Quantum turbulence: From the Kolmogorov cascade to sound emission, passing by Kelvin waves and vortex reconnections.

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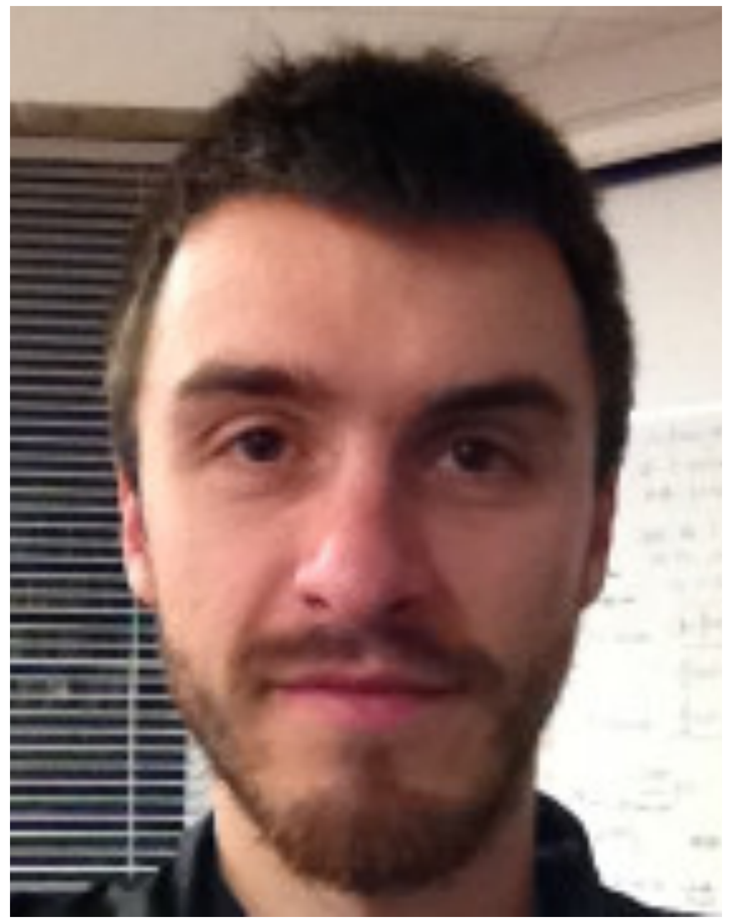
In collaboration with ...

University of East Anglia.
Norwich, UK

Davide Proment



Alberto Villois
(Now in Torino)

Umberto Giuriato
Former PhD. Student

Nicolas Müller
Former PhD. Student

International Exchanges Cost Share Scheme

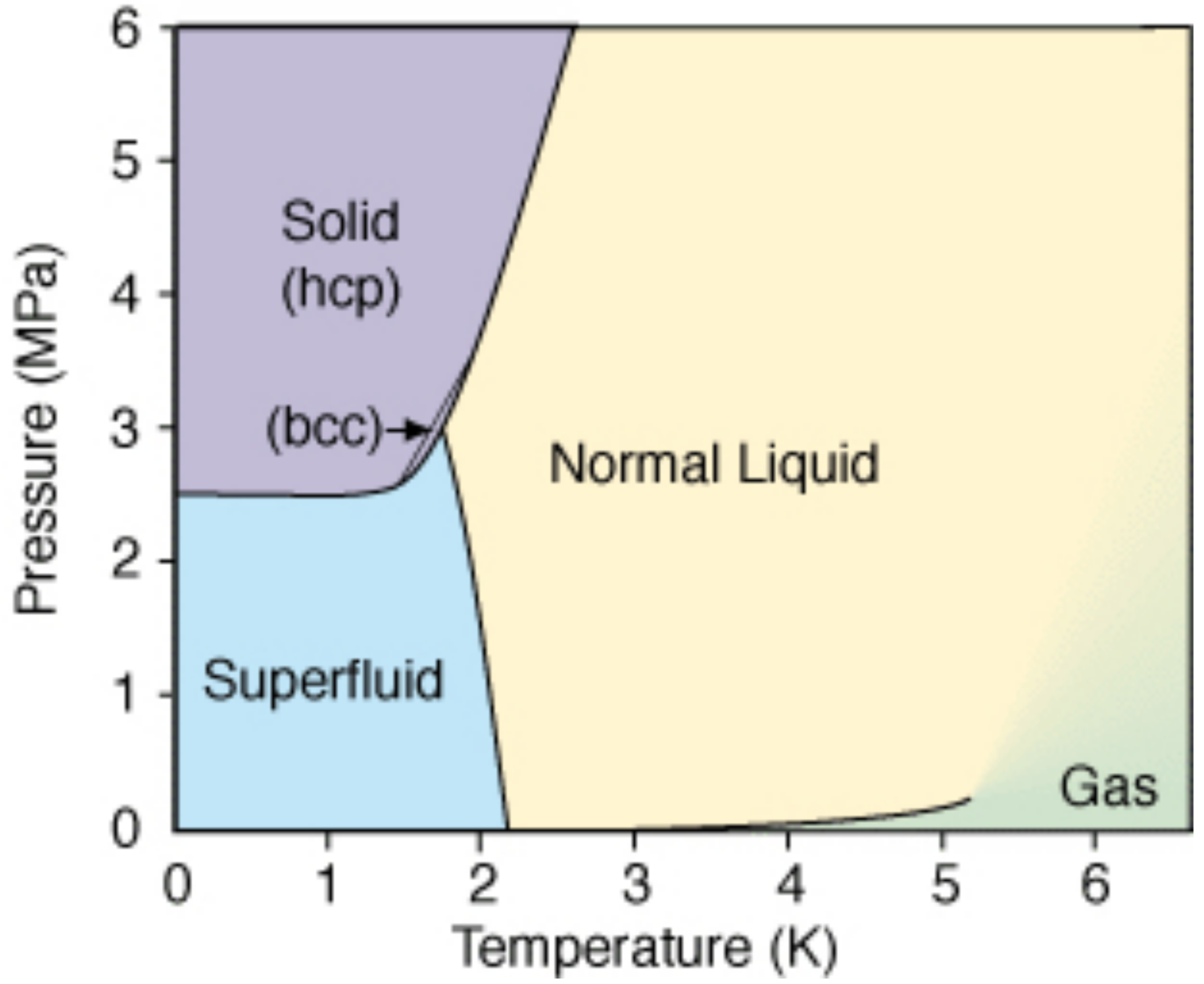
Superfluids

Superfluid ^{4}He

$$T \sim 2K$$

They have no viscosity !

