratio = 2)

"The combined local and nonlocal cascade"

Local dynamics from N-S

$$\frac{d\mathbf{A}}{dt} = -\mathbf{A}^2 + ..\mathbf{F}$$





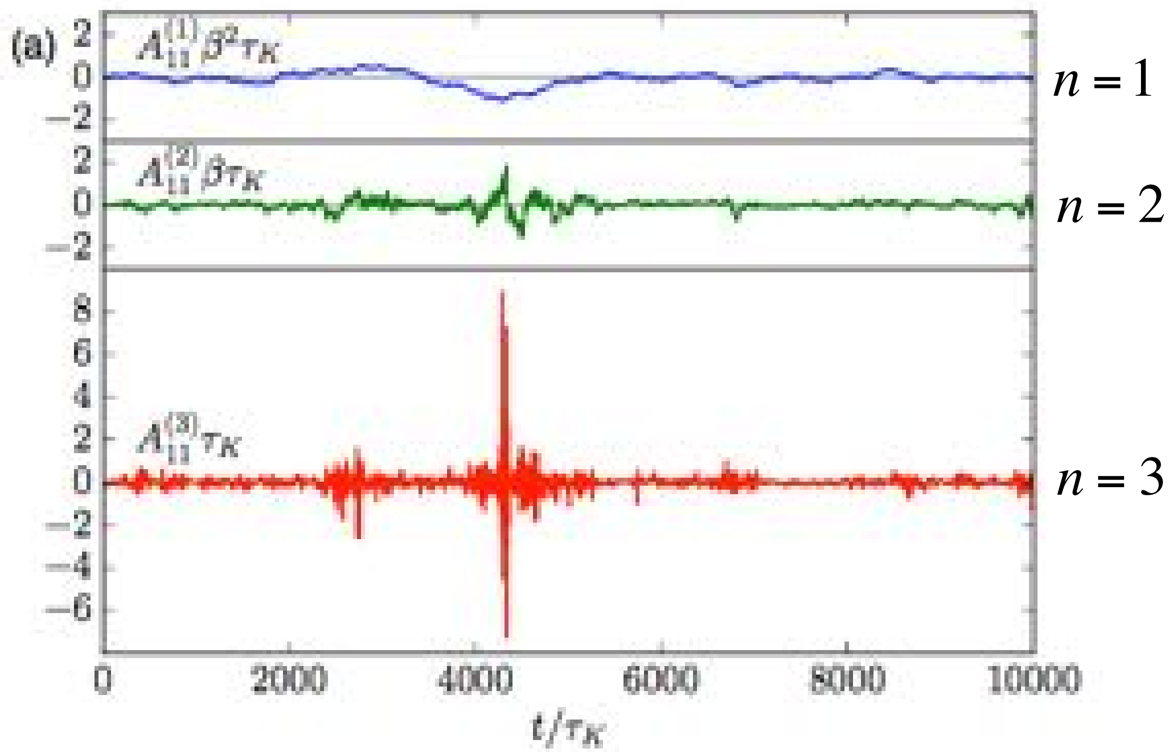
Nonlocal dynamics, modulated time-scales, scale separation

Results: Time scale modulation

β=10

$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \tau_n) - \frac{\dot{\tau}_n}{\tau_n} A_{ij} \right] dt + dF_{ij}(\tau_n)$$

$$\tau_n(t) = \frac{1}{\beta}\left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)} \right)^{-1/2}$$

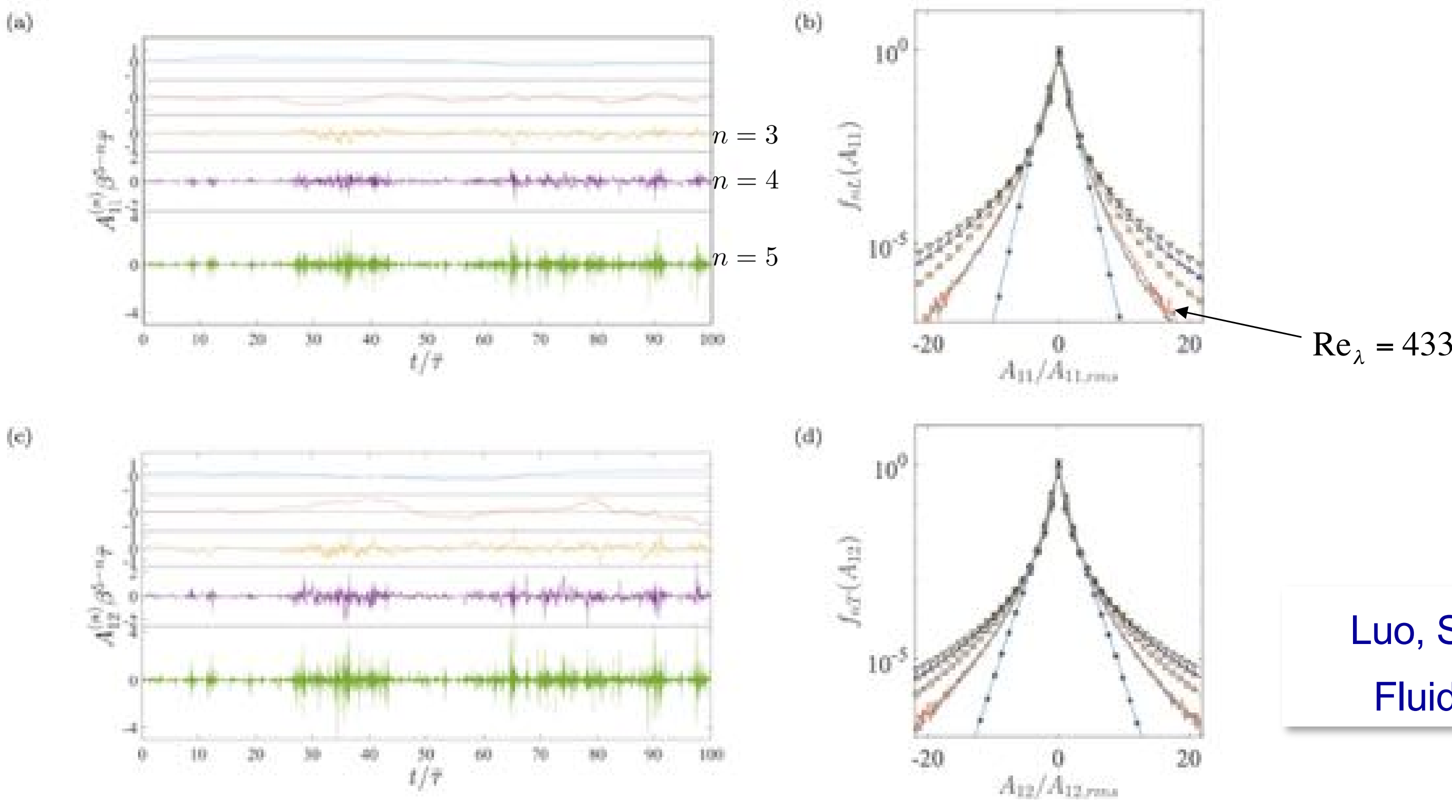


- N x 9 x SDE solver

Results: Widening tails in gradient PDF

β=10

$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \tau_n) - \frac{\dot{\tau}_n}{\tau_n} A_{ij} \right] dt + dF_{ij}(\tau_n) \qquad \tau_n(t) = \frac{1}{\beta} \left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)} \right)^{-1/2}$$

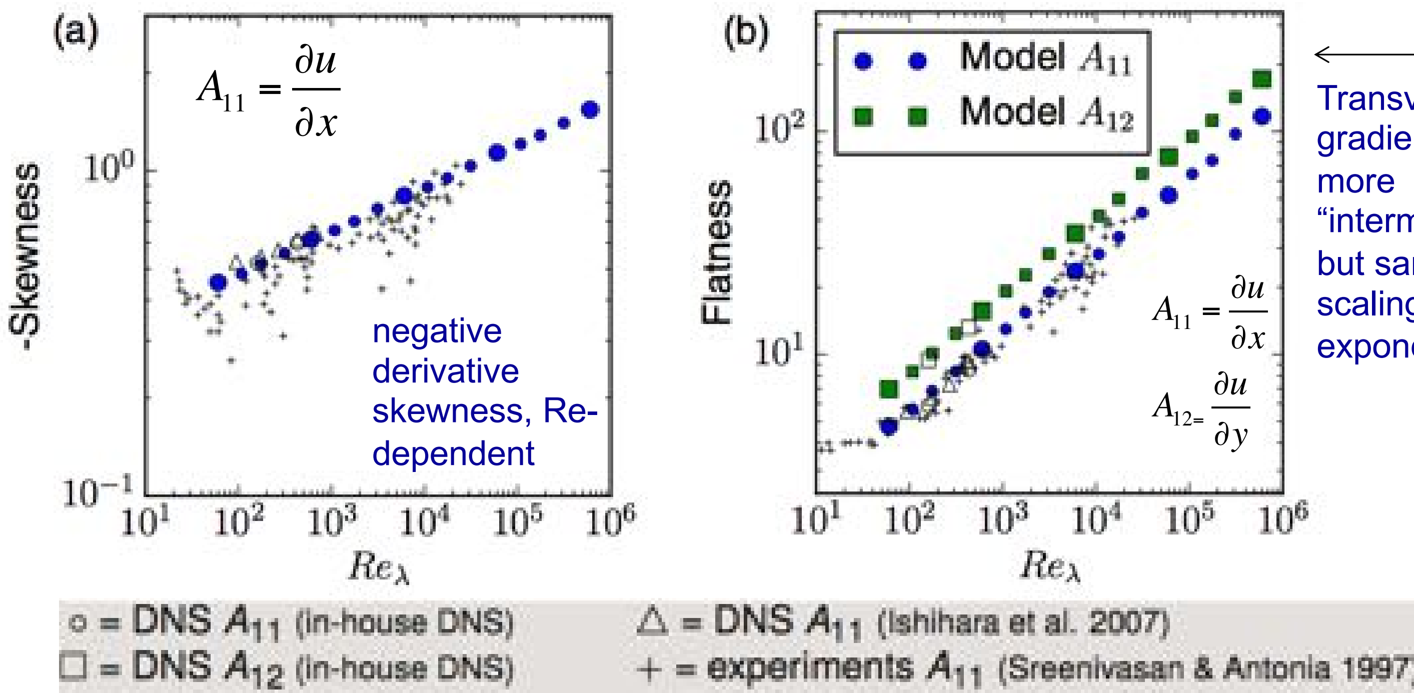


Luo, Shi & CM, Phys Rev. Fluids **7**, 04609 (2022)

Reynolds number scaling of –S and F

Choosing β =10

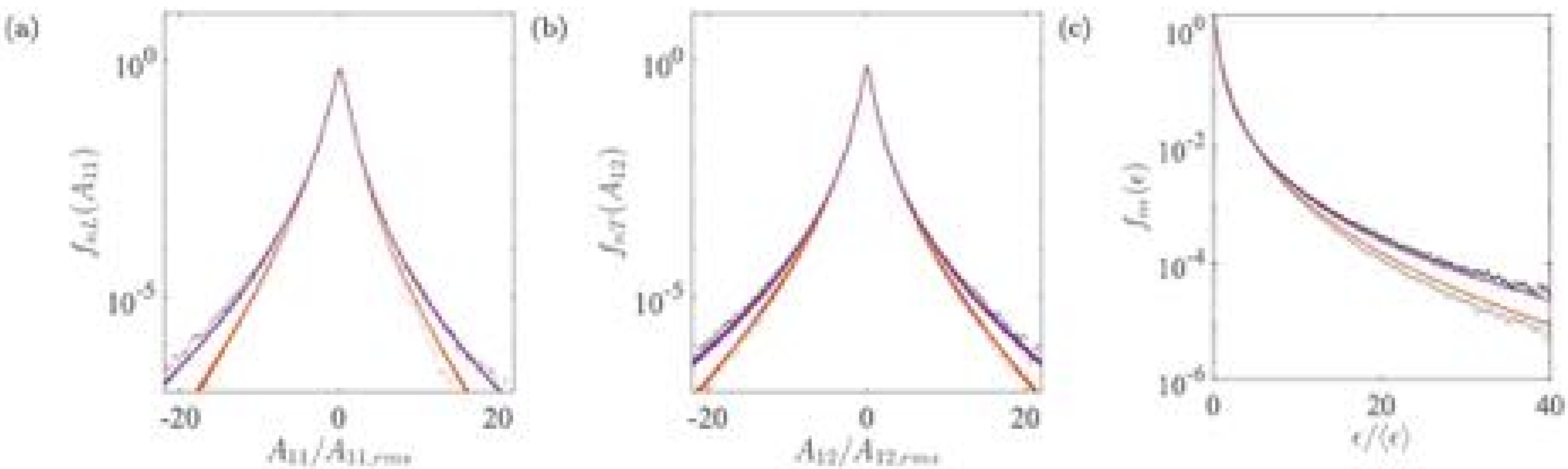
$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \tau_n) - \frac{\dot{\tau}_n}{\tau_n} A_{ij} \right] dt + dF_{ij}(\tau_n) \qquad \tau_n(t) = \frac{1}{\beta}\left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)}\right)^{-1/2}$$



← Transverse gradient more "intermittent" but same scaling exponent

Detailed comparison with DNS at 2 Reynolds numbers:

Luo, Shi & CM, Phys Rev. Fluids **7**, 04609 (2022)



R_λ=430 (dots: JHTDB 1024^3 DNS data. Line = model, at n=1.85)

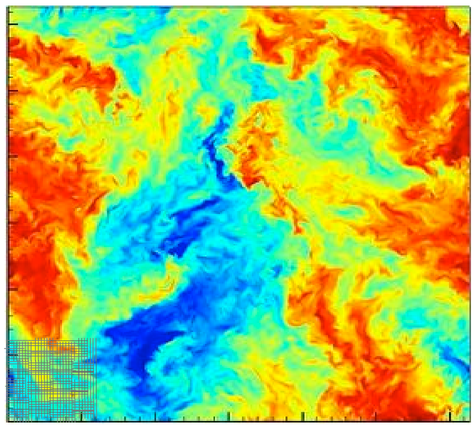
R_λ=1,300 (JHTDB, data from PK Yeung, Georgia Tech, 8193^3 DNS. Line = model, at n=2.33)

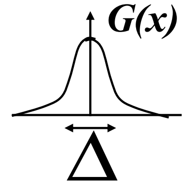
Engineering application: Large-Eddy-Simulation (LES)

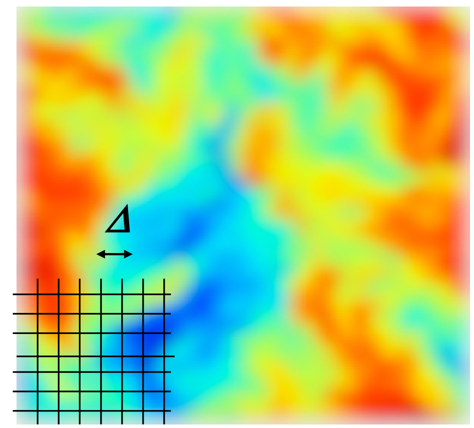
Coarse-graining (local homogeneization) for more affordable simulations

$u_1(x, y, z_0, t_0)$ $\tilde{u}_1(x, y, z_0, t_0)$

$4x10^9$ d.o.f.







10^5 d.o.f.

DNS **LES**

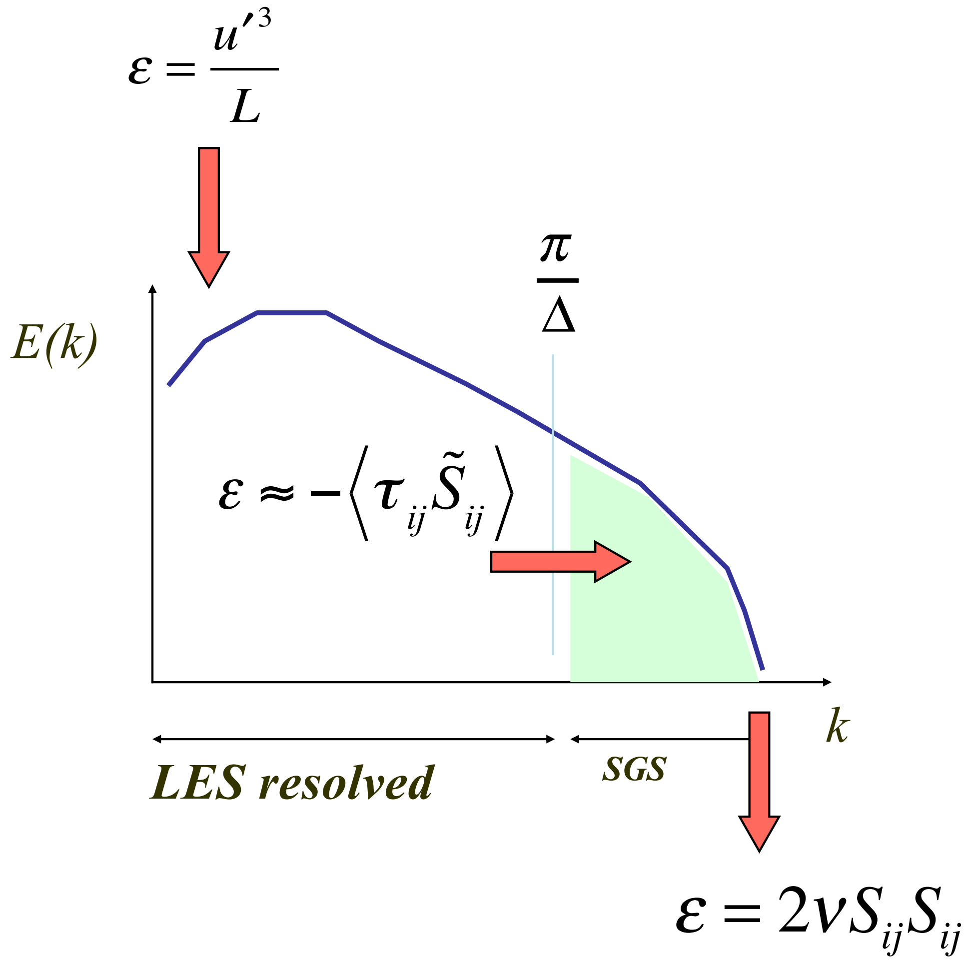
$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j$$

$$\frac{\partial u_j}{\partial x_j} = 0$$

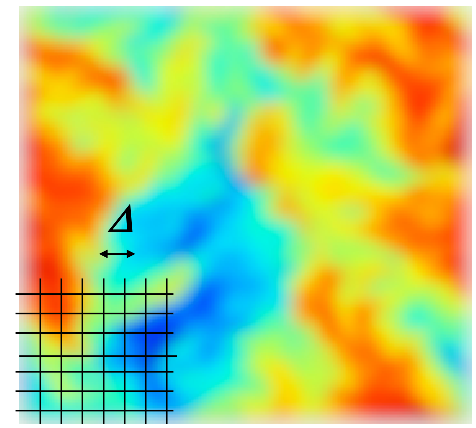
$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k} \tau_{jk}$$

SGS stress tensor: $\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$

Dissipation of kinetic energy in LES



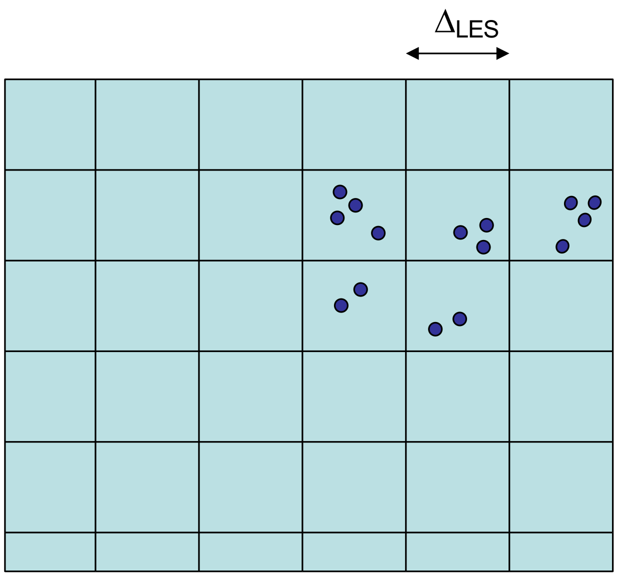
$\tilde{u}_1(x,y,z_0,t_0)$

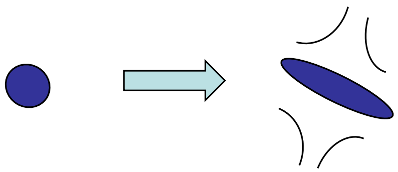


$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

What about LES dealing with "small-scale dominated physics" ?

Large Eddy Simulation (LES) including predictions of small droplet deformation statistics & geometry (consider only $d_p < \eta_K$)

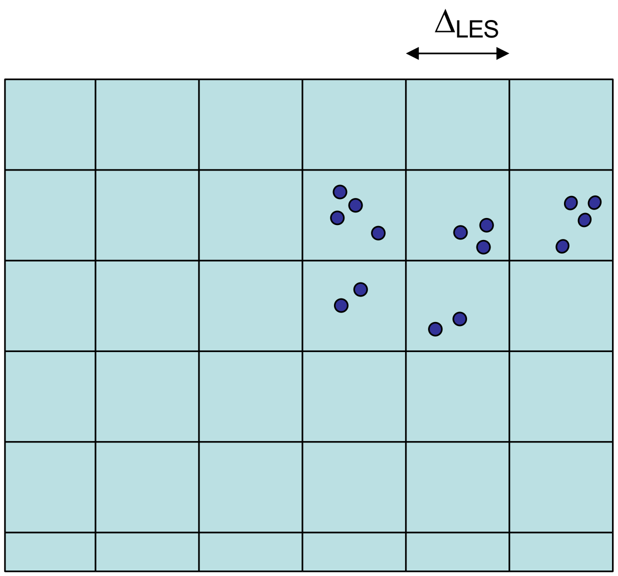


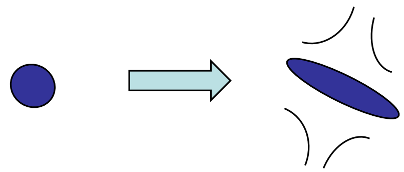


$$\left|\frac{\partial u_i}{\partial x_j}\right| \sim \frac{1}{\tau_K} \sim \sqrt{\frac{\varepsilon}{\nu}}$$

What about LES dealing with "small-scale dominated physics" ?

Large Eddy Simulation (LES) including predictions of small droplet deformation statistics & geometry (consider only $d_p < \eta_K$)





$$\left|\frac{\partial u_i}{\partial x_j}\right| \sim \frac{1}{\tau_K} \sim \sqrt{\frac{\varepsilon}{\nu}}$$

Traditional approach to model strain-rates responsible for droplet deformations:

$$\varepsilon \sim \left\langle \varepsilon \right\rangle_{LES} \sim \nu_{sgs} \left| \mathbf{S} \right|^3 (\mathbf{x}, t)$$

Lacks intermittency and "directional information" (geometry)

Applications in LES:

Small scale intermittency of turbulence generates extreme events, difficult to characterize based on local averages

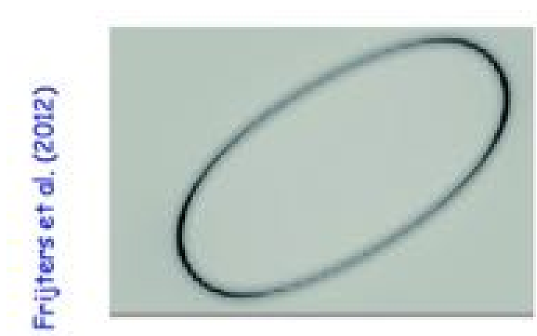
Small scale intermittency affects:

Johnson & CM (JFM, 2018)

- **droplet deformations in turbulence**
- flame structure (quenching) in combustion
- micro-organism motility and nutrient uptake
- polymer stretching–relaxation dynamics

Here, focus on small (sub-Kolmogorov) droplets

- The sub-Kolmogorov size of the droplets implies that: only viscous drag induced by the shear can distort the droplet shape (no inertial forces)
- The distortion is resisted by the surface tension that tends to restore the spherical shape
- Initial study by Taylor (1932) with drops in a laminar flow

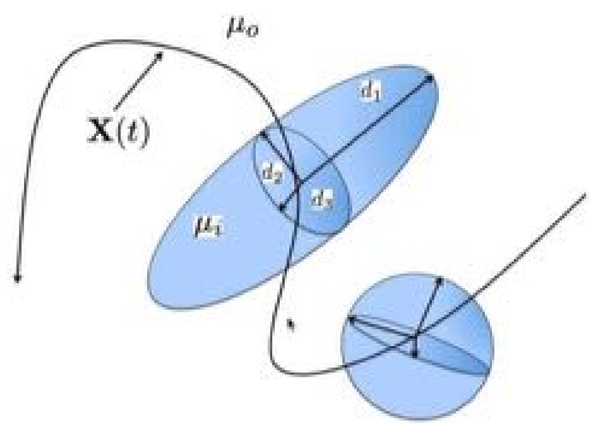


Equilibrium configuration **or** **droplet break-up**

See Kolmogorov (1949) and Hinze (1955), Lasheras et al. (2002),

Lagrangian model for sub-Kolmogorov droplets

- We model fluid with small but finite viscosity using standard DNS and LES
- Droplets of immiscible viscous fluid
- Surface tension at the interface
- Droplet shape approximated as triaxial ellipsoids (OK for initial, small, deformation)



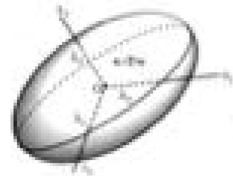
The Maffettone Minale model:

Maffettone, Minale, J. Non-Newtonian Fluid Mech., 78 (1998) 227–241

ELLIPSOIDAL DROP DETERMINED BY A
SYMMETRIC POSITIVE DEFINED SECOND RANK TENSOR

M's eigenvalues: squared semiaxis of the ellipsoid $M_{ij}(X(t), t) = \rho_d \int_V (r_i - X_i(t))(r_j - X_j(t))\, dV,$

$$G_t = \left\langle \left(\frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle^{1/2}.$$



$$Ca = \frac{\mu_o R G_t}{\Lambda} = \tau G_t.$$

$$\frac{dM_{ij}}{dt} = \Omega_{ik}M_{kj} - M_{ik}\Omega_{kj} + f_2(S_{ik}M_{kj} + M_{ik}S_{kj}) - \frac{f_1}{\tau_c}(M_{ij} - g(II_M, III_M)\delta_{ij}).$$

$$\frac{dM_{ij}}{dt'} = [f_2(S'_{ik}M_{kj} + M_{ik}S'_{kj}) + \Omega'_{ik}M_{kj} - M_{ik}\Omega'_{kj}] - \frac{f_1}{Ca}\left(M_{ij} - 3\frac{III_M}{II_M}\delta_{ij}\right).$$

$$A_{ij} = \frac{\partial u_i}{\partial x_j} \qquad S_{ij} = \frac{1}{2}\left(A_{ij} + A_{ji}\right) \qquad \Omega_{ij} = \frac{1}{2}\left(A_{ij} - A_{ji}\right)$$

Consider for now

$\lambda = 1 \rightarrow$

$f_1 = 0.457, \quad f_2 = 1$

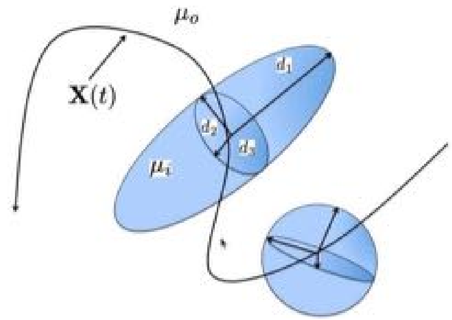
$$f_1 = \frac{40(\lambda+1)}{(2\lambda+3)(19\lambda+16)}, \quad f_2 = \frac{5}{2\lambda+3}. \qquad \lambda = \mu_1/\mu_2$$

G.I. Taylor,
Proc. R. Soc. A 138 (1932) 41–48.

Evolution of small droplets, small deformation (ellipsoidal):

We require time evolution of unfiltered turbulent velocity gradient tensor along particle trajectories

$$A_{ij} = \frac{\partial u_i}{\partial x_j}$$



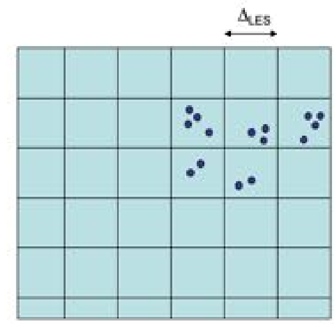
$$S_{ij} = \frac{1}{2}\left(A_{ij} + A_{ji}\right)$$

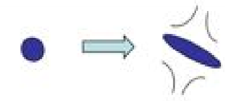
$$\Omega_{ij} = \frac{1}{2}\left(A_{ij} - A_{ji}\right)$$

$$\Rightarrow \quad M_{ij}(t)$$

for “representative particle method” (RPM) in a mixed Lagrangian Eulerian LES method

- We want to do LES of turbulent flow at high Re
- We want to keep track of deformation and orientation of “droplet microstructure”, even if unresolved in LES
- We will use hybrid Euler-Lagrangian approach
- We will follow droplets (Lagrangian) using M&M equation





$$\left|\frac{\partial u_i}{\partial x_j}\right| \sim \frac{1}{\tau_K} \sim \sqrt{\frac{\varepsilon}{\nu}}$$

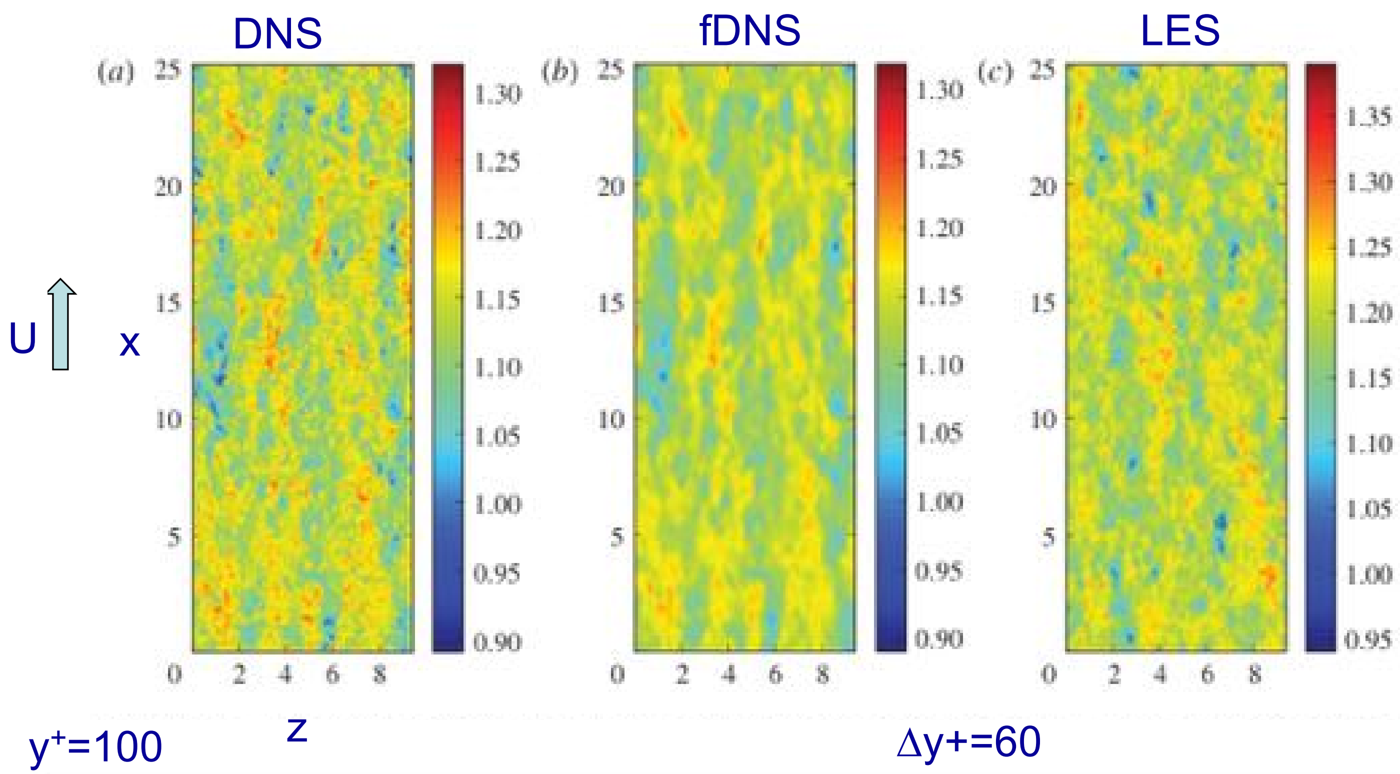
$$\frac{dM_{ij}}{dt} = \Omega_{ik}M_{kj} - M_{ik}\Omega_{kj} + f_2(S_{ik}M_{kj} + M_{ik}S_{kj}) - \frac{f_1}{\tau_e}(M_{ij} - g(II_M, III_M)\delta_{ij}).$$

- But for this we need “unfiltered” velocity gradient tensor history along droplet trajectories, unavailable from LES

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

- We use our SGS modeling of Largangian history of gradients
- **application to LES of channel flow**

Application to drops in channel (1-way coupling):



Case	$\Delta_x/dx = \Delta_z/dz$	$N_x \times N_y \times N_z$	dx^+, dy_c^+, dz^+	N_t	dt_{sim}	dt_{DB}	U_{bulk}	$\tau_{\eta,bulk}$
DNS	1	2048 × 512 × 1536	12.3, 6.2, 6.1	4000	0.0013	0.0065	1.00	0.141
fDNS	32	128 × 32 × 96	196, 98.5, 98.2	250	(N/A)	0.104	1.00	(N/A)
LES	16	128 × 32 × 96	196, 98.5, 98.2	250	0.0104	0.104	1.04	0.139

Applications: channel flow, DNS & LES

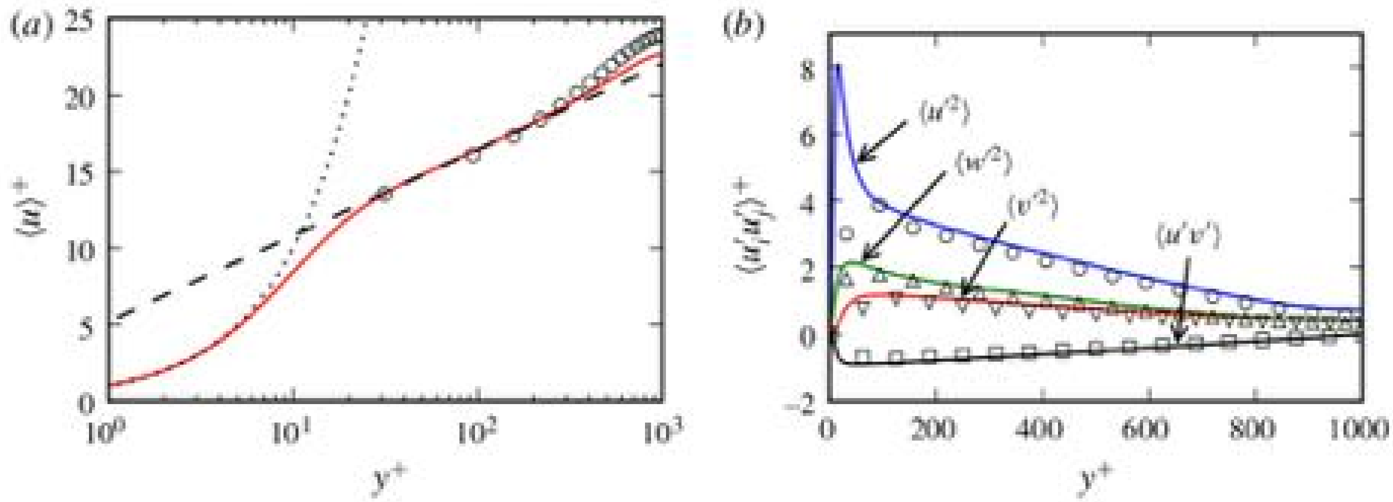


FIGURE 3. (Colour online) Mean velocity (*a*) and Reynolds stress tensor components (*b*) profiles for DNS (continuous lines) and LES (symbols). In (*a*), the well-established linear, $\langle u^+\rangle = y^+$ (dotted line), and log-law, $\langle u^+\rangle = \ln y^+/0.41 + 5.2$ (dashed line), profiles are also shown for reference. For the Reynolds stress components in (*b*): $\langle u'^2\rangle$ (◯), $\langle v'^2\rangle$ (▽), $\langle w'^2\rangle$ (△), and $\langle u'v'\rangle$ (□).

LES: Lagrangian scale dependent dynamic SGS model
Algebraic equilibrium wall model

LES + "representative particles":

- Langevin equation, $dA_{ij} =$

$$\left[-\left((\mathbf{A}^2)_{ij} - \frac{C_{ij}^{-1}}{C_{kk}^{-1}}\mathrm{tr}\left(\mathbf{A}^2\right)\right) - \left(G_{ij} - \frac{C_{ij}^{-1}}{C_{kk}^{-1}}\mathrm{tr}\left(\mathbf{G}\right)\right) + V_{ij}\right] dt + dF_{ij}$$

so far: 1 level RDGF model

- 9 ODEs for droplet conformation tensor

$$\frac{\mathrm{d}\mathcal{D}_{ij}}{\mathrm{d}t} = \Omega_{ik}\mathcal{D}_{kj} + f_2(\hat{\mu}) S_{ik}\mathcal{D}_{kj} - \frac{f_1(\hat{\mu})}{2\tau_d}\left(\mathcal{D}_{ij} - \frac{3\mathrm{III}}{\mathrm{II}}\mathcal{D}_{ji}^{-1}\right)$$

Better conditioned matrix ODE

Equivalent to Maffetone – Minale model for $\boldsymbol{M} = \mathcal{D}\mathcal{D}^{\mathrm{T}}$

information about the semi-axes can be extracted from the deformation using a singular value decomposition, $\mathcal{D} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{T}}$, where $\boldsymbol{U}$ is a unitary matrix comprised of the singular vectors indicating the semi-axis directions and $\boldsymbol{\Sigma}$ is a diagonal matrix whose elements are the associate singular values $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$, i.e., the length of the semi-axes (Greene & Kim 1987). The total extent of deformation away from a spherical droplet is commonly measured using a deformation parameter, $D = (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$.

LES and initial conditions: sprinkle droplets on channel center-plane

Stochastic Dispersion Model: (Fede et al. 2006)

Tracer particle tracking:

$$\frac{dX_i}{dt} = u_i(X_i(t), t) = \widetilde{u}_i + u_i'.$$

$$du_i' = \frac{\partial \tau_{ij}}{\partial x_j} dt - u_j' \frac{\partial \widetilde{u}_i}{\partial x_j} dt - \left(\frac{1}{2} + \frac{3}{4} C_0 \right) \frac{\Pi}{k_r} u_i' dt + \sqrt{C_0 \Pi} dW_i,$$

where

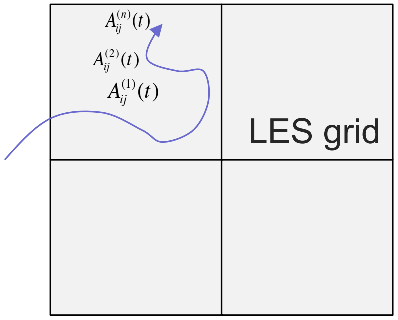
$$\Pi = (C_s \Delta)^2 |\widetilde{S}|^3, \qquad k_r = 2 C_y \Delta^2 |\widetilde{S}|^2,$$

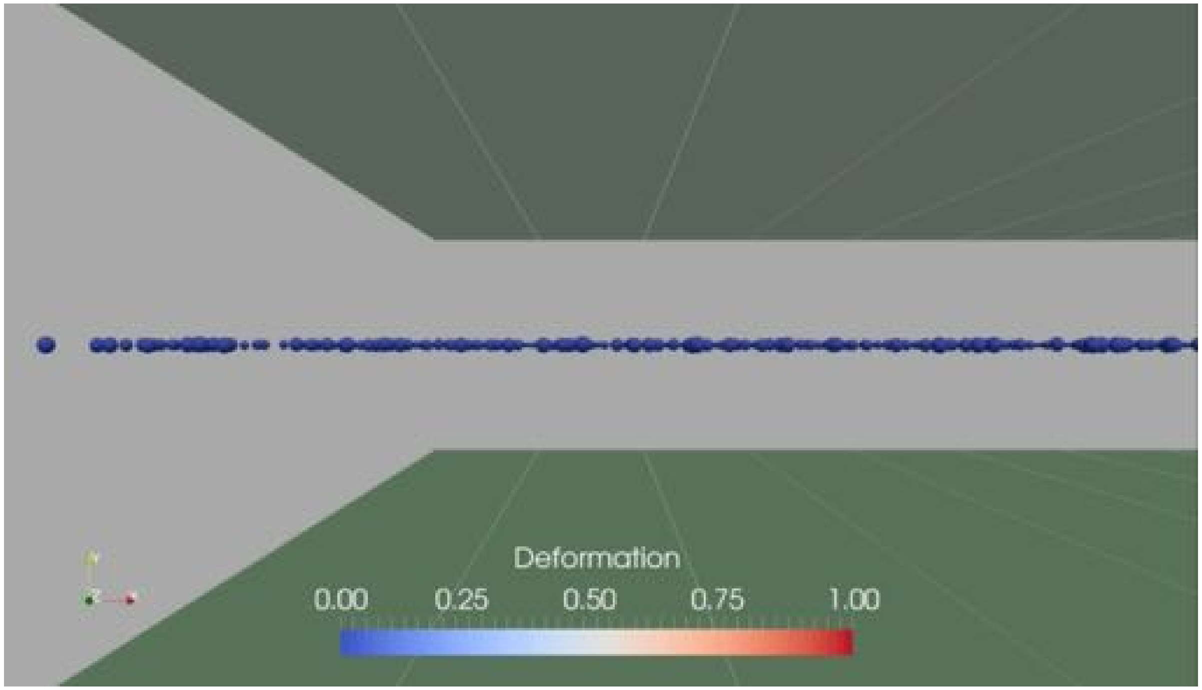
$$\frac{\Pi}{k_r} = C' |\widetilde{S}|, \qquad C' = \frac{C_s^2}{2C_y} = \frac{1}{\mathcal{S}_s} \left(\frac{2}{3C_k} \right)^{3/2} \approx 0.21,$$

derived using spectral cutoff theory (following Lilly 1967).

LES of deforming droplets in channel flow:

- DNS + (9+3) x N x Lagr. SDE for M&M model
- LES + (18+3) x N x Lagr. SDEs)
- Ensemble of N=172,000
- See Johnson & CM (JFM 2018)





Sample signals:

$$D = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3},$$

DNS

LES

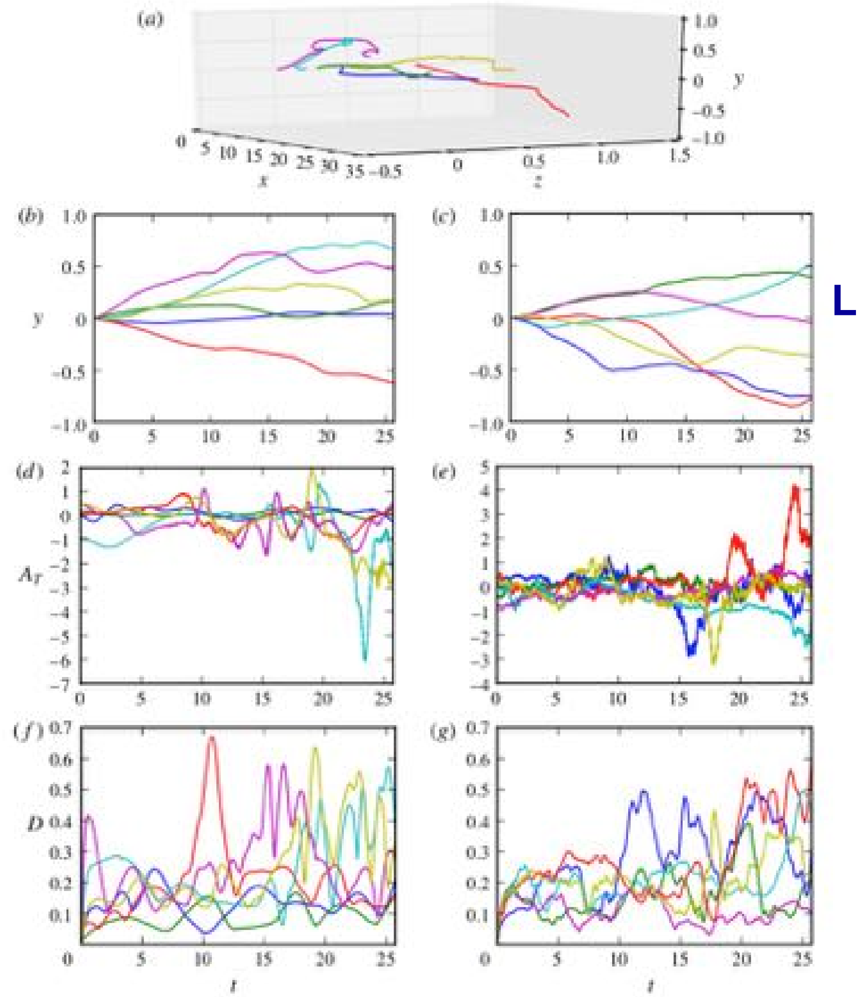


FIGURE 5. (Colour online) Sample time histories of particle locations in three dimensions (*a*), the wall-normal location (*b*,*c*), the transverse velocity gradient (*d*,*e*) and deformation magnitude parameter D (*f*,*g*) from the DNS (*b*,*d*,*f*) and LES-RDGF (*c*,*e*,*g*) results for 6 independent Lagrangian trajectories. The droplets shown are have $Ca = 1.0$ and $\hat{\mu} = 1.0$.

Time evolution of concentration of particles:

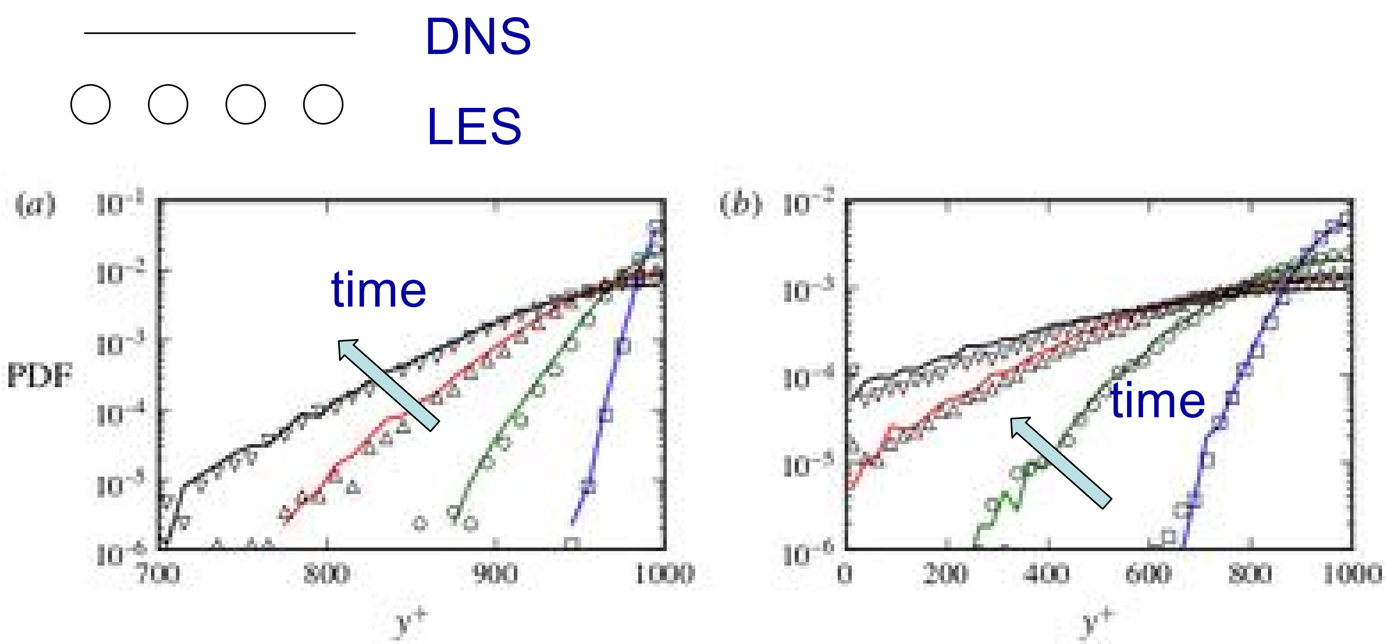
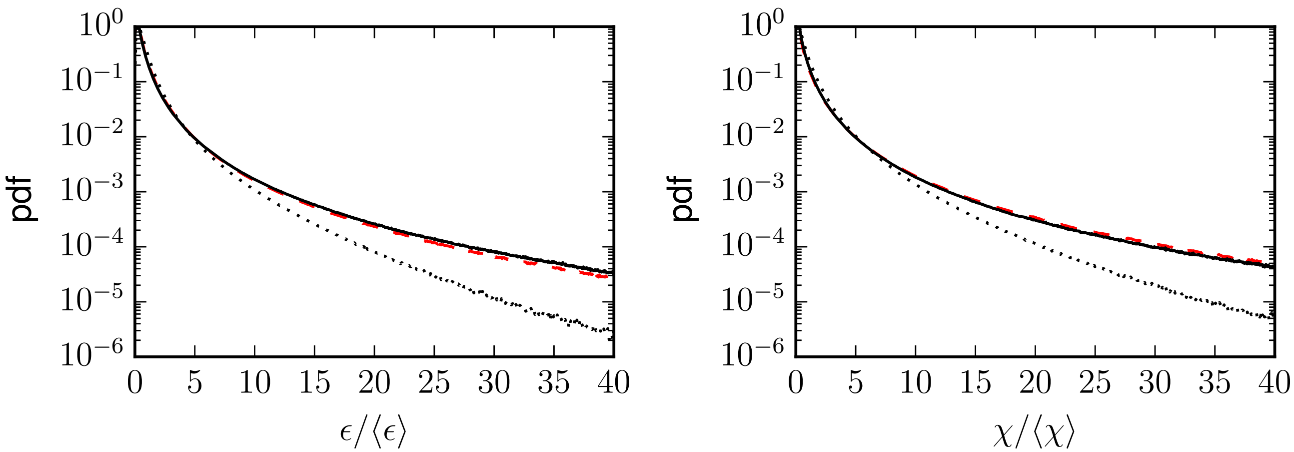


FIGURE 6. (Colour online) Distribution of particles at different times after being released from the centreline at $t = 0$. Continuous lines show the distributions from DNS while symbols show the results from LES with stochastic model for subgrid velocity. (*a*) $t = 0.26$ (□), $t = 0.78$ (◯), $t = 1.56$ (△), $t = 2.34$ (▽). (*b*) $t = 2.6$ (□), $t = 7.8$ (◯), $t = 15.6$ (△), $t = 23.4$ (▽).

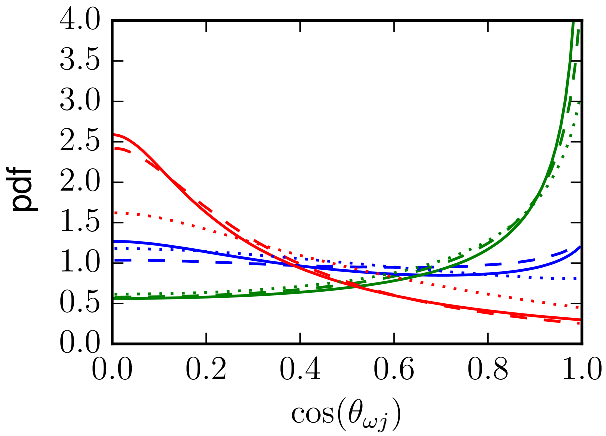
Dissipation and Enstrophy Statistics

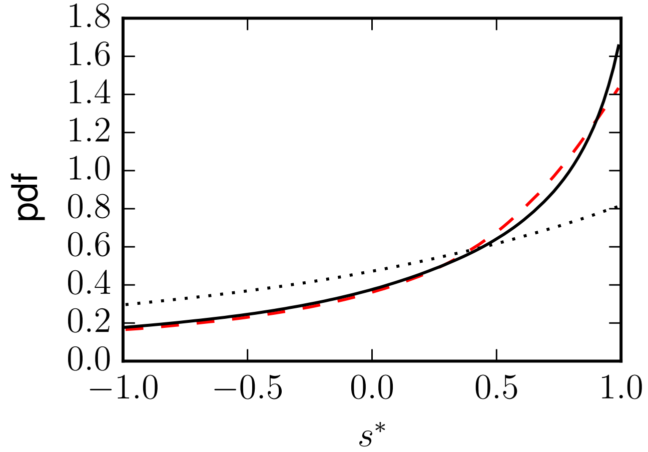
- ensemble histogram from all data $t \in [0, L_t]$
- includes $\hat{\epsilon}$ fluctuations from LES and internal fluctuations from RDGF



Dashed line: DNS
Solid line: LES-RDGF
Dotted line: LES, no model

Velocity Gradient Geometry Statistics





$$\cos(\theta_{\omega j}) = \widehat{\omega} \cdot \widehat{\mathbf{e}}_j,$$

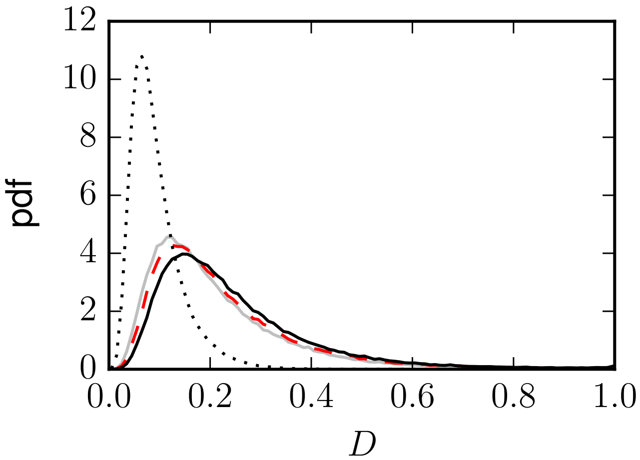
$$s^* = -\frac{3\sqrt{6}\Lambda_1\Lambda_2\Lambda_3}{(\Lambda_1 + \Lambda_2 + \Lambda_3)^{3/2}},$$

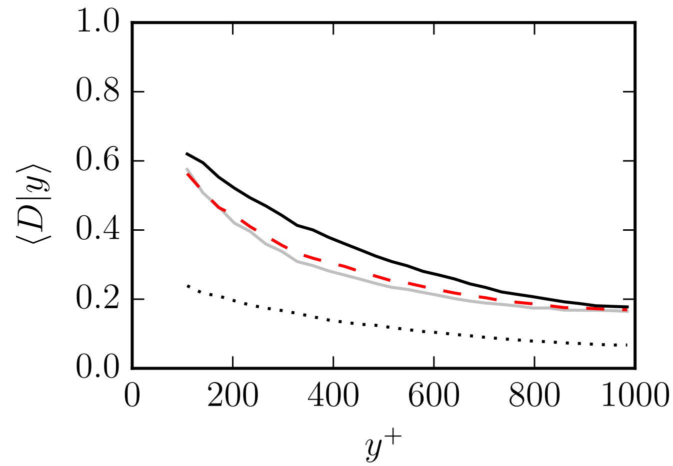
Dashed line: DNS
Solid line: LES-RDGF
Dotted line: LES, no model

Droplet deformation results:

$$D = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3},$$

Droplet Deformation at $Ca = 1.0$ ($t = 23.4$)



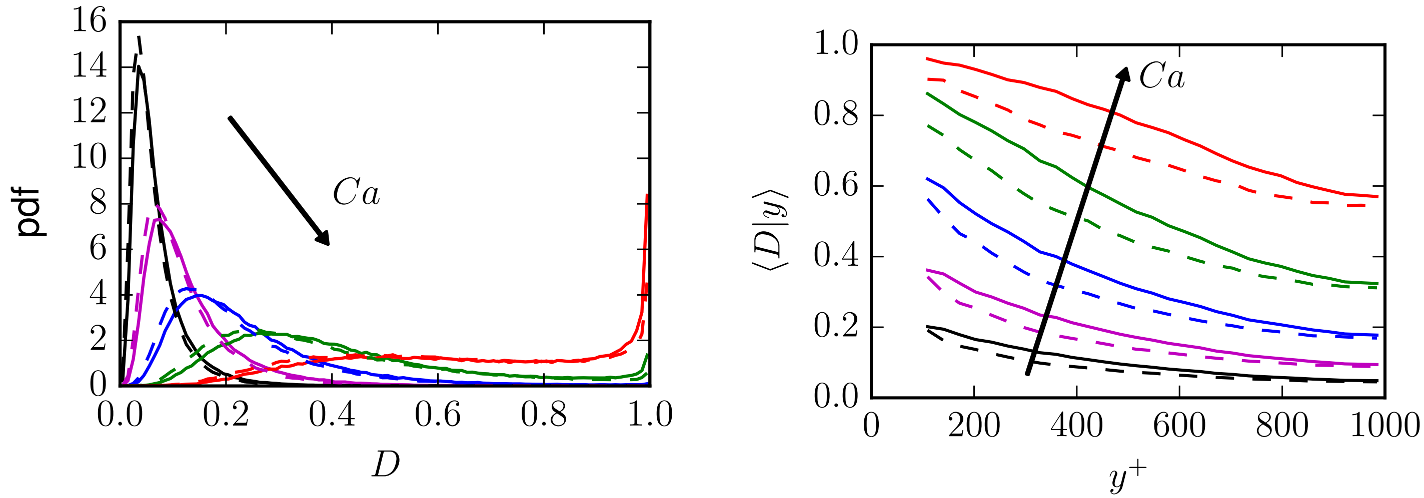


Red: baseline DNS
Gray: *a priori* (fDNS-RDGF)
Black: *a posteriori* (LES-RDGF)
Dotted: LES, no model

Dependence on surface tension (Capillary number):

Effect of Ca Number on Droplet Deformation ($t = 23.4$)

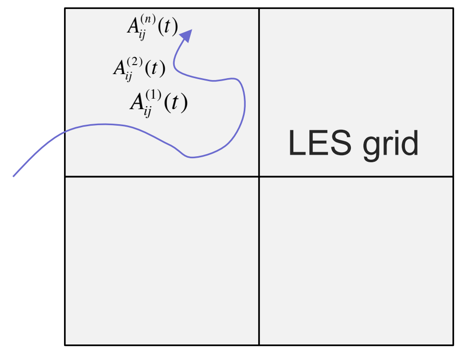
$$Ca = 0.25,\ 0.5,\ 1.0,\ 2.0,\ 4.0$$

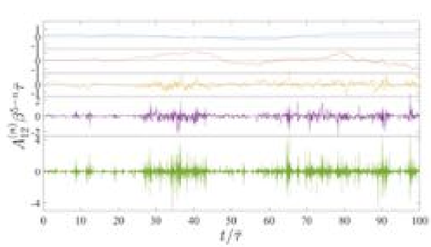


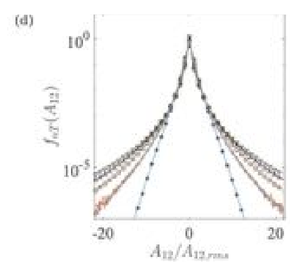
Solid lines: LES-RDGF
Dashed lines: DNS

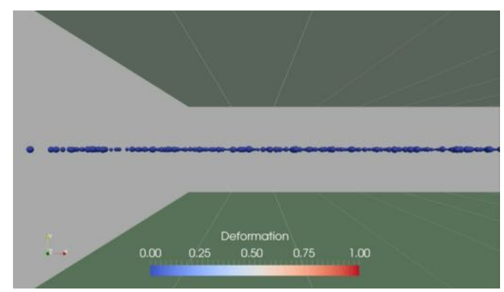
Conclusions:

- **Enriching LES with small-scale models**
- **Lagrangian stochastic model for A_{ij}**
- **Applications to model droplet deformation statistics in turbulent channel flow**
- Refs:
  - Johnson & CM (J Fluid Mech. 2016, 2017, 2018)
  - Luo, Shi & CM (Phys. Rev. Fluids **7**, 2022)









Thank you!