On the role of mutual friction in He-II Turbulence

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Outline :

PART A: brief introduction on superfluid He^4 (called He-II)

- Fluid-Mechanical approach of He-II dynamics ; two-fluid approach
- Coarse-grained dynamics
- HVBK closure conditions

PART B: Quantum Turbulence under HVBK closure conditions

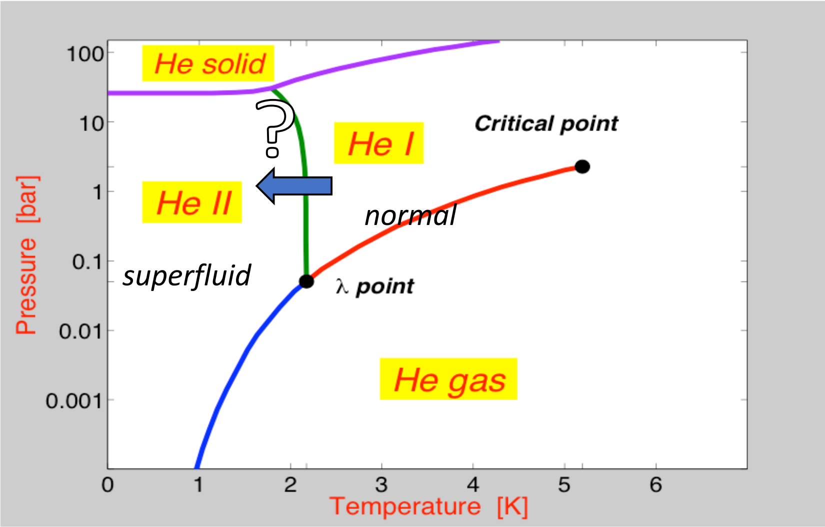
- Scale-by-scale energy cascade budget of He-II mixture
- Introduction of effective characteristic scales and Reynolds numbers
- Intermittency and anomalous scaling laws ; temperature dependence

PART A

Brief introduction on superfluid He^4 (called He-II)

Superfluid He^4 – general features

Superfluid He^4 is an archetype of quantum fluid



A superfluid is not a « perfect » Euler fluid with kinematic viscosity $\nu \rightarrow 0$ but behaves as if it consists of two components (co-penetrating fluids) :

- **inviscid superfluid component** (quantum ground state : vortex line)

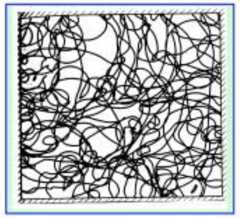
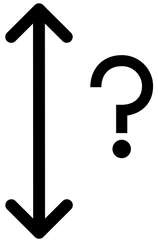


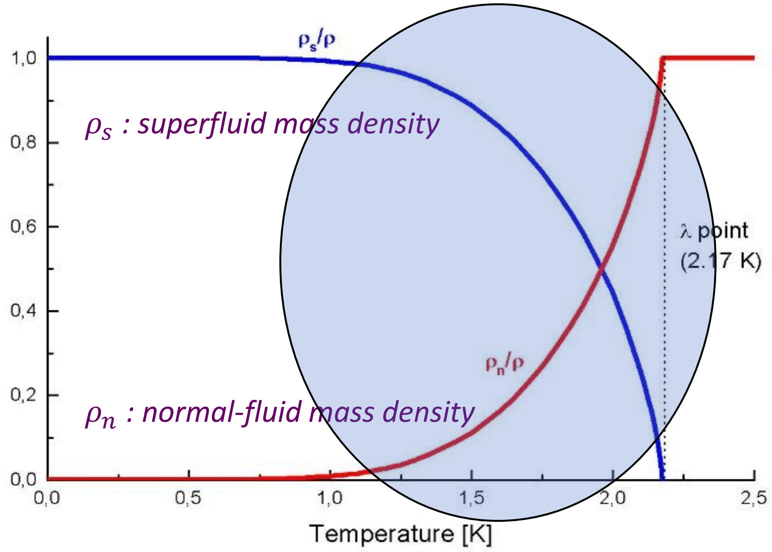
Fig. 12: A turbulent tangle of vortex lines.



$$\rho = \rho_n + \rho_s$$

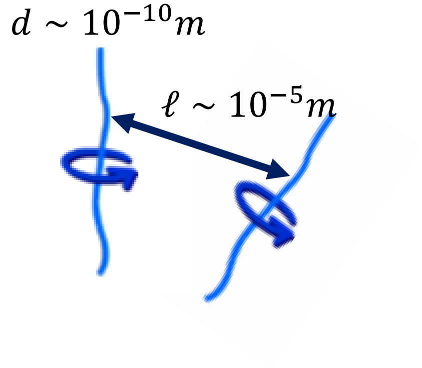
$$\rho u = \rho_n u^n + \rho_s u^s$$

- **normal fluid component** (excited states : phonons and rotons)

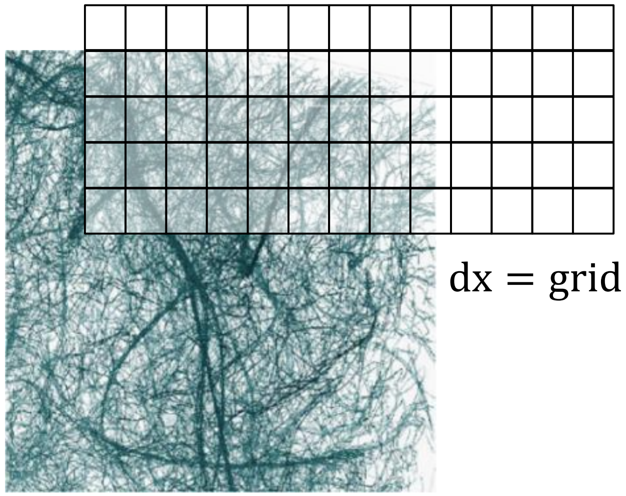


Length scales in the problem

d = vortex core **nm** Kelvin waves Vortex reconnection **μm** ℓ = intervortex distance << dx = resolution **mm** flow dimension



Kivotides, 2006

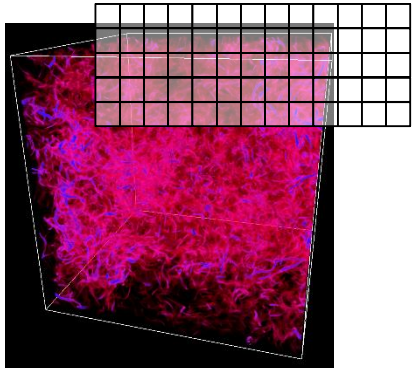


dx = grid resolution $\gg \ell$ = intervortex distance

Coarse-grained dynamics of He-II at $1\mathrm{K} < T < T_\lambda = 2.17\mathrm{K}$

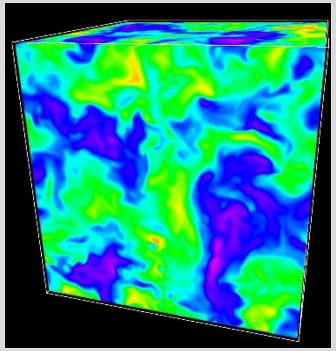
- superfluid component:

coarse-graining
dx >> intervortex distance



Vortex tangle → Euler fluid

+ subgrid scale dissipation :
vortex reconnection, etc.



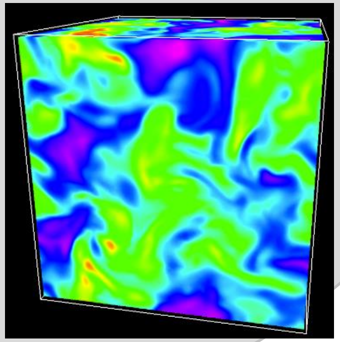
microscopic = semi-classical approach, e.g. Gross Pitaevskii

macroscopic

with mutual coupling

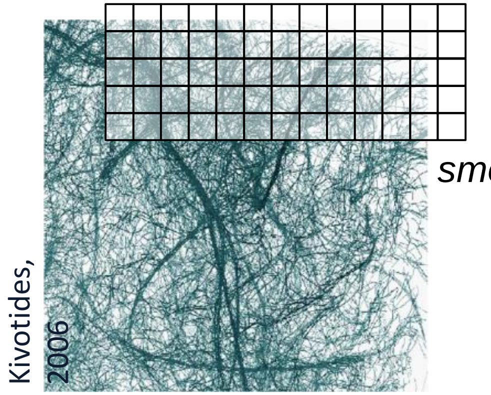
- normal-fluid component:

Navier-Stokes fluid
with viscosity ν_n

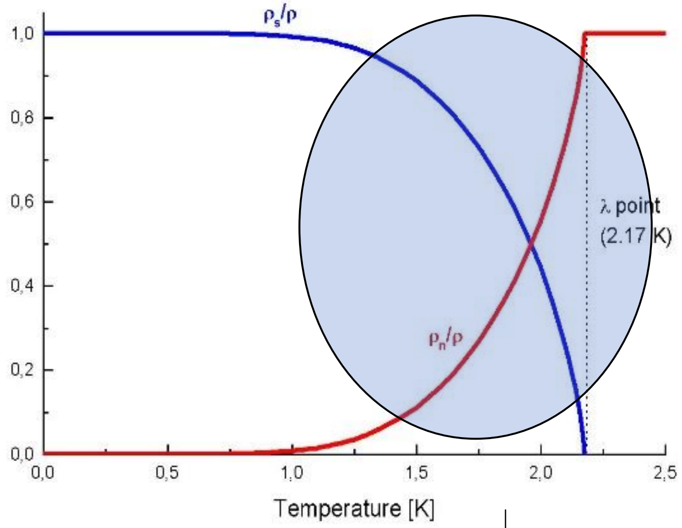


HVBK closure condition for the mutual coupling
Hall-Vinen-Bekharevich-Khalatnikov (1956, 1961)

- **He-II** is considered **isothermal and incompressible**



smoothed superfluid velocity field



$$\rho_n \frac{\mathrm{D}\boldsymbol{u}^n}{\mathrm{D}t} = -\nabla p_n + \boldsymbol{F}_{\mathrm{hvbk}} + \nu_n \nabla^2 \boldsymbol{u}^n$$

$$\rho_s \frac{\mathrm{D}\boldsymbol{u}^s}{\mathrm{D}t} = -\nabla p_s - \boldsymbol{F}_{\mathrm{hvbk}} \quad + \text{"subgrid dissipation"}$$

$\boldsymbol{\omega^s} = \boldsymbol{\nabla} \times \boldsymbol{u}^s$ ***superfluid vorticity accounts for the underlying tangle of quantized vortices***

superfluid vorticty

empirical parameters

$$\boldsymbol{F}_{\mathrm{hvbk}}\left(\frac{\rho_n\rho_s}{\rho}, \boldsymbol{\omega}^s, \boldsymbol{u}^n - \boldsymbol{u}^s, B, B'\right) \approx -\frac{B}{2}\frac{\rho_n\rho_s}{\rho}\frac{\boldsymbol{\omega}^s}{|\boldsymbol{\omega}^s|} \times \left(\boldsymbol{\omega}^s \times (\boldsymbol{u}^s - \boldsymbol{u}^n)\right) + \frac{B'}{2}\frac{\rho_n\rho_s}{\rho}\boldsymbol{\omega}^s \times (\boldsymbol{u}^s - \boldsymbol{u}^n) \approx -\frac{B}{2}\frac{\rho_n\rho_s}{\rho}|\boldsymbol{\omega}^s|(\boldsymbol{u}^n - \boldsymbol{u}^s)$$

mass density

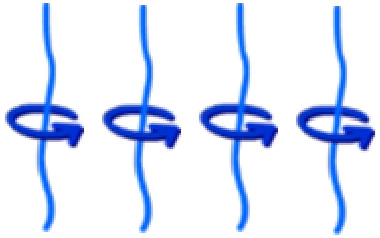
slip velocity

mutual friction **+** **Magnus effect** + vortex tension

most relevant for
Quantum Turbulence

Limitations and alternatives to HVBK closure condition

HVBK was originally designed for **regular pattern of quantized vortices with parallel orientation**



$$\boldsymbol{F}_{\mathrm{hvbk}} \approx -\frac{B}{2}\frac{\rho_n \rho_s}{\rho}\kappa L(\boldsymbol{u}^n - \boldsymbol{u}^s)$$

$$L \approx \frac{|\boldsymbol{\omega}^s|}{\kappa}$$ vortex line density

In the context of QT, it is expected to capture **only the *locally polarized* contribution of the superfluid vortex tangle** : $|\boldsymbol{\omega}^s| = \kappa L_{\parallel}$
Randomly oriented quantized vortices also participate to mutual friction : $\kappa L = \kappa(L_{\parallel} + L_{\chi})$ but are not taken into account

Vinen's model accounts for ***locally non-polarized* superfluid vortex tangle :**

$$\boldsymbol{F}_{\mathrm{Vinen}} = -\alpha(T)\, \rho_s\, \kappa\, L_{\chi}(\boldsymbol{u}^n - \boldsymbol{u}^s)$$

$$\frac{\mathrm{d}L_{\chi}}{\mathrm{dt}} = \alpha_V |\boldsymbol{u}^n - \boldsymbol{u}^s|\, L_{\chi}^{\frac{3}{2}} - \beta_V\, L_{\chi}^2$$

production   decay

Attempts to **unify Vinen's model and HVBK closure condition** (T. Lipniacki) :

$$\boldsymbol{q} = \frac{\nabla \times \boldsymbol{u}^s}{\kappa L}$$ measure of local anisotropy of the superfluid vortex tangle

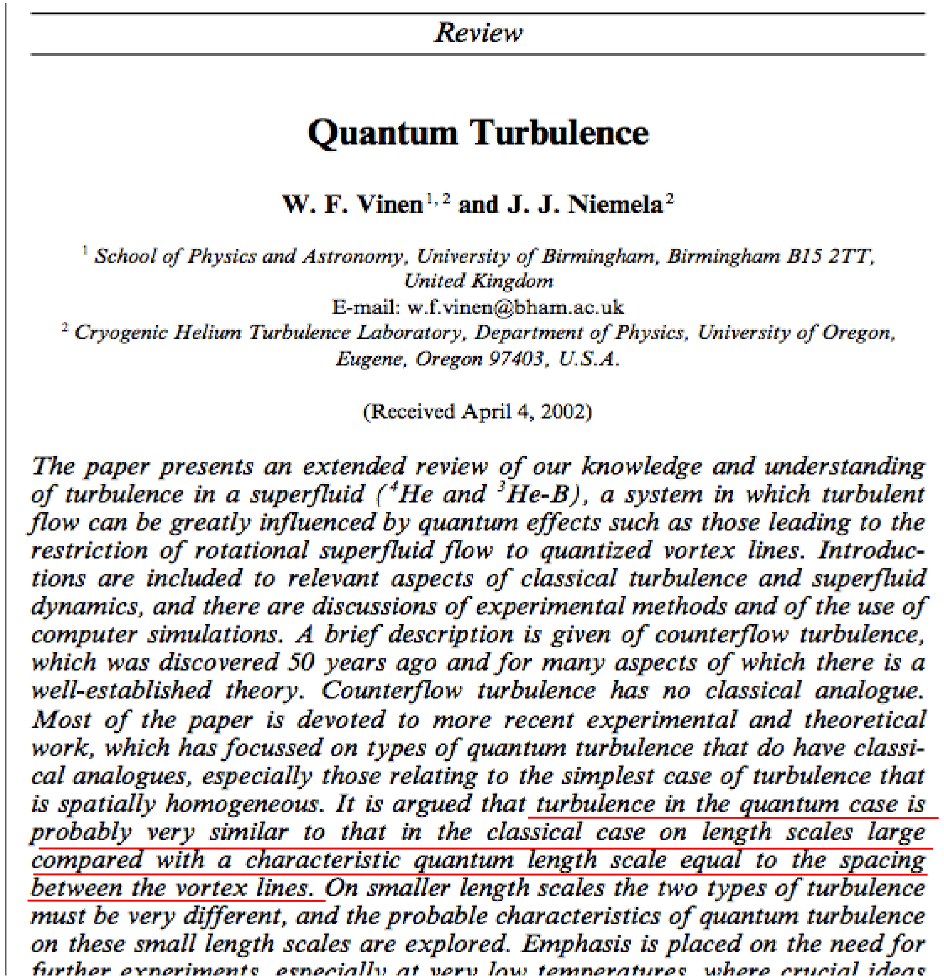
$q = 1$: polarized vortices
$q = 0$: isotropic, unpolarized tangle

PART B

Quantum Turbulence under HVBK closure conditions

Mainstream consensus
Quantum Turbulence is **probably** very similar to Classical Turbulence

To what extent : scale-by-scale comparison ; dependence on temperature ?



Review

Quantum Turbulence

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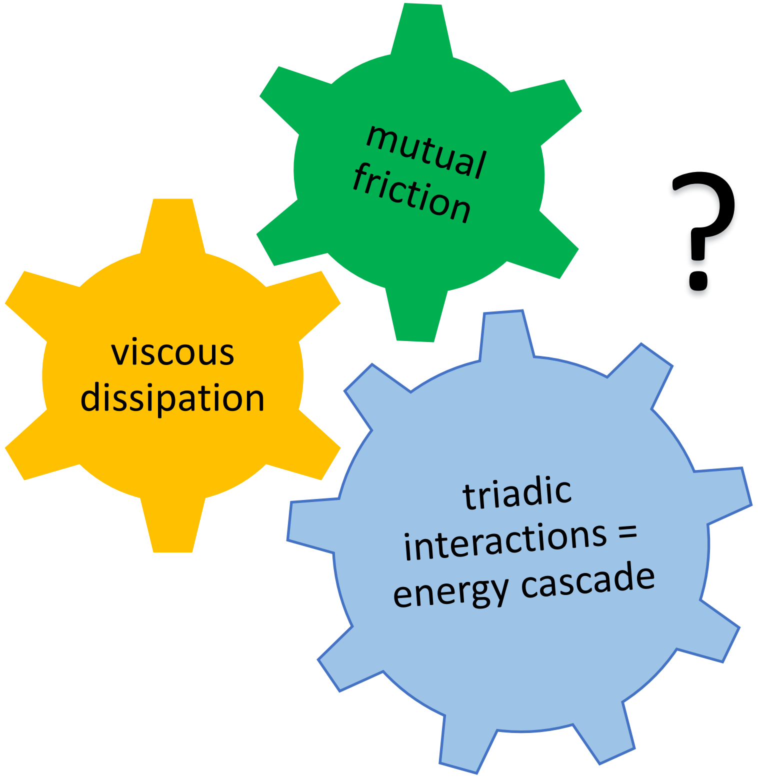
The paper presents an extended review of our knowledge and understanding of turbulence in a superfluid (^{4}He and ^{3}He-B), a system in which turbulent flow can be greatly influenced by quantum effects such as those leading to the restriction of rotational superfluid flow to quantized vortex lines. Introductions are included to relevant aspects of classical turbulence and superfluid dynamics, and there are discussions of experimental methods and of the use of computer simulations. A brief description is given of counterflow turbulence, which was discovered 50 years ago and for many aspects of which there is a well-established theory. Counterflow turbulence has no classical analogue. Most of the paper is devoted to more recent experimental and theoretical work, which has focussed on types of quantum turbulence that do have classical analogues, especially those relating to the simplest case of turbulence that is spatially homogeneous. It is argued that turbulence in the quantum case is probably very similar to that in the classical case on length scales large compared with a characteristic quantum length scale equal to the spacing between the vortex lines. On smaller length scales the two types of turbulence must be very different, and the probable characteristics of quantum turbulence on these small length scales are explored. Emphasis is placed on the need for further experiments, especially at very low temperatures, where crucial ideas



Coarse-grained dynamics of QT under HVBK closure conditions
Mathematical framework of our study

No temperature effect

Momentum equation for normal fluid and superfluid components:

$$\frac{\partial \boldsymbol{u}^n}{\partial t} + (\boldsymbol{u}^n \cdot \nabla)\boldsymbol{u}^n = -\frac{1}{\rho_n}\nabla \mathrm{p}_n + \frac{\rho_s}{\rho}\boldsymbol{F}^{ns} + \nu_n \nabla^2 \boldsymbol{u}^n \qquad \nabla \cdot u^n = 0$$

$$\frac{\partial \boldsymbol{u}^s}{\partial t} + (\boldsymbol{u}^s \cdot \nabla)\boldsymbol{u}^s = -\frac{1}{\rho_s}\nabla \mathrm{p}_s - \frac{\rho_n}{\rho}\boldsymbol{F}^{ns} + \nu_s \nabla^2 \boldsymbol{u}^s \qquad \nabla \cdot u^s = 0$$

to account for dissipation beyond inter-vortex scale: reconnection, Kelvin waves, sound emission, etc.

$\frac{\rho_n \rho_s}{\rho^2}\boldsymbol{F}^{ns}$ is the **mutual coupling force per unit mass of He-II** with

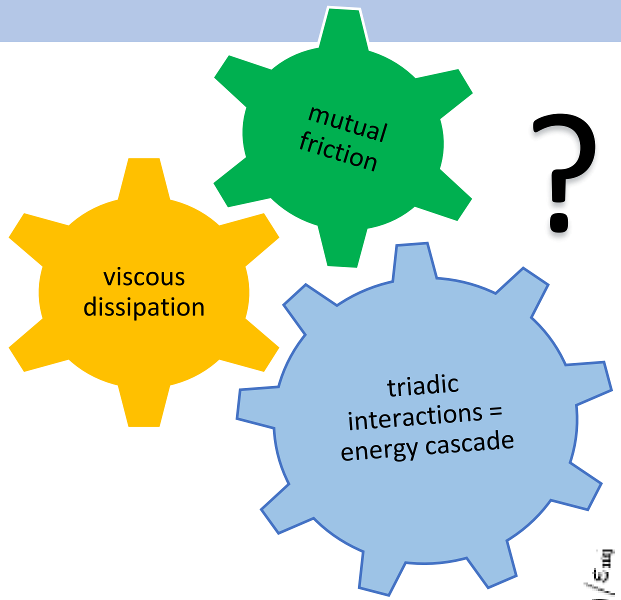
$$\boldsymbol{F}^{ns} = -\frac{B}{2}|\boldsymbol{\omega}^s|\,(\boldsymbol{u}^n - \boldsymbol{u}^s)$$

for the HVBK closure condition ; $\boldsymbol{\omega}^s = \nabla \times \boldsymbol{u}^s$ is the coarse grained superfluid vorticity

$$\boldsymbol{F}^{ns} = -\frac{B}{2}\kappa L\,(\boldsymbol{u}^n - \boldsymbol{u}^s)$$

in a *mean-field approximation* with $\boldsymbol{L}$ **being the (uniform) vortex line density**

Wavenumber-by-wavenumber energy budget of QT



$$\frac{\rho_s}{\rho_n} = 1 \quad \text{at} \quad T = 1.96\text{K}$$

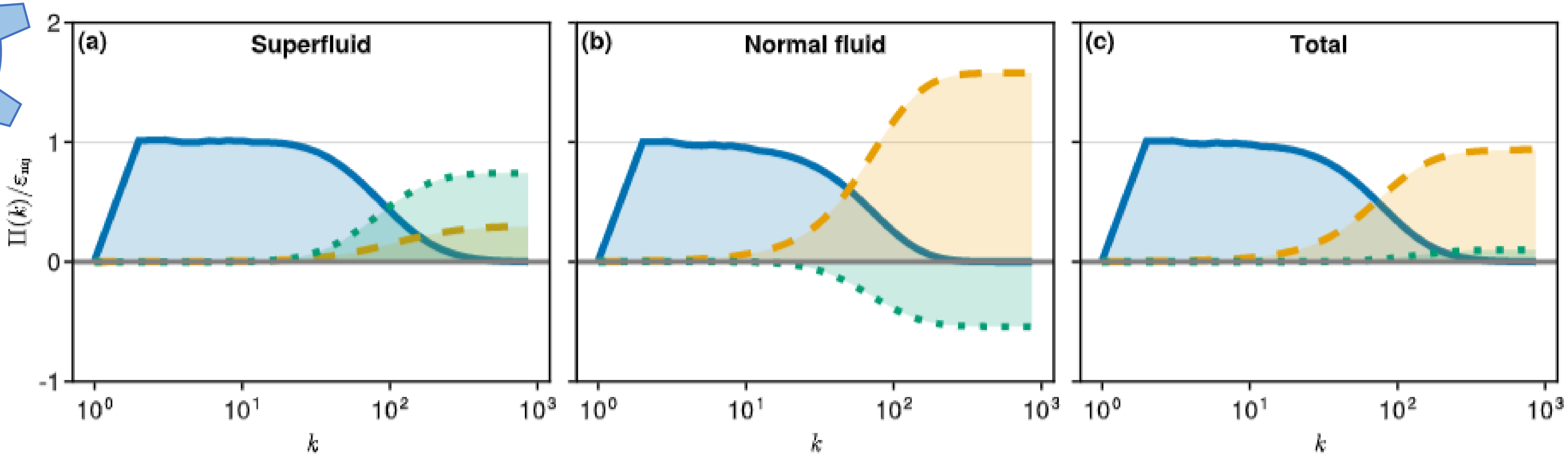
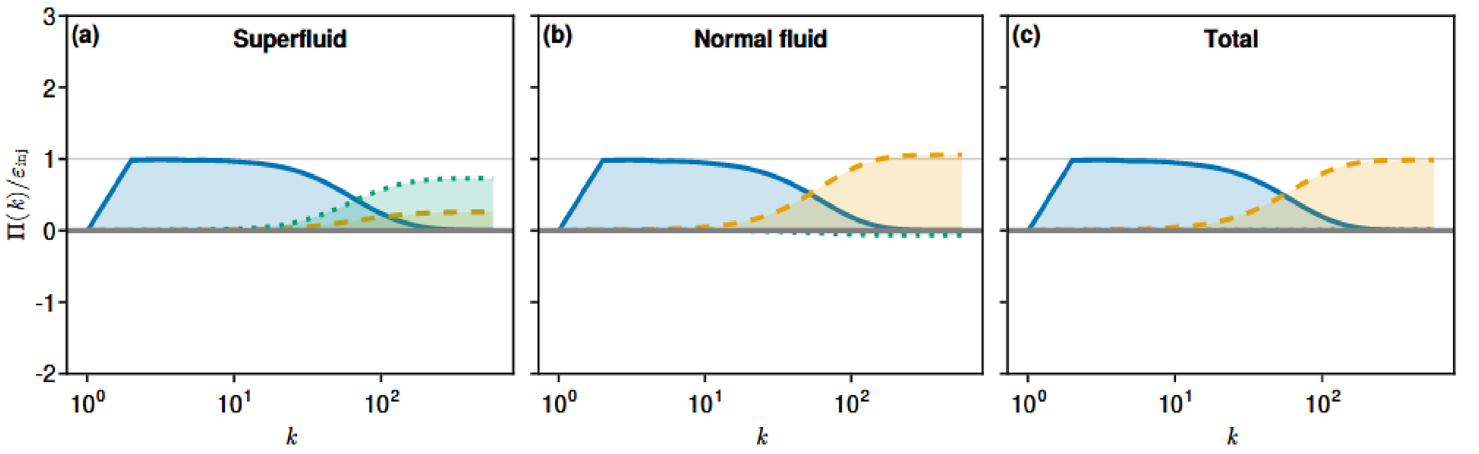


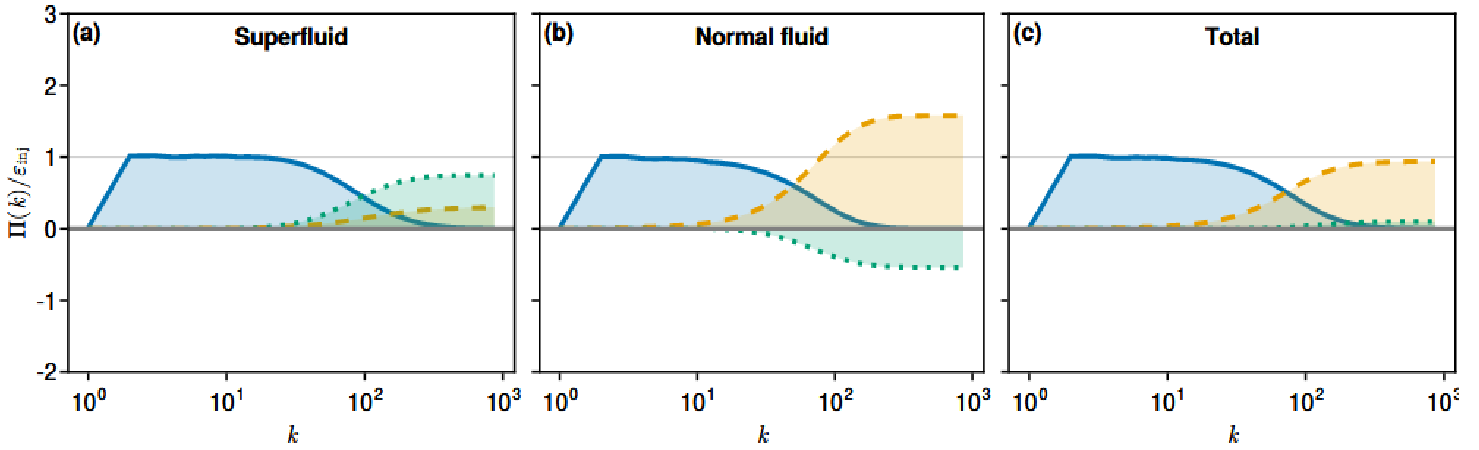
FIG. 1. Scale-by-scale spectral energy fluxes in (a) the superfluid component, (b) the normal fluid component, and (c) the two-fluid mixture. Blue solid lines, energy flux $\Pi(k)$ through wave number k; orange dashed lines, viscous dissipation $\mathcal{D}(k)$; green dotted lines, energy transfers due to mutual friction. Results obtained from HVBK simulations at $T = 1.96$ K (case ▸ in Table I).

$$\frac{\nu_n}{\nu_s} = 5$$

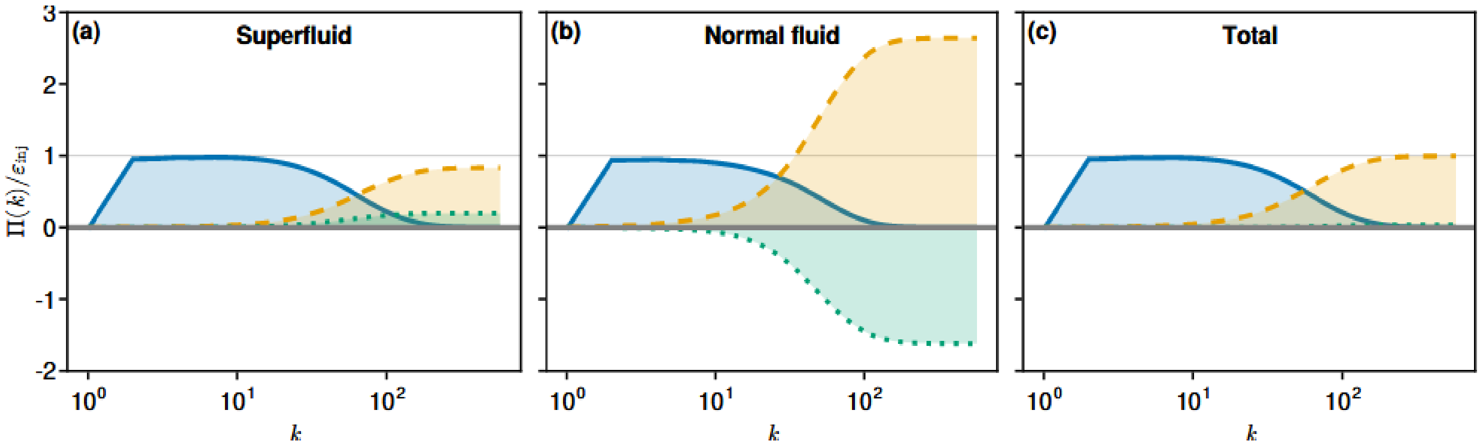
$$\frac{\rho_s}{\rho_n} = 0.1$$



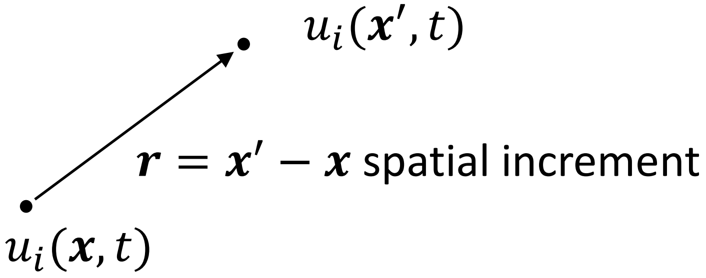
$$\frac{\rho_s}{\rho_n} = 1$$



$$\frac{\rho_s}{\rho_n} = 10$$



Scale-by-scale energy budget in classical turbulence



$\delta u_i(\boldsymbol{x}, \boldsymbol{r}, t) \equiv u_i(\boldsymbol{x}', t) - u_i(\boldsymbol{x}, t)$ velocity increment

$\delta u_\parallel(\boldsymbol{x}, \boldsymbol{r}, t) = \delta \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{r}, t) \cdot \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$ longitudinal velocity increment

Under the assumption of **stationary homogeneous and isotropic turbulence**,
an **exact scale-by-scale energy budget** can be derived from the Navier-Stokes equations,

$$\epsilon = \epsilon_{\mathrm{inj}}$$

ϵ refers to the mean dissipation rate (per unit mass)

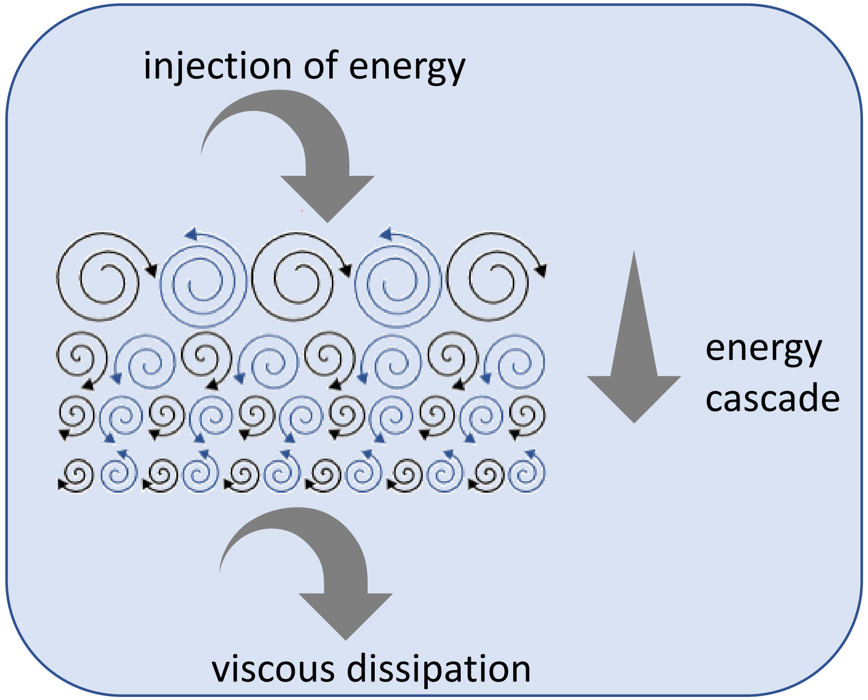
$$< |\delta u_i|^2(r)\delta u_\parallel(r) > = -\frac{4}{3}\epsilon_{\mathrm{inj}}\, r + 2\nu \frac{\mathrm{d} < |\delta u_i|^2(r) >}{\mathrm{dr}}$$

energy cascade ***viscous dissipation***

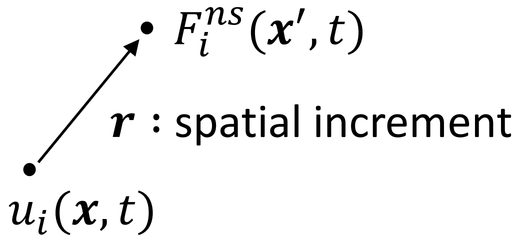
for $r \ll L_0$: energy injection scale

This eventually yields the famous ***Kolmogorov's four-fifth law***

$$< \delta u_\parallel^3(r) > = -\frac{4}{5}\epsilon_{\mathrm{inj}}\, r + 6\,\nu \frac{\mathrm{d} < \delta u_\parallel^2(r) >}{\mathrm{dr}}$$



Scale-by-scale energy budget in Quantum Turbulence for **normal fluid and superfluid components individually**



Normal fluid:

$$\epsilon_{\mathrm{inj}}^{n} = \epsilon^{n} - \frac{\rho_s}{\rho} < u_i^n F_i^{ns} >$$

$$< |\delta u_i^n|^2(r)\delta u_{\parallel}^n(r) > = -\frac{4}{3}\epsilon_{\mathrm{inj}}^{n} r - \frac{\rho_s}{\rho}\frac{2}{r^2}\int_0^r (< u_i^{n\prime} F_i^{ns} > + < u_i^n F_i^{ns\prime} >) r'^2 dr' + 2\nu_n \frac{\mathrm{d}}{\mathrm{dr}} < |\delta u_i^n|^2(r) >$$

for $r \ll L_0$: energy injection scale

Superfluid:

$$\epsilon_{\mathrm{inj}}^{s} = \epsilon^{s} + \frac{\rho_n}{\rho} < u_i^s F_i^{ns} >$$

$$< |\delta u_i^s|^2(r)\delta u_{\parallel}^s(r) > = -\frac{4}{3}\epsilon_{\mathrm{inj}}^{s} r + \frac{\rho_n}{\rho}\frac{2}{r^2}\int_0^r (< u_i^{s\prime} F_i^{ns} > + < u_i^s F_i^{ns\prime} >) r'^2 dr' + 2\nu_s \frac{\mathrm{d}}{\mathrm{dr}} < |\delta u_i^s|^2(r) >$$

for $r \ll L_0$: energy injection scale

Scale-by-scale energy budget in Quantum Turbulence for the **two-fluid He-II mixture**

$$\frac{\rho_n}{\rho} < |\delta u_i^n|^2 \delta u_\parallel^n > + \frac{\rho_s}{\rho} < |\delta u_i^s|^2 \delta u_\parallel^s > =$$

$$-\frac{4}{3}\left(\frac{\rho_n}{\rho}\epsilon_{\text{inj}}^n + \frac{\rho_s}{\rho}\epsilon_{\text{inj}}^s\right) r$$

$$+\,2\left(\frac{\rho_n}{\rho}\nu_n \frac{\mathrm{d}}{\mathrm{dr}} < |\delta u_i^n|^2 > + \frac{\rho_s}{\rho}\,\nu_s \frac{\mathrm{d}}{\mathrm{dr}} < |\delta u_i^s|^2 >\right)$$

$$-\boldsymbol{\frac{\rho_s\,\rho_n}{\rho^2}\frac{2}{r^2}\int_0^r < u_i^{ns'} F_i^{ns} + u_i^{ns} F_i^{ns'} > r'^2 dr'}$$

with $u_i^{ns} \equiv u_i^n - u_i^s$

$$\bar{\epsilon}_{\text{inj}} = \frac{\rho_n}{\rho}\epsilon_{\text{inj}}^n + \frac{\rho_s}{\rho}\epsilon_{\text{inj}}^s$$

$$\bar{\nu} = \frac{\rho_n}{\rho}\nu_n + \frac{\rho_s}{\rho}\,\nu_s$$

Supplementary term compared to Classical Turbulence
At which scale is it effective?

$$\overline{S_3}(r) = -\frac{4}{3}\,\bar{\epsilon}_{\text{inj}} r + 2\bar{\nu}\,\frac{\mathrm{d}\,\overline{S_2}(r)}{\mathrm{dr}} + \varphi_{ns}(r)$$

mass-density weighted scale-by-scale energy budget of He-II

Mutual coupling – energy budget

$$\varphi_{ns}(r) = -\frac{\rho_s\,\rho_n}{\rho^2}\frac{2}{r^2}\int_0^r < u_i^{ns'}F_i^{ns} + u_i^{ns}F_i^{ns'} > r'^2 dr'$$

$$\epsilon_{ns} = \frac{\rho_s\,\rho_n}{\rho^2} < u_i^{ns}F_i^{ns} >$$

$$\varphi_{ns}(r) = -\frac{4\epsilon_{ns}}{r^2}\int_0^r \frac{< u_i^{ns'}F_i^{ns} + u_i^{ns}F_i^{ns'} >}{2 < u_i^{ns}F_i^{ns} >} r'^2 dr'$$

If **HVBK**: $F_i^{ns} = -\frac{B}{2}|\boldsymbol{\omega}^s|\, u_i^{ns}$ with $|\boldsymbol{\omega}^s| \simeq \sqrt{< |\boldsymbol{\omega}^s|^2 >} = \kappa L$

mean-field approximation

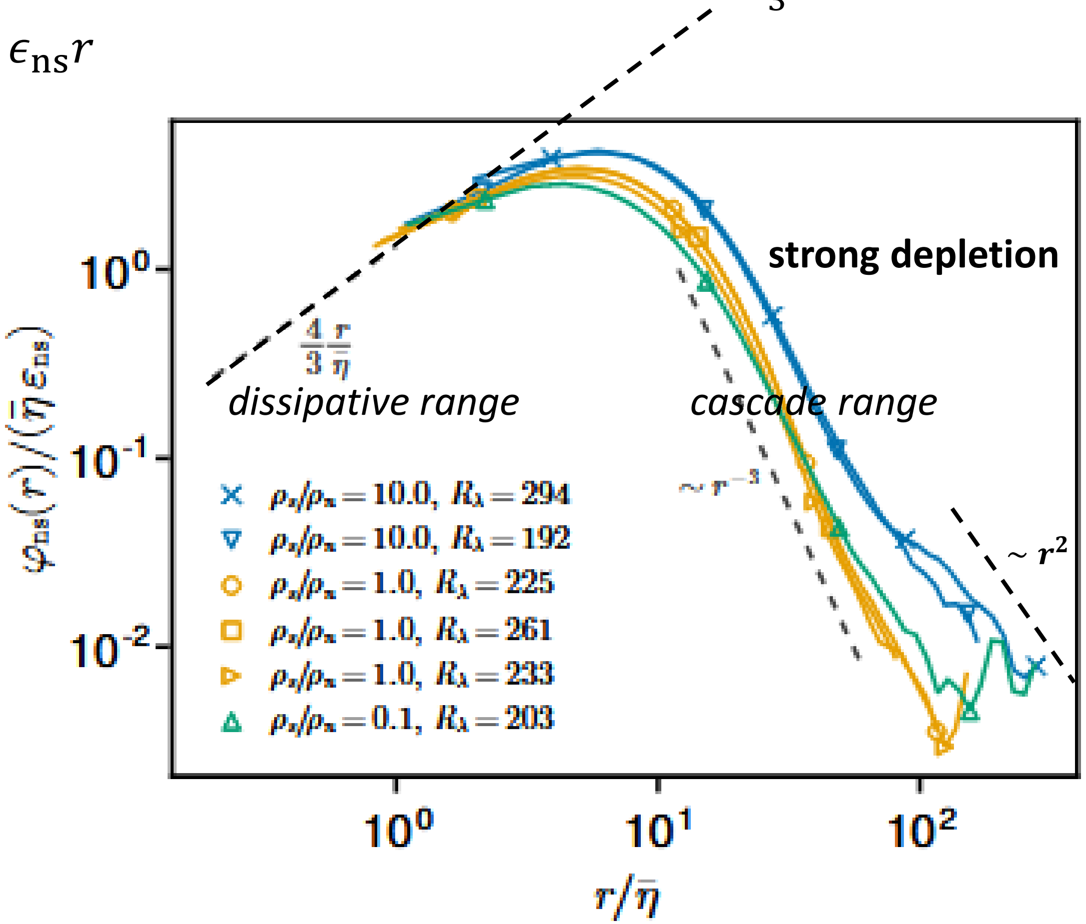
$$\varphi_{ns}(r) = 2\epsilon_{ns}\,\frac{2}{r^2}\int_0^r \frac{< u_i^{ns'}u_i^{ns} >}{< |u^{ns}|^2 >}\, r'^2 dr'$$

Mutual coupling – scale behaviour

$$\varphi_{ns}(r) = 2\epsilon^{\rm ns}\frac{2}{r^2}\int_0^r \frac{< u_i^{ns\prime} u_i^{ns} >}{< |u^{ns}|^2 >}\, r'^2 dr'$$

As $r \to 0$: $\varphi_{ns}(r) \to \frac{4}{3}\epsilon_{\rm ns} r$

$\frac{4}{3}\epsilon_{\rm ns} r \quad (\epsilon_{ns} < \epsilon_{\rm inj})$



→ $\varphi_{ns}(r)$ **negligible compared to** $\frac{4}{3}\epsilon_{\rm inj} r$

frictional dissipation is negligible compared to energy cascade flux

at large scales $\varphi_{ns}(r) \sim \frac{1}{r^2}$

Effective Kolmogorov's scale for He-II

In **classical turbulence** : Kolmogorov's scale is obtained by equaling

- $S_2(r) \sim < \left(\frac{\partial u_x}{\partial x}\right)^2 > r^2 \sim \frac{\epsilon}{\nu} r^2$ in the ``dissipative range" where velocity field is regular (smooth)
- and $S_2(r) \sim \epsilon^{2/3}\, r^{2/3}$ in the ``inertial range'' where velocity field is irregular

This yields $\boldsymbol{\eta} \sim \left(\frac{\boldsymbol{\nu}^3}{\boldsymbol{\epsilon}}\right)^{\mathbf{1/4}}$

Analogously in **quantum turbulence** : effective Kolmogorov's scale would be obtained by equaling

- $\overline{S_2}(r) \sim \frac{<\rho_n \nu_n \left(\frac{\partial u_x^n}{\partial x}\right)^2 + \rho_s \nu_s \left(\frac{\partial u_x^s}{\partial x}\right)^2>}{\rho_n \nu_n + \rho_s \nu_s} r^2 \sim \frac{\bar{\epsilon}_\nu}{\bar{\nu}} r^2$ in the ``dissipative range'' where velocity field is regular
- and $\overline{S_2}(r) \sim \bar{\epsilon}_{\text{inj}}^{2/3}\, r^{2/3}$ in the ``inertial range'' where velocity field is regular

This yields

$$\bar{\eta} \sim \left(\frac{\bar{\nu}^3}{\bar{\epsilon}_{\text{inj}}}\right)^{\frac{1}{4}} \times \left(\frac{\bar{\epsilon}_{\text{inj}}}{\bar{\epsilon}_{\text{inj}} - \epsilon_{ns}}\right)^{\frac{3}{4}}$$

by considering $\bar{\epsilon}_{\text{inj}} = \bar{\epsilon}_\nu + \epsilon_{\text{ns}}$

Or equivalently :

enhanced viscosity that accounts for mutual friction

$$\boldsymbol{\bar{\eta}} \sim \left(\frac{\boldsymbol{\bar{\nu}_{\text{eff}}}^3}{\boldsymbol{\bar{\epsilon}_{\text{inj}}}}\right)^{\frac{\mathbf{1}}{\mathbf{4}}} \quad \text{with} \quad \boldsymbol{\bar{\nu}_{\text{eff}}} = \boldsymbol{\bar{\nu}} \times \left(\frac{\boldsymbol{\bar{\epsilon}_{\text{inj}}}}{\boldsymbol{\bar{\epsilon}_{\text{inj}} - \epsilon_{\text{ns}}}}\right)$$

Effective Reynolds number for He-II

Effective Reynolds number:

$$\mathbf{Re} = \frac{\boldsymbol{u}_{\mathbf{rms}} \boldsymbol{L}_{\mathbf{0}}}{\overline{\boldsymbol{\nu}}_{\mathbf{eff}}} \sim \left(\frac{\boldsymbol{L}_{\mathbf{0}}}{\overline{\boldsymbol{\eta}}}\right)^{\frac{3}{4}}$$

$$\bar{\eta} \sim \left(\frac{\bar{\nu}_{\mathrm{eff}}{}^{3}}{\bar{\epsilon}_{\mathrm{inj}}}\right)^{\frac{1}{4}} \text{ with } \bar{\nu}_{\mathrm{eff}} = \bar{\nu}\left(\frac{\bar{\epsilon}_{\mathrm{inj}}}{\bar{\epsilon}_{\mathrm{inj}} - \epsilon_{\mathrm{ns}}}\right) = \bar{\nu}\left(1 + \frac{\epsilon_{\mathrm{ns}}}{\epsilon_{\bar{\nu}}}\right)$$

Effective Taylor microscale $\bar{\lambda}$ is defined by

$$\bar{\boldsymbol{\epsilon}}_{\mathbf{inj}} = \mathbf{15}\, \overline{\boldsymbol{\nu}}_{\mathbf{eff}} \left(\frac{\boldsymbol{u}_{\mathbf{rms}}}{\bar{\boldsymbol{\lambda}}}\right)^{\mathbf{2}}$$

or equivalently

$$\bar{\boldsymbol{\epsilon}}_{\boldsymbol{\nu}} = \mathbf{15} \overline{\boldsymbol{\nu}} \left(\frac{\boldsymbol{u}_{\mathbf{rms}}}{\bar{\boldsymbol{\lambda}}}\right)^{\mathbf{2}}$$

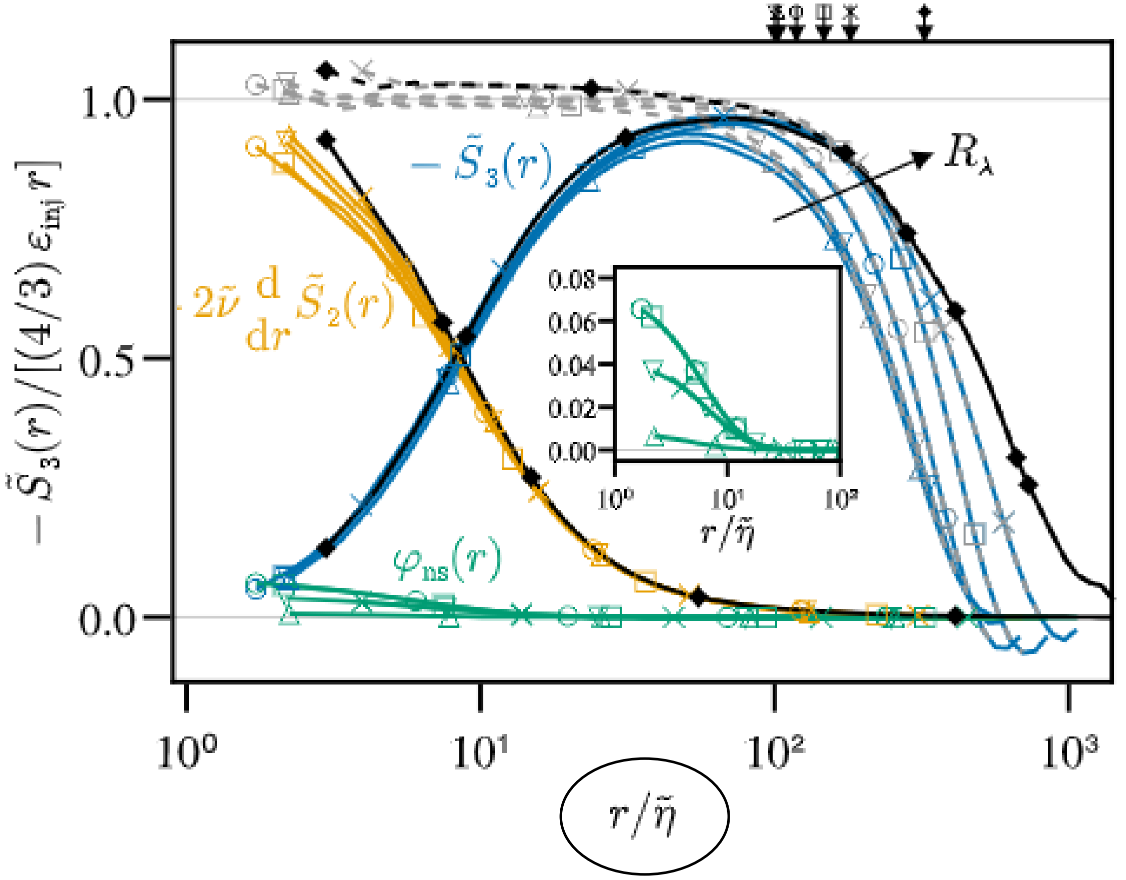
$$\boldsymbol{R}_{\bar{\lambda}} = \frac{\boldsymbol{u}_{\mathbf{rms}} \bar{\boldsymbol{\lambda}}}{\overline{\boldsymbol{\nu}}_{\mathbf{eff}}} \sim \sqrt{\mathbf{Re}}$$

Numerical Results from Pseudo-Spectral Simulations

$$\frac{4}{3}\,\bar{\epsilon}_{\rm inj} r = -\overline{S_3}(r) + 2\bar{\nu}\,\frac{\mathrm{d}\,\overline{S_2}(r)}{\mathrm{dr}} + \varphi_{ns}(r)$$

TABLE I. Simulation parameters.

Key	T (K)	ρ_s/ρ_n	ν_s/ν_n	B	N	$k_{\max}\tilde{\eta}$	$\tilde{\lambda}/\tilde{\eta}$	$L/\tilde{\lambda}$	R_λ
×	1.44	10	0.2	2.0	1152	1.6	33.7	19.6	294
▽	1.44	10	0.2	2.0	1152	2.9	27.2	12.7	191
○	1.96	1	0.2	1.0	1728	3.6	29.0	14.5	217
□	1.96	1	0.2	1.0	1728	2.9	31.2	16.7	251
▷	1.96	1	**0.1**	1.0	1728	3.5	30.0	14.6	233
△	2.157	0.1	0.2	2.16	1152	2.8	27.4	12.9	194
♦	NS	0	–	–	2048	1.4	44.6	34.2	510



Mutual friction for He-II mixture is essentially a dissipative process that adds to viscous dissipation at very scale scales

Controversy on intermittency in Quantum Turbulence
Experimental results

<u>Experimental studies :</u>

- ``Local investigation of superfluid turbulence''
  J. Maurer and P. Tabeling
  Europhys. Lett. **43** (1), pp. 29-34 (1998)

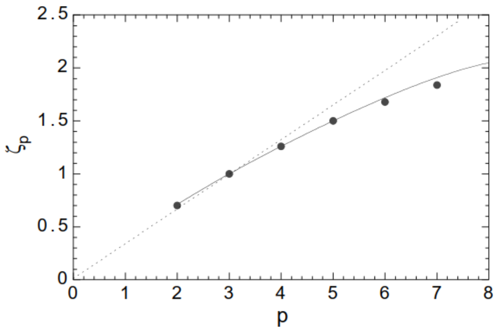
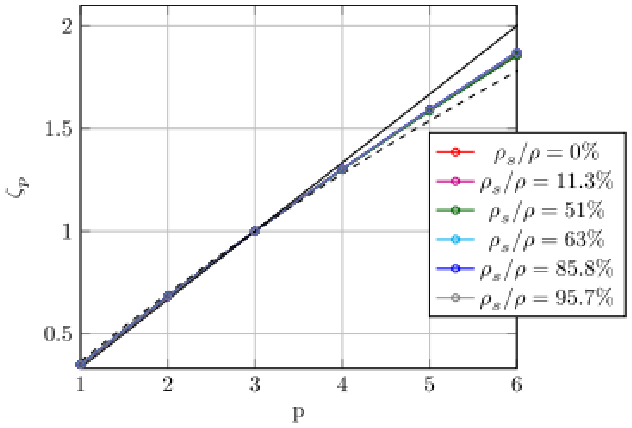


Fig. 5. – Exponents of the structure functions of the absolute values of the longitudinal veloci increments, up to $p = 7$, for $T = 1.4$ K (black disks); the full line represents the current values fou in normal fluid turbulence; the dashed line is the Kolmogorov line.



- ``Intermittency of QT with superfluid fraction from 0% to 96%''
  E. Rusaouen, B. Chabaud, J. Salort, P-E Roche
  Phys. Fluids **29**, 105108 (2017)

*``… **No evidence of temperature dependence is found on these scaling exponents** in the upper part of the inertial cascade, where turbulence is well developed and fully resolved by the probe…''*

- ``Intermittency enhancement in quantum turbulence in superfluid 4He''
  Emil Varga, Jian Gao, Wei Guo, and Ladislav Skrbek
  Phys. Rev. Fluids **3**, 094601 (2018)

*``… **measurements reveal temperature-dependent intermittency corrections** (on transverse velocity structure functions) **that peak in the vicinity of 1.85K** in excellent agreement with recent theoretical predictions …''*

Controversy on intermittency in Quantum Turbulence
Numerical results

Numerical studies :

- ``Enhancement of Intermittency in Superfluid Turbulence''
Laurent Boué, Victor L'vov, Anna Pomyalov, and Itamar Procaccia
Phys. Rev. Lett. **110**, 014502 (2013)

- "Multiscaling in superfluid turbulence: A shellmodel study"
V. Shukla and R. Pandit
Phys. Rev. E **94**, 043101 (2016)

- ``Turbulent statistics and intermittency enhancement in coflowing superfluid 4He''
L. Biferale, D. Khomenko, V. L'vov, A. Pomyalov, I. Procaccia, and G. Sahoo
Phys. Rev. Fluids **3**, 024605 (2018)

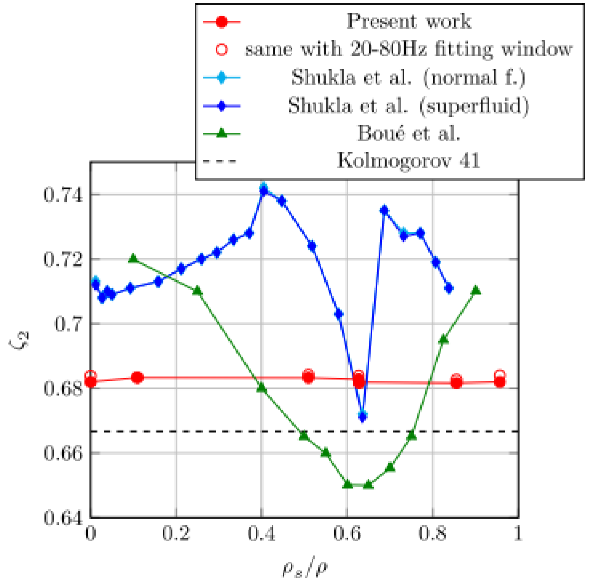


FIG. 10. Exponents of the second order structure function as a function of the superfluid fraction. For explanation on open symbols, see the text.

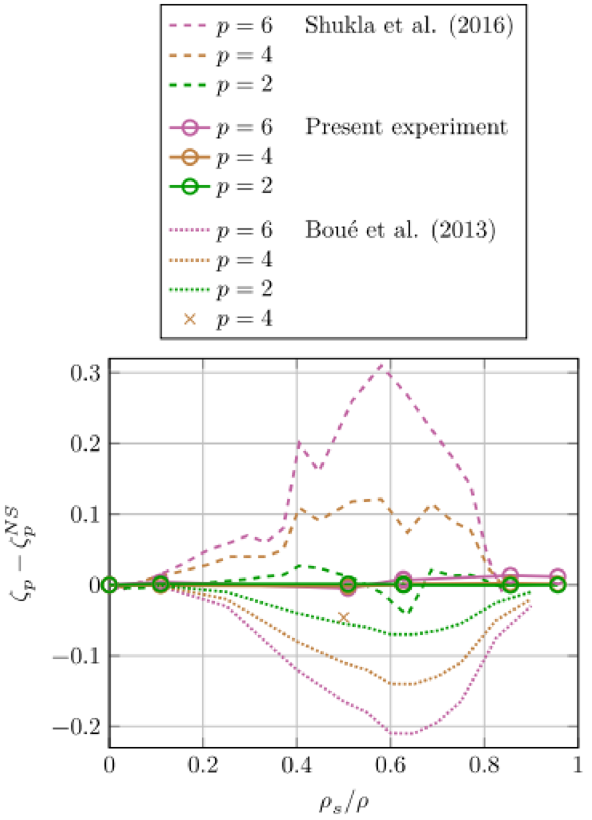


FIG. 11. Superfluid correction of the intermittency exponents. Note that the dotted line for orders $p = 4$ and $p = 6$ have been calculated from an analytical formula provided in the original paper.

``…The energy transfer by mutual friction between components is particularly efficient **in the temperature range between 1.8 and 2 K, leading to enhancement of small-scale intermittency for these temperatures…''**

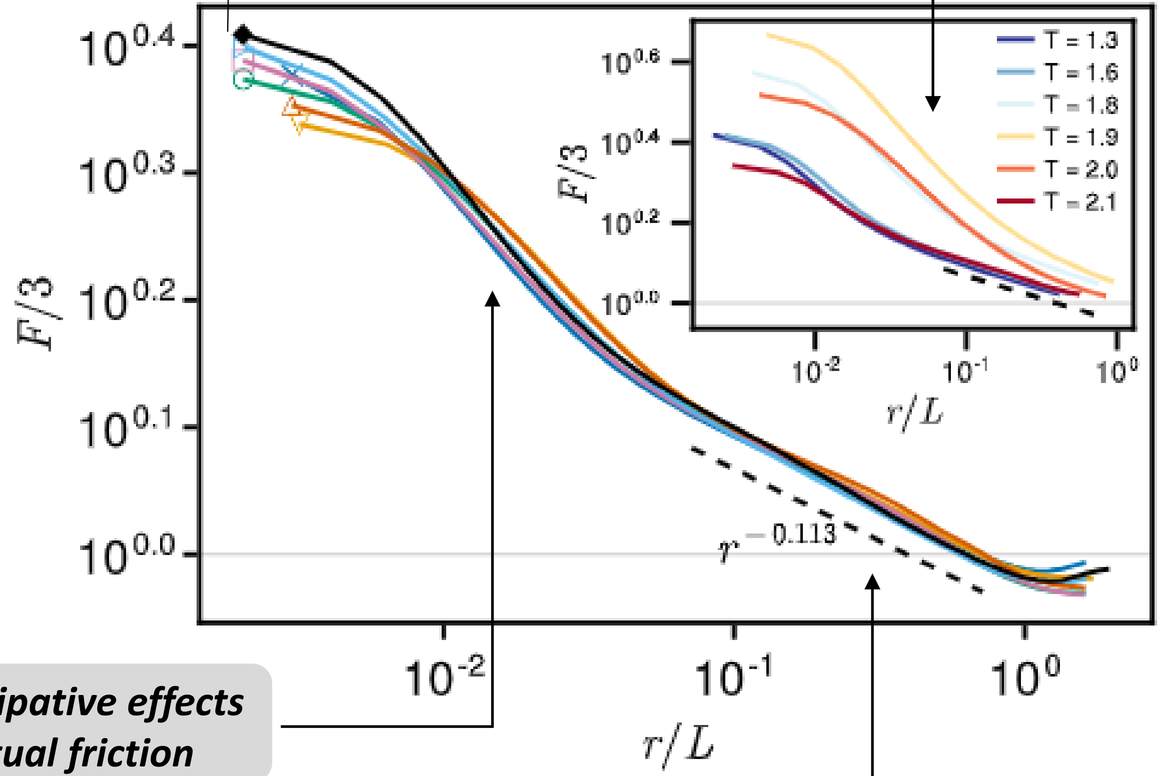
Flatness of longitudinal increments for normal fluid and superfluid components

TABLE I. Simulation parameters.

Key	T (K)	ρ_s/ρ_n	ν_s/ν_n	B	N	$k_{max}\tilde{\eta}$	$\tilde{\lambda}/\tilde{\eta}$	$L/\tilde{\lambda}$	R_λ
×	1.44	10	0.2	2.0	1152	1.6	33.7	19.6	294
▽	1.44	10	0.2	2.0	1152	2.9	27.2	12.7	191
○	1.96	1	0.2	1.0	1728	3.6	29.0	14.5	217
□	1.96	1	0.2	1.0	1728	2.9	31.2	16.7	251
▷	1.96	1	**0.1**	1.0	1728	3.5	30.0	14.6	233
△	2.157	0.1	0.2	2.16	1152	2.8	27.4	12.9	194
◆	NS	0	–	–	2048	1.4	44.6	34.2	510

classical turbulence

Inset = *results from Biferale et al.*

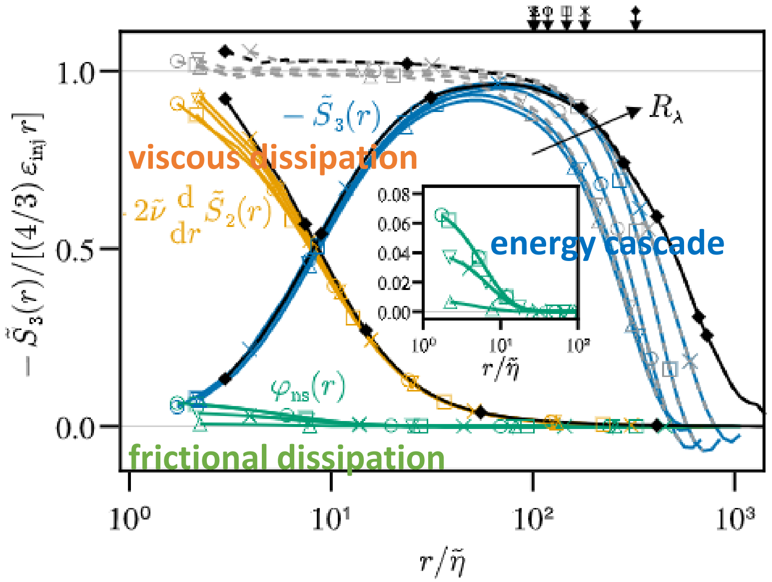


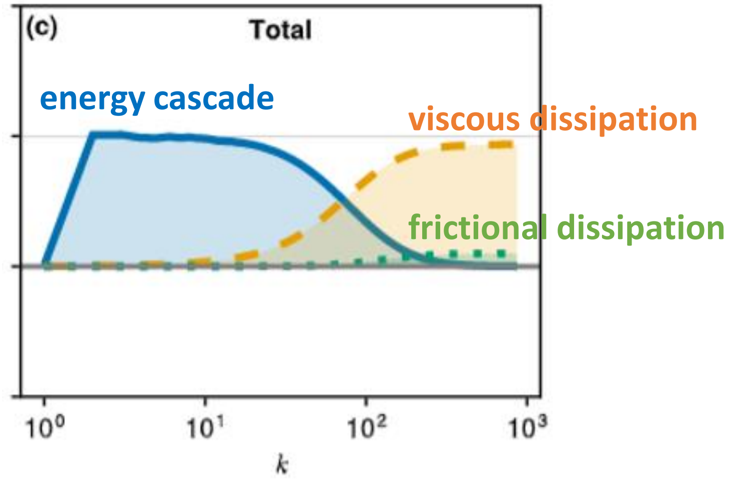
Enhancement of intermittency due to dissipative effects depends on temperature through the mutual friction

Universal scaling exponent for all temperatures similar to classical turbulence

Conclusion – Take Home messages

- **Mutual friction for He-II mixture** is essentially **a dissipative process that adds to viscous dissipation at very scale scales**





This allows us to define **an effective Kolmogorov scale** $\bar{\eta} \sim \left(\frac{\bar{\nu}_{\text{eff}}^{3}}{\bar{\epsilon}_{\text{inj}}}\right)^{\frac{1}{4}}$ with $\bar{\nu}_{\text{eff}} = \bar{\nu} \times \left(\frac{\bar{\epsilon}_{\text{inj}}}{\bar{\epsilon}_{\text{inj}} - \epsilon_{\text{ns}}}\right)$

and **an effective Reynolds number** $\text{Re} = \frac{u_{\text{rms}} L_0}{\bar{\nu}_{\text{eff}}} \sim \left(\frac{L_0}{\bar{\eta}}\right)^{\frac{3}{4}}$

Conclusion – Take Home messages

❑ **In the inertial range, the energy cascade remains the dominant dynamical process :**
- **the two fluid components are locked**
- **scaling properties agree with that of classical turbulence for all temperatures**

Fully consistent with Zhang Z, Danaila I, Lévêque E, Danaila L.
``Higher-order statistics and intermittency of a two-fluid Hall-Vinen-Bekharevich-Khalatnikov quantum turbulent flow'' *J. Fluid Mech.*. 2023;962:A22