Helium phase diagram



Picture from: Low Temperature Laboratory, Aalto University webpage

Kapitsa, Allen and Misener, 1937

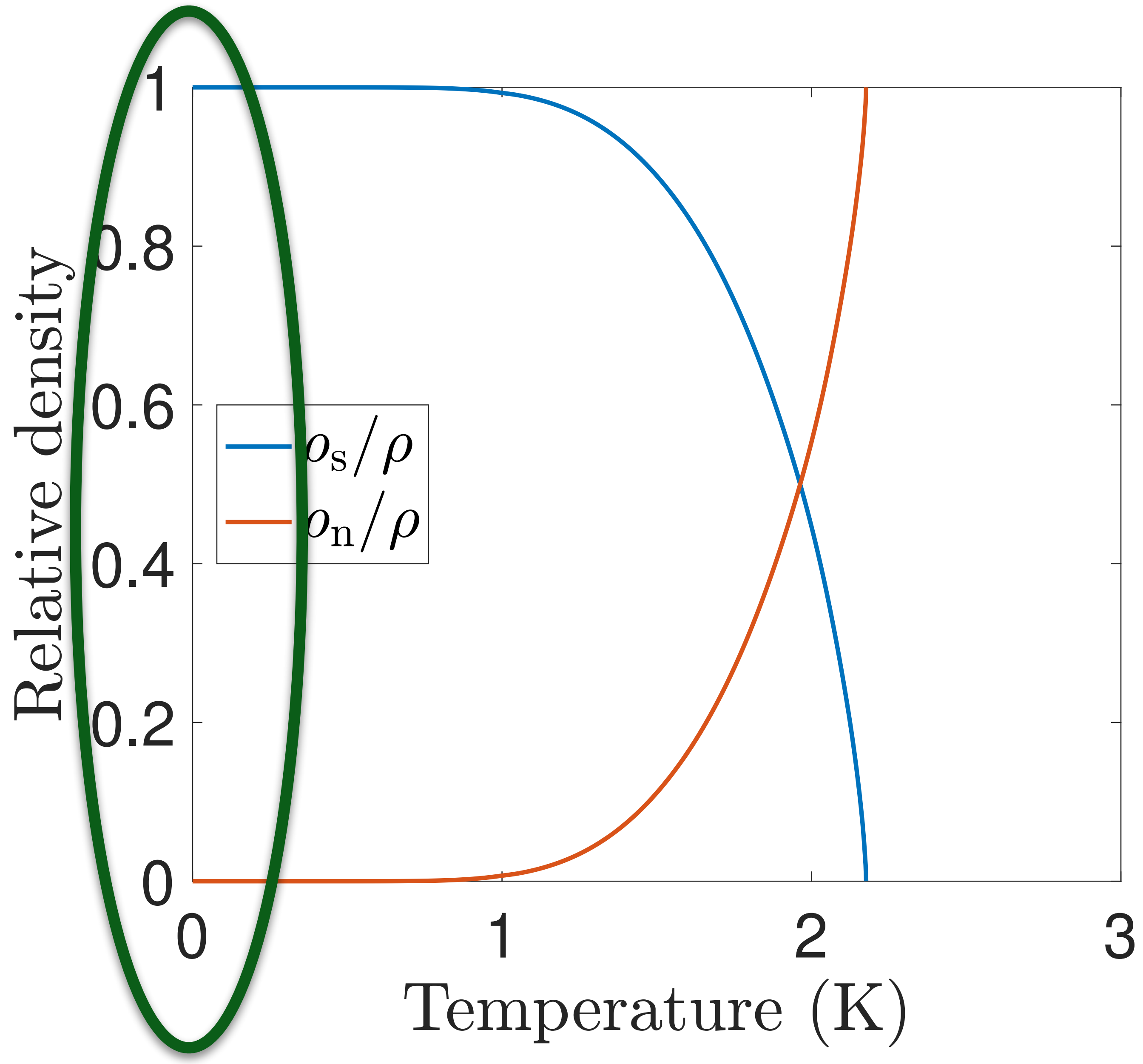
Landau-Tiszla description of superfluid helium

Two immiscible fluids:

- ✦*normal* (viscous) fluid of density ρ_n
- ✦*superfluid* of density ρ_s

$$\rho = \rho_\mathrm{n} + \rho_\mathrm{s}$$

$$\mathbf{P} = \rho_\mathrm{n}\mathbf{v}_\mathrm{n} + \rho_\mathrm{s}\mathbf{v}_\mathrm{s}$$



Today's talk

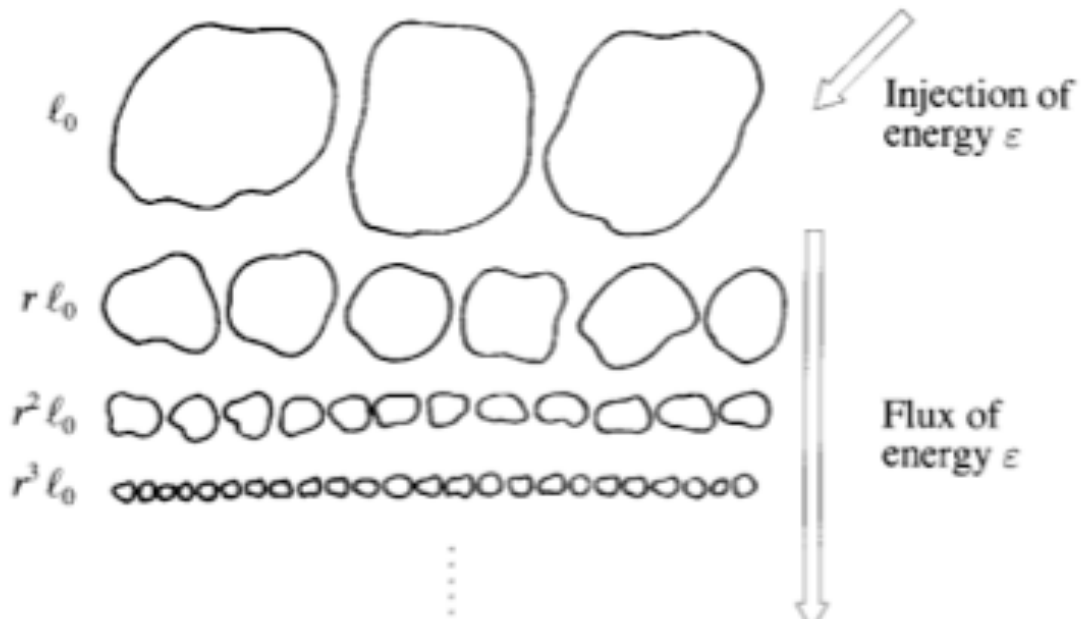
Length scales of superfluid turbulence

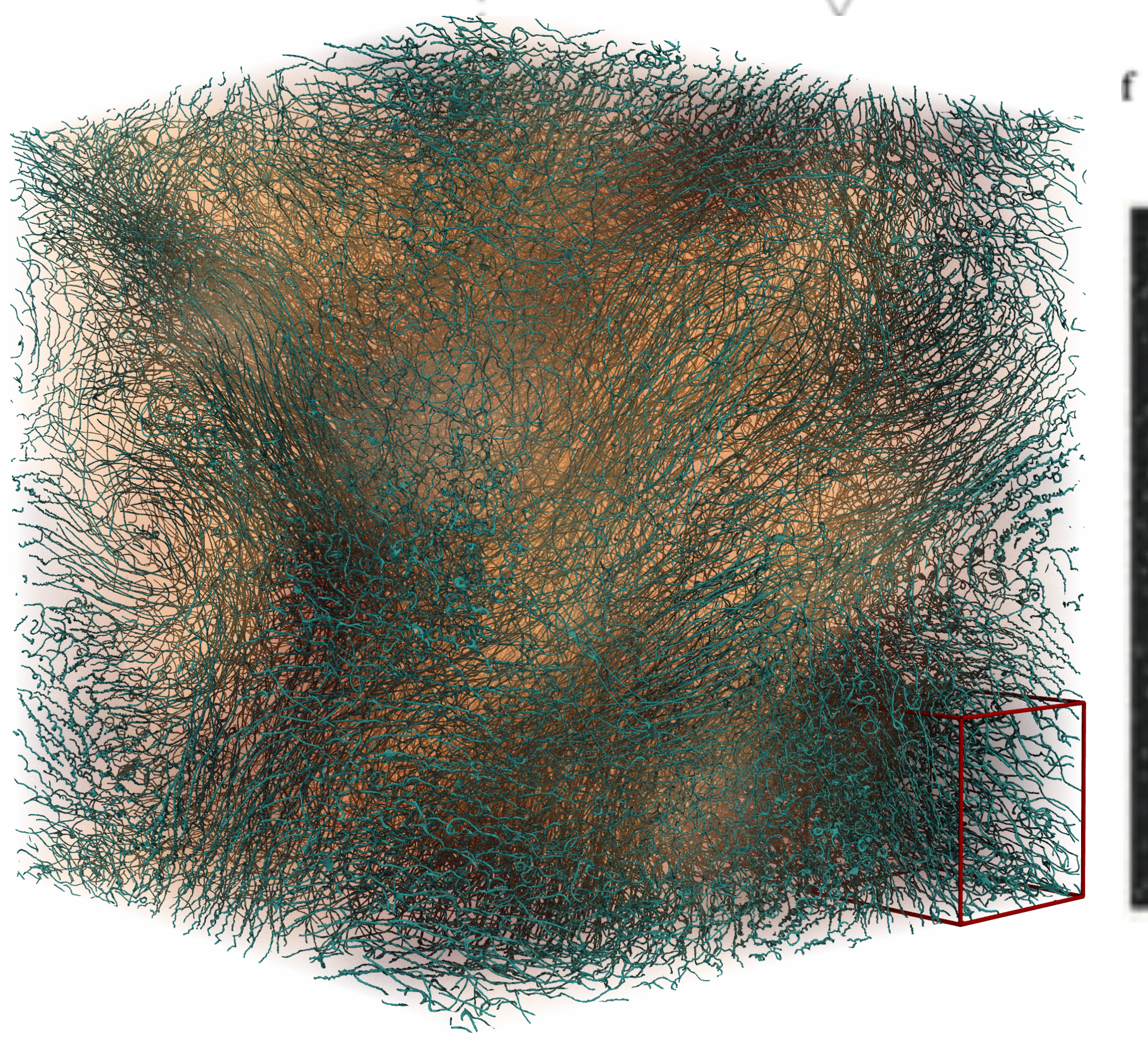
energy injection
$\sim m$

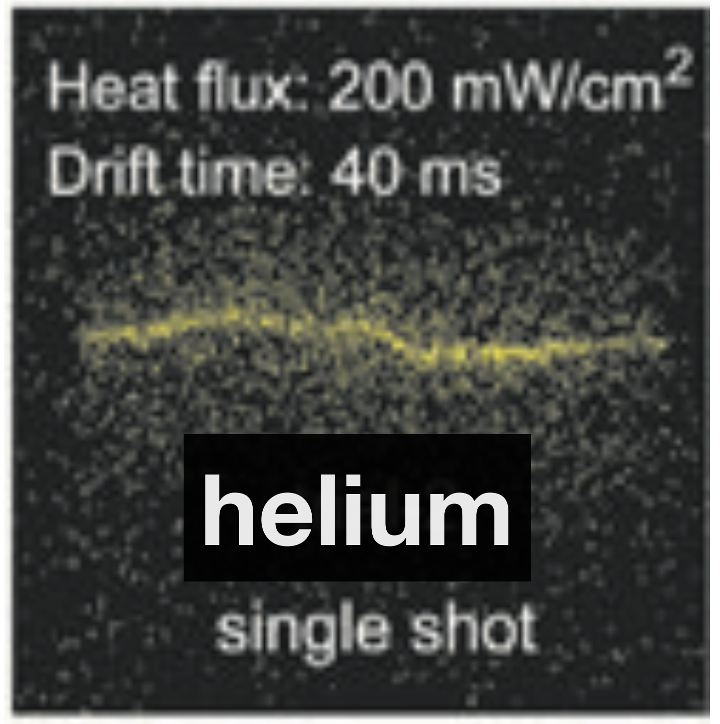
ℓ_{I}

SHREK (France)

Classical (Kolmogorov) turbulence



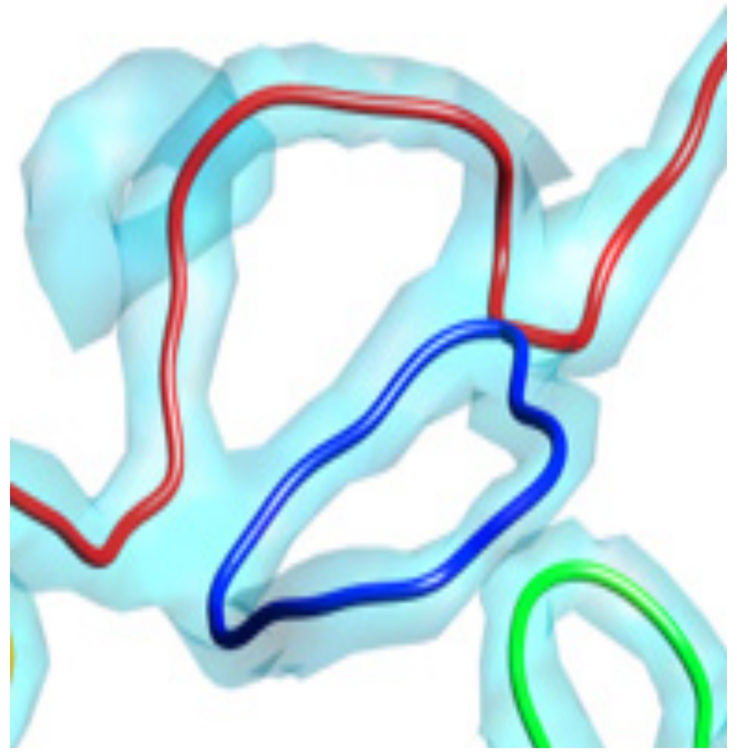




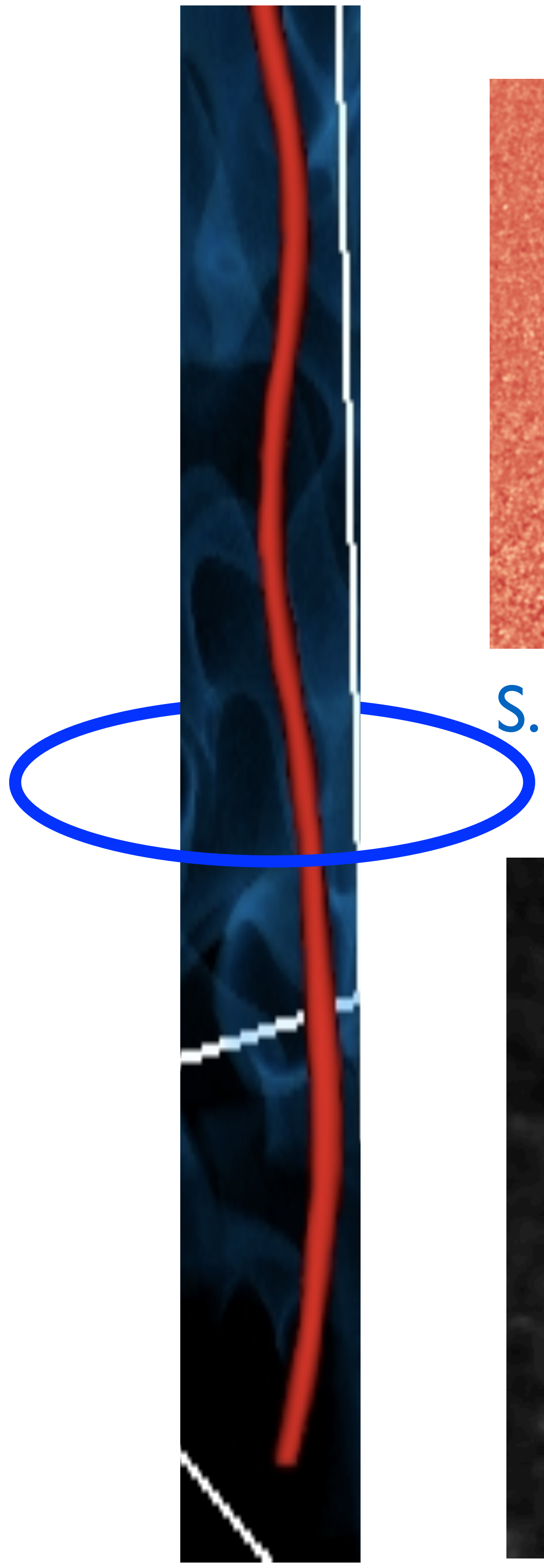
Wei Guo's group

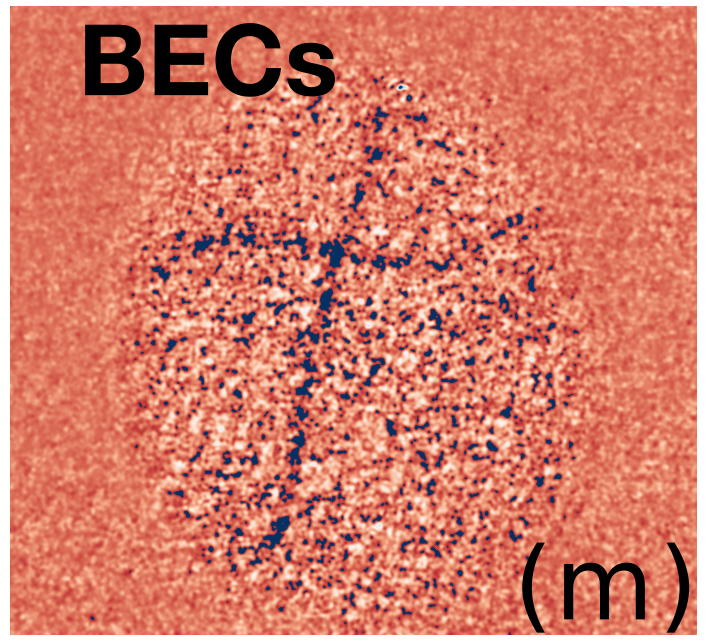
inter-vortex distance
$\sim 10^{-5} m$

ℓ

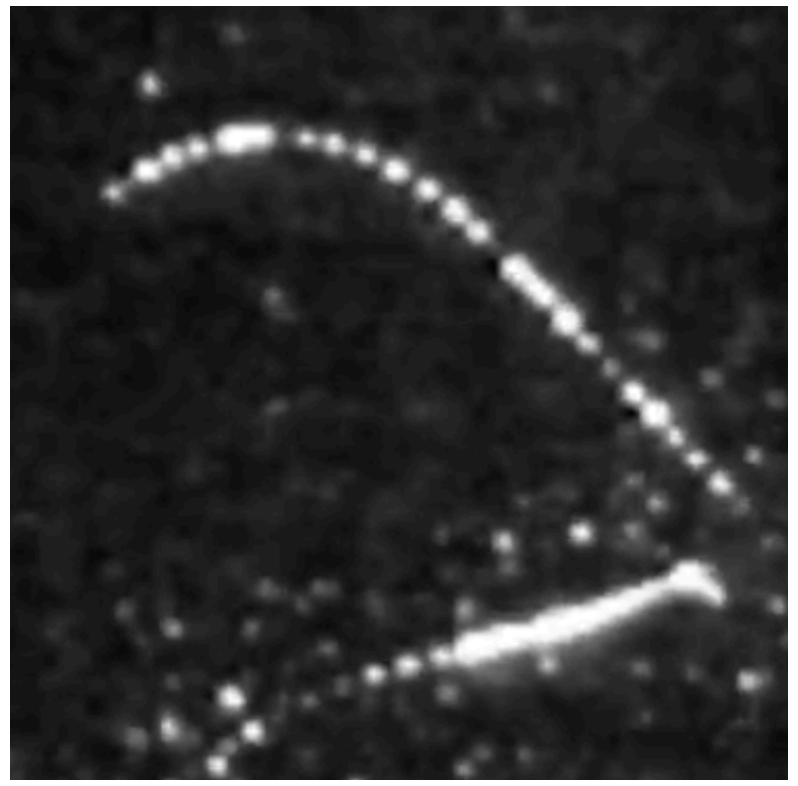


Kelvin wave cascade & vortex reconnections





S.Serafini et al. PRL 2015

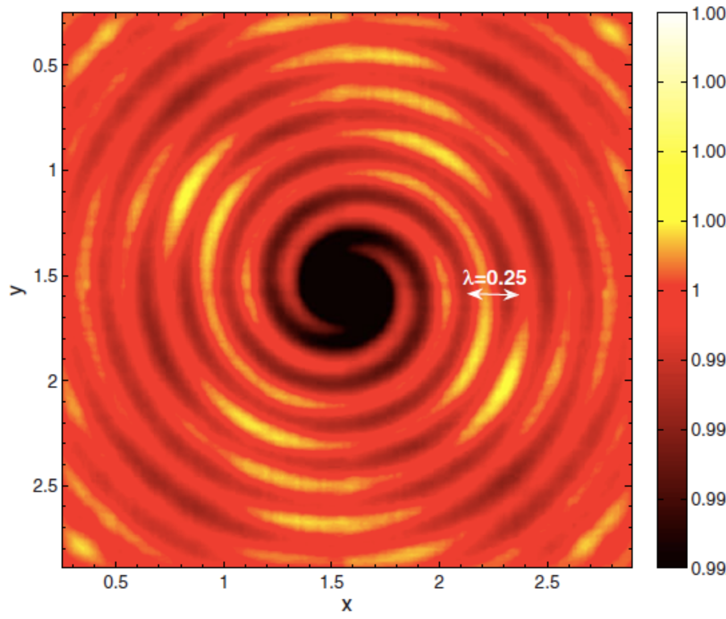


G. Bewley et al. Nature 2006.

coherence length vortex core size
$\sim \AA$

ξ

sound emission

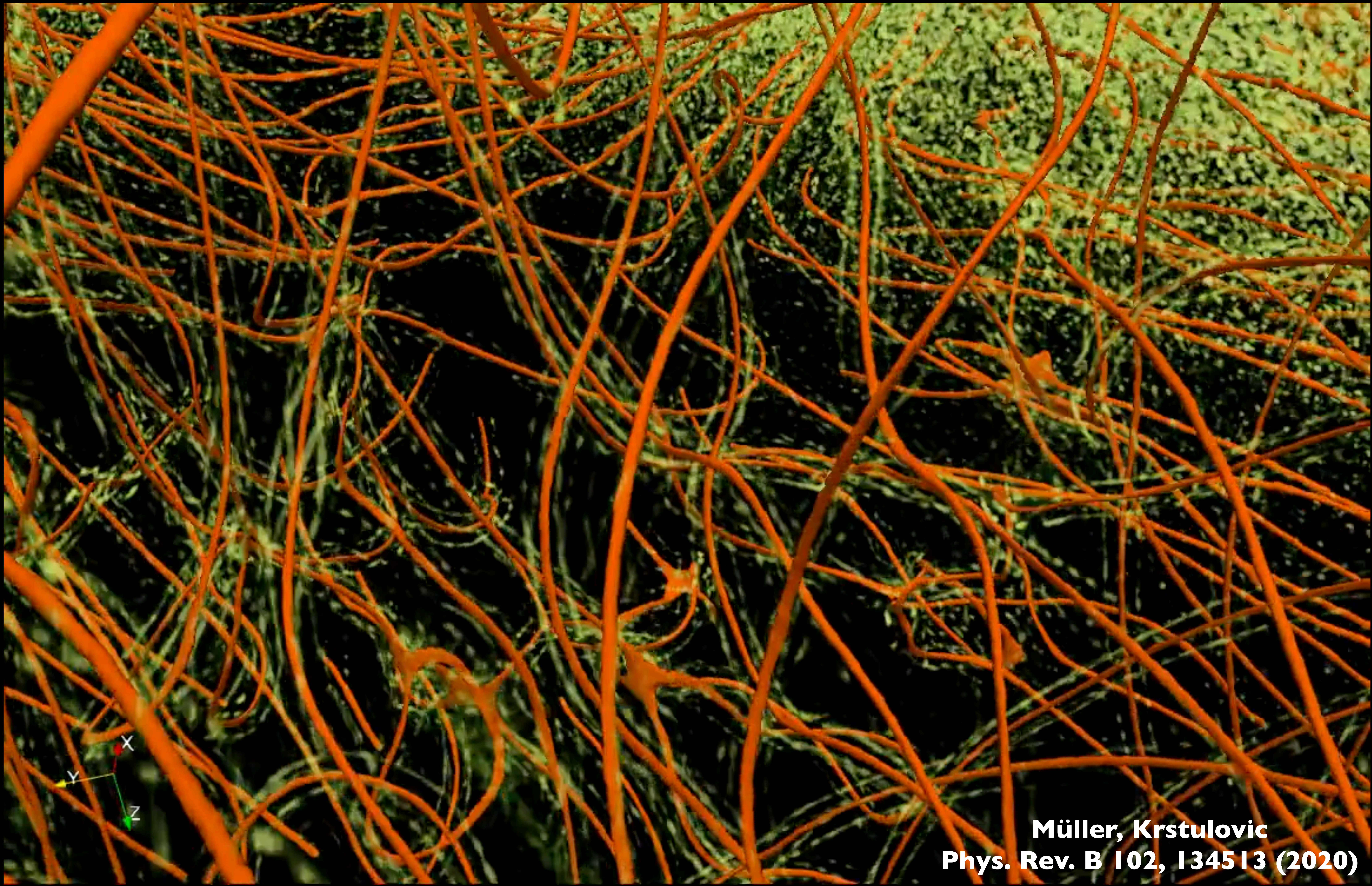


Experiments: Maurer et al. (1998), Salort et al. (2010), Tang et al. (2021), …
Simulations in GP: Nore et al. (1997), Kobayashi et al. (2005), …
Simulations in vortex-filament method: Baggaley et al. (2012), …

Quantum vortices

Density fluctuations

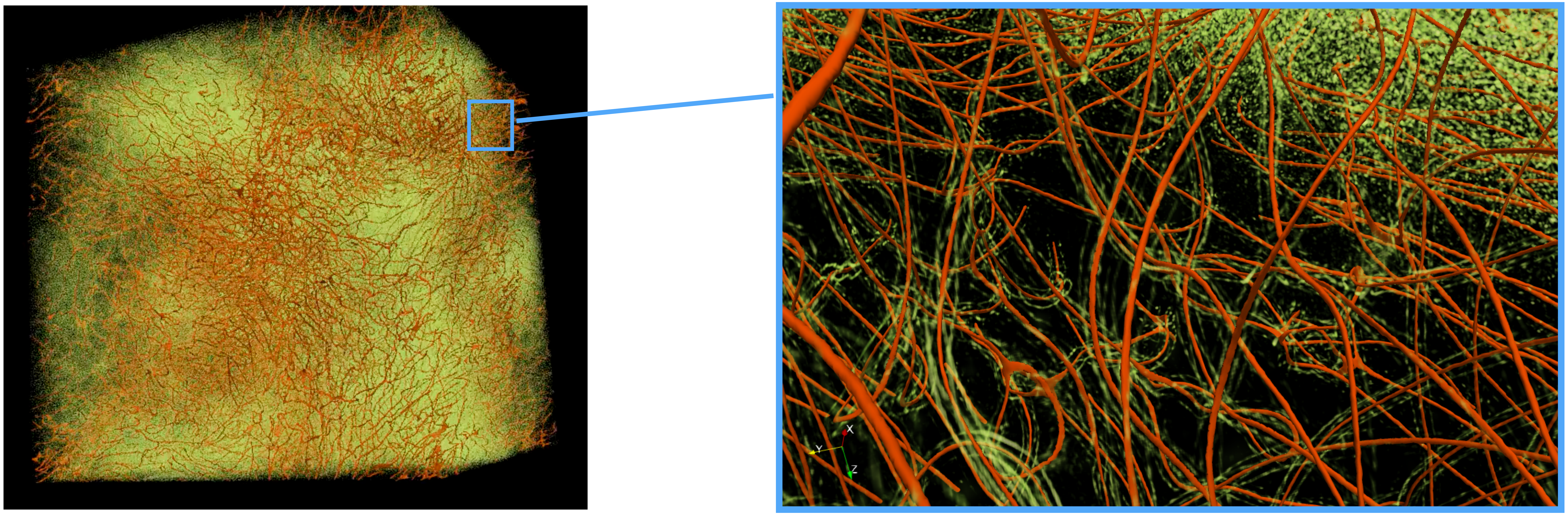
3D Gross-Pitaevskii



Müller, Krstulovic
Phys. Rev. B 102, 134513 (2020)

Quantum vortices and turbulence

- At "zero-temperature", a superfluid has no viscosity
- Compressible fluid (and dispersive)
- Described by a complex order parameter (wave function)
- Quantum vortices (filaments) are naturally present in turbulent states



Numerical simulation of Gross-Pitaevskii (a.k.a NLS)

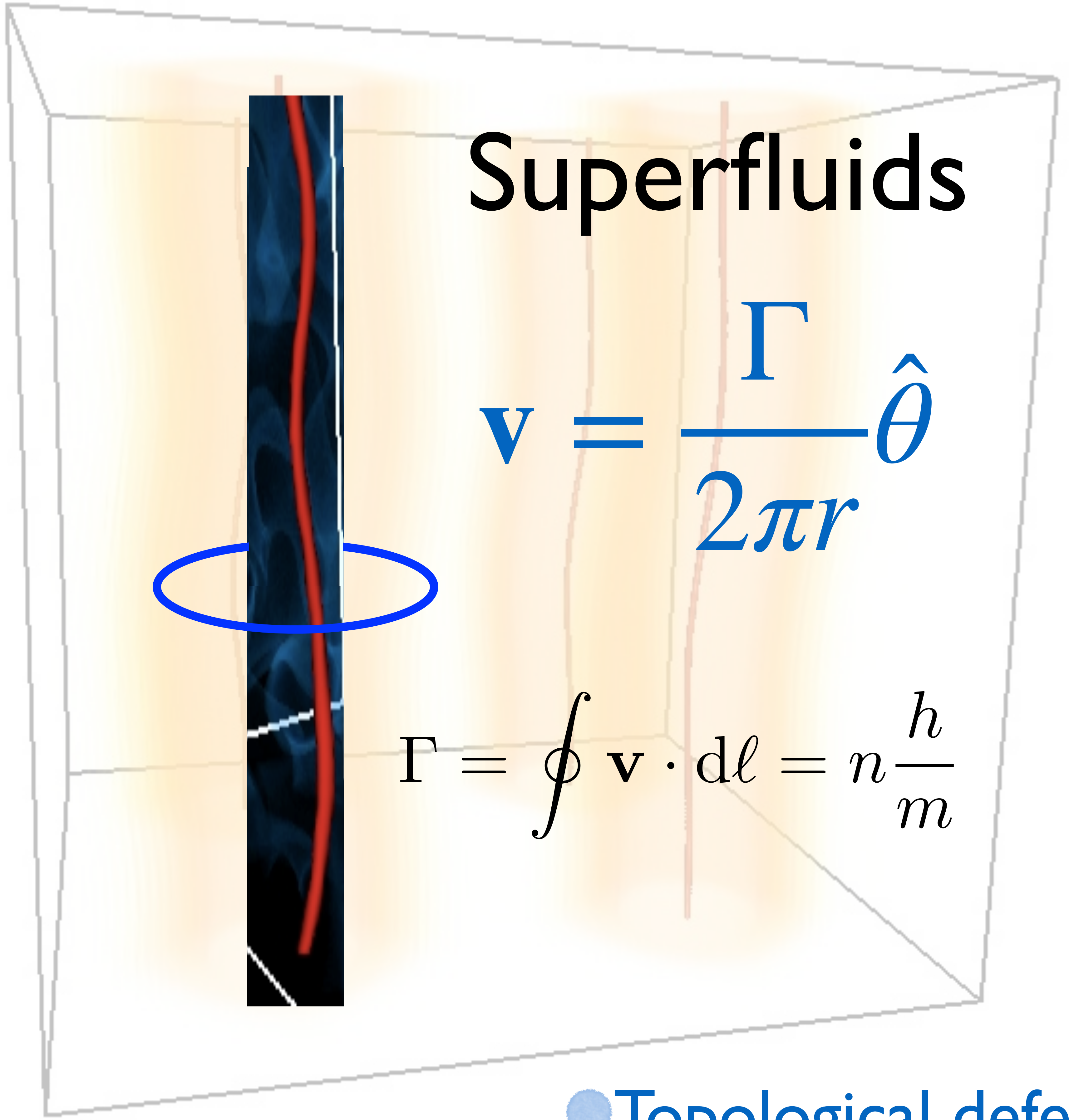
Müller, Krstulovic
Phys. Rev. B 102, 134513 (2020)

Quantum vortices

- Finite core-size
- Continuous circulation

vortex core

$$\xi \to 0$$



- Topological defects
- Quantised circulation

$\xi \sim$ Å (^{4}He)

$\xi \sim \mu m$ (BEC)

Modeling superfluid helium

Multi-scale physics

Scales

Classical fluid (Navier-Stokes)

Modeling superfluid helium

Multi-scale physics

Scales

vortex core size

ξ

mean inter-vortex

ℓ

$T = 0$

Temperature

$T = 2.17K$

Gross-Pitaevskii based model

Vortex filament
&
Coupled Navier-Stokes and vortex filament method
(FOUCAULT)

Projected Gross-Pitaevskii

Hall-Vinen-Bekarevich-Khalatnikov
(HVBK)

Classical fluid (Navier-Stokes)

Modeling superfluid helium

Multi-scale physics

Scales

vortex core size

ξ

mean inter-vortex distance

ℓ

$T = 0$

Gross-Pitaevskii based model

Temperature

$T = 2.17K$

Classical fluid (Navier-Stokes)

The Gross-Pitaevskii equation

Modelling low-temperature superfluids

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g|\psi|^2\psi,} \qquad g = \frac{4\pi a\hbar^2}{m}$$

Linearising about a flat state:

$$\psi = A_0 e^{-i\frac{\mu}{\hbar}t} + \delta\psi$$

Bogoliubov dispersion relation:

$$\omega(k) = c\,k\sqrt{1 + \frac{1}{2}\xi^2 k^2}$$

$$\omega(k) = \sqrt{\frac{g|A_0|^2}{m}k^2 + \frac{\hbar^2}{4m^2}k^4}.$$

Speed of sound $c = \sqrt{g|A_0|^2/m}$

Coherence length $\xi = \sqrt{\hbar^2/2m|A_0|^2 g}$

Hydrodynamics?

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g|\psi|^2\psi,}$$

Speed of sound $c = \sqrt{g|A_0|^2/m}$

Coherence length $\xi = \sqrt{\hbar^2/2m|A_0|^2 g}$

Madelung transformation

$$\psi(\mathbf{x},t) = \sqrt{\frac{\rho(\mathbf{x},t)}{m}}\exp\left[i\frac{m}{\hbar}\phi(\mathbf{x},t)\right] = \sqrt{\frac{\rho(\mathbf{x},t)}{m}}\exp\left[i\frac{\phi(\mathbf{x},t)}{\sqrt{2}c\xi}\right]$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\,\nabla\,\phi) = 0,$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 = c^2(1-\rho) + c^2\xi^2\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}.$$

density of particles

$$\rho = |\psi|^2$$

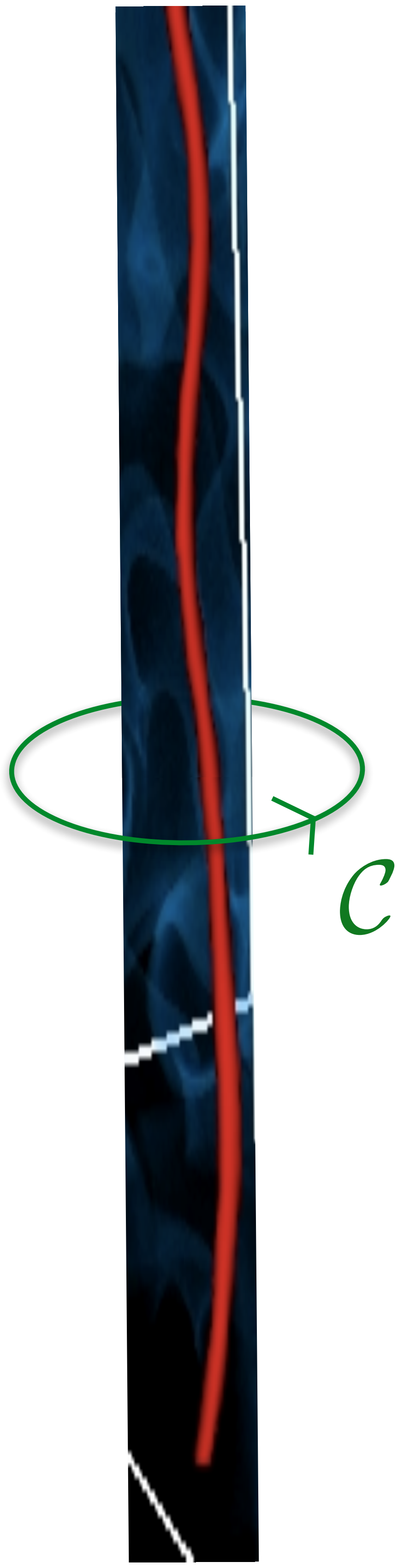
$\mathbf{v} = \nabla\phi$ is a potential flow

Quantum vortices

$\mathbf{v} = \nabla\phi$ is a potential flow but:

Vortices are topological defects: $\psi(\mathbf{x}) = \mathbf{0}$

$$\Gamma = \oint_{\mathcal{C}} \nabla\phi \cdot \mathrm{d}\ell = \phi^{+} - \phi^{-}$$

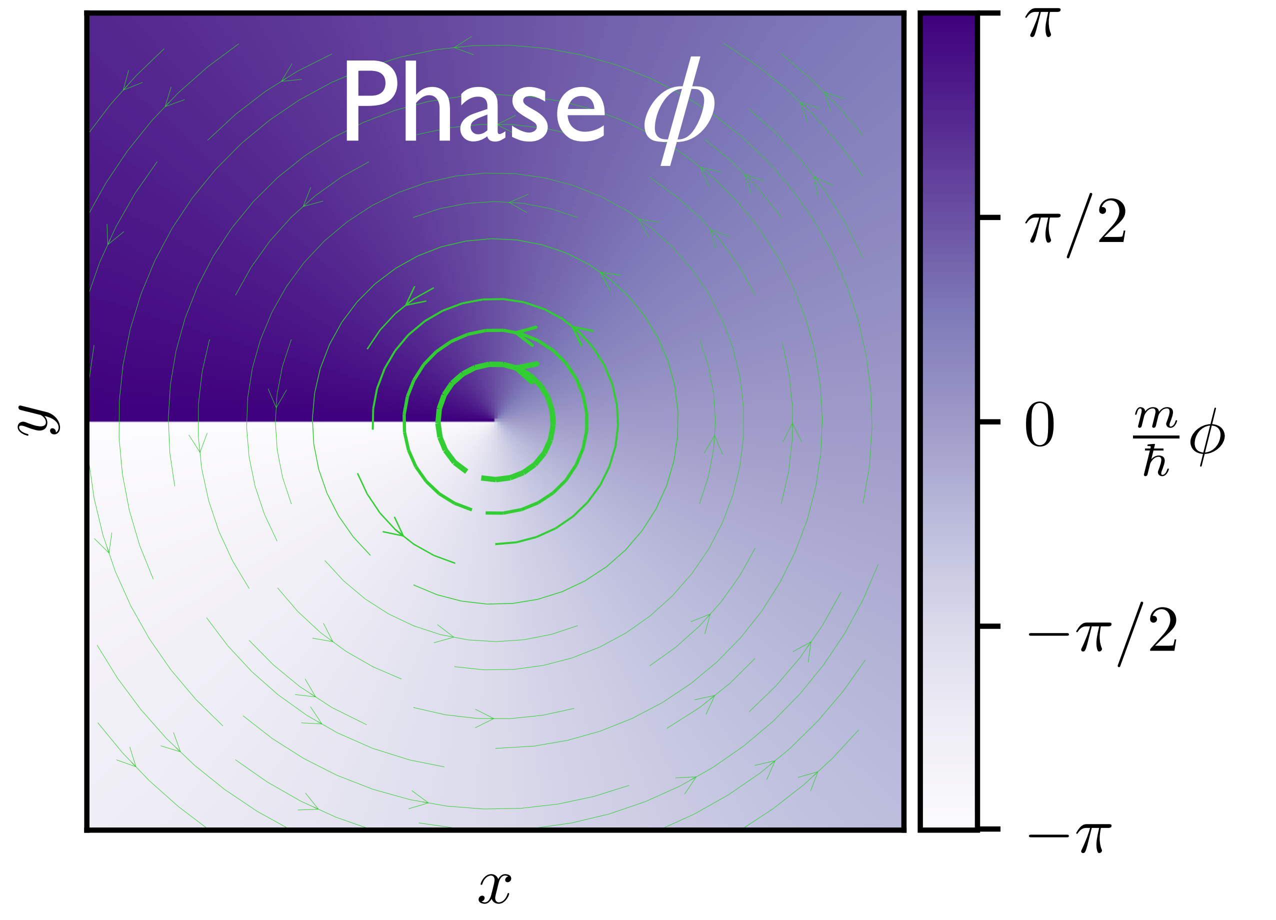


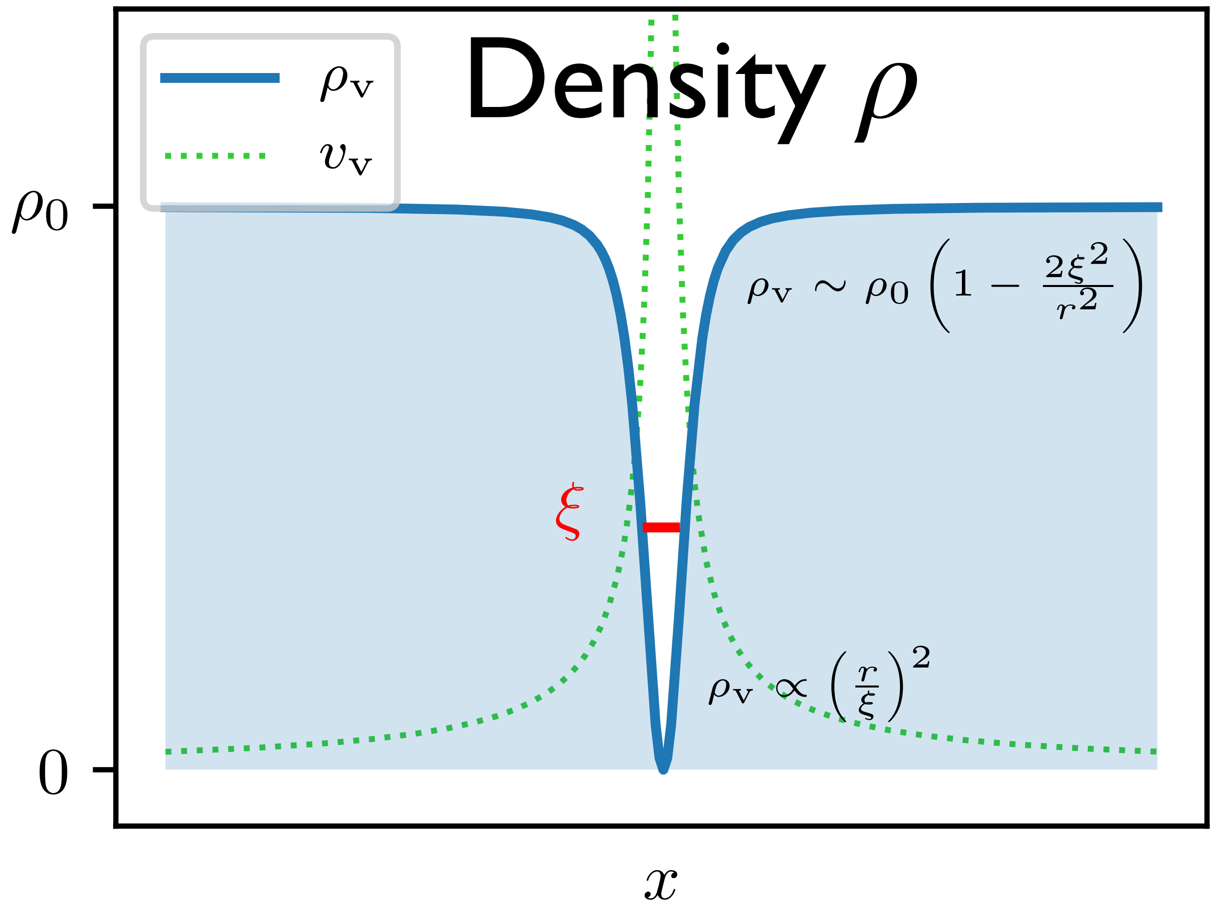
$$\boxed{\Gamma = n\frac{h}{m} = n2\pi\sqrt{2}c\xi, \quad \text{with}\, n \in \mathbb{Z}}$$

Points in $2D$ and lines in $3D$

$$\boxed{\mathbf{v} \sim \frac{1}{r} \Rightarrow \nabla \times \mathbf{v} \sim \delta(\mathbf{r})}$$

$$\mathbf{w}(\mathbf{x}) = \nabla \times \mathbf{v} = \frac{h}{m}\oint \delta(\mathbf{x} - \mathbf{s}(\ell))\frac{\mathrm{d}\mathbf{s}(\ell)}{\mathrm{d}\ell}\mathrm{d}\ell$$





Quantum vortices

$\mathbf{v} = \nabla\phi$ is a potential flow but:

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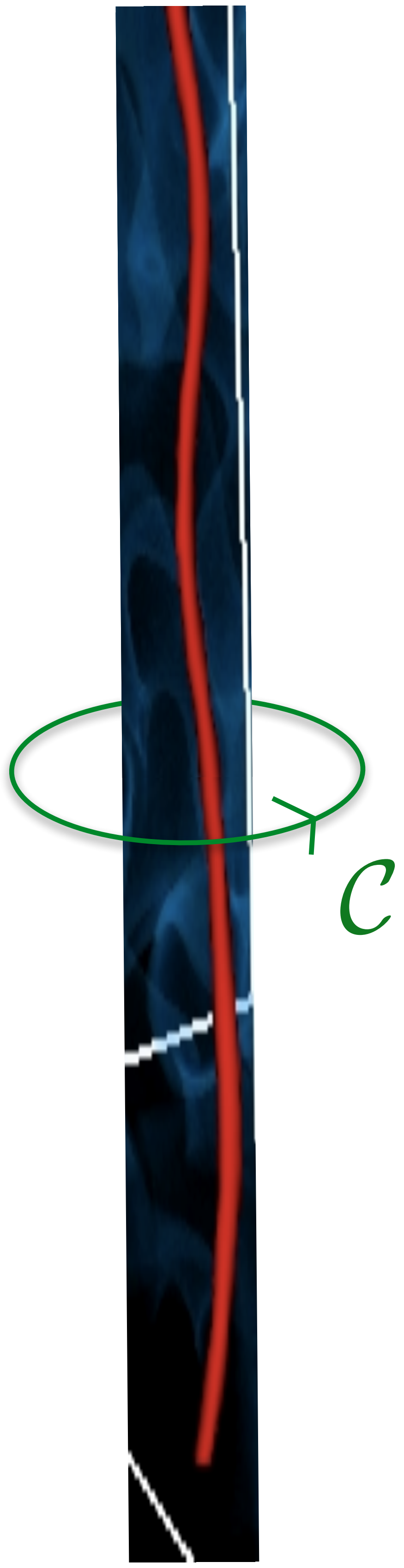
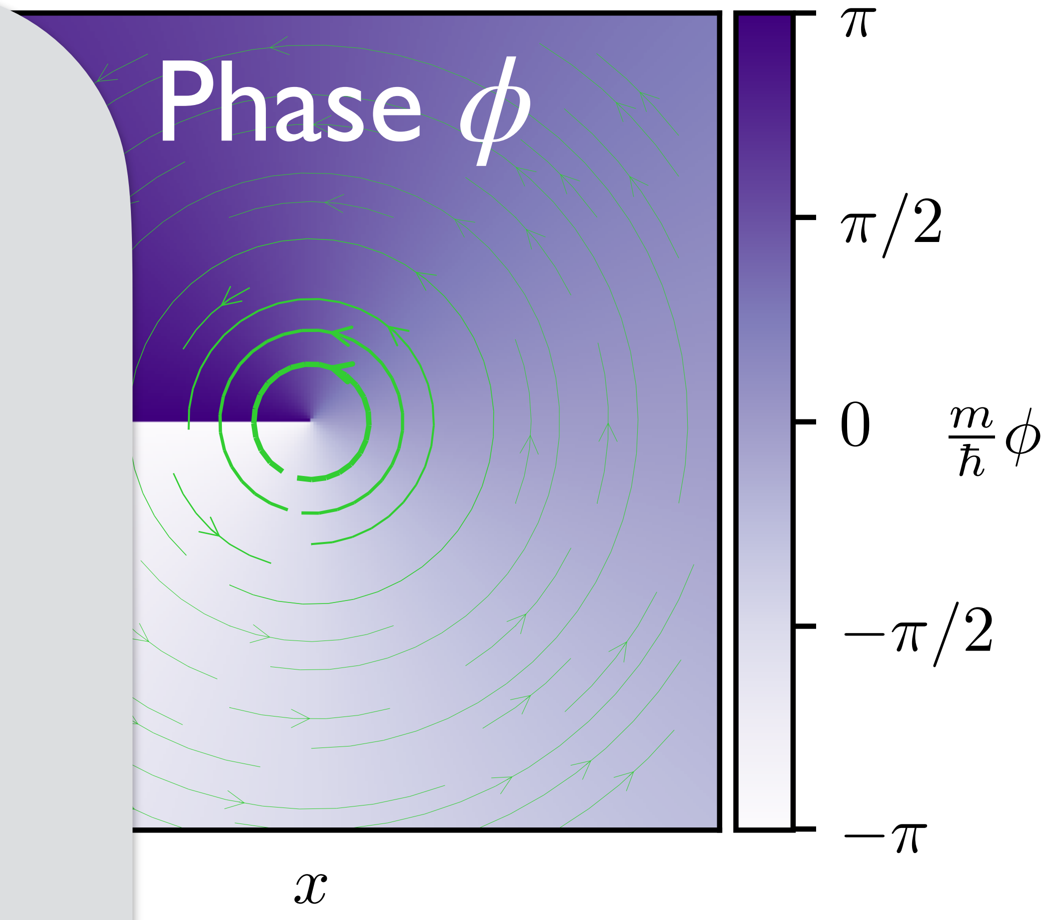




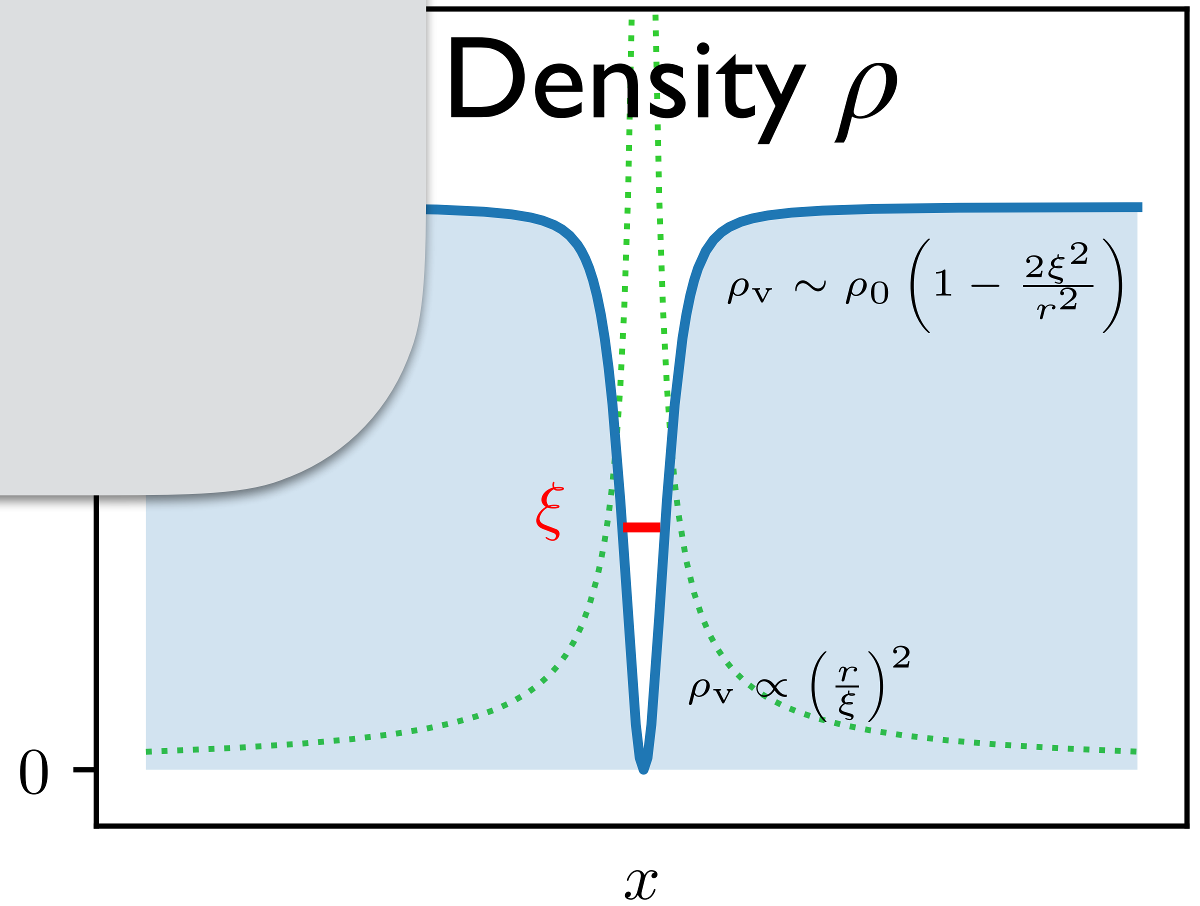
Poin

Gross-Pitaevskii model

Collection of "ideal" vortex filaments:

- Velocity field $\vec{v} = \dfrac{\Gamma}{2\pi r}\hat{\theta}$
- Core size ξ (small)

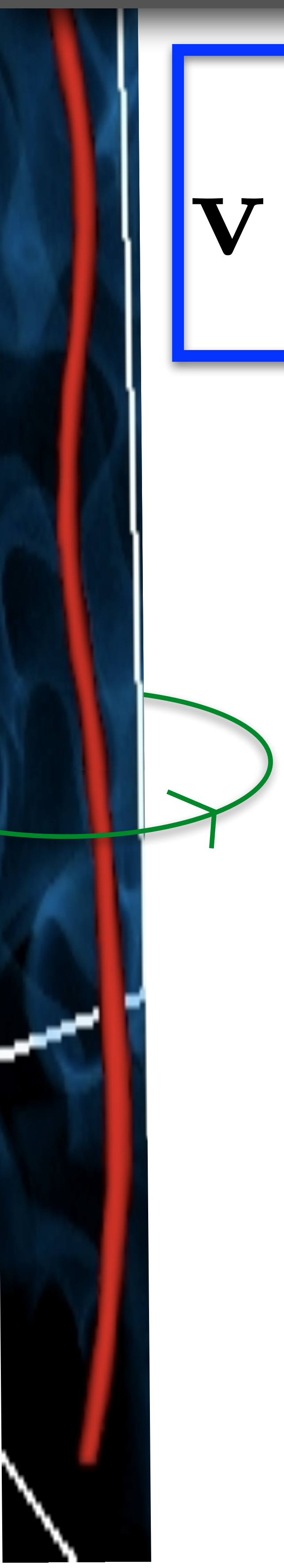
Euler equation + small-scale dispersion regularisations



$$\mathbf{w}(\mathbf{x}) = \nabla \times \mathbf{v} = \frac{h}{m}\oint \delta(\mathbf{x} - \mathbf{s}(\ell))\frac{\mathrm{d}\mathbf{s}(\ell)}{\mathrm{d}\ell}\mathrm{d}\ell$$

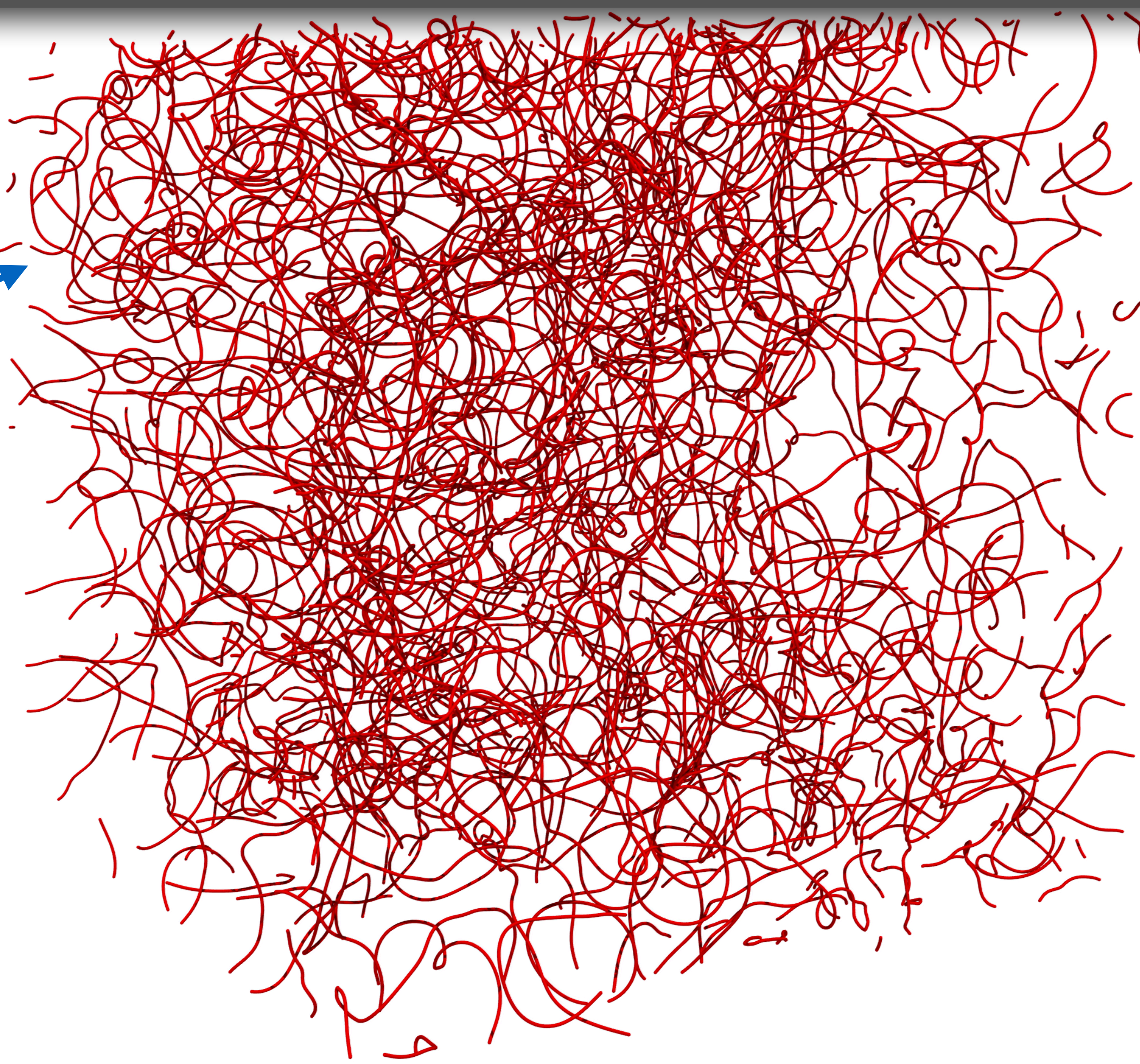
Vortex filament method

At $T = 0$



$$\boxed{\mathbf{v} \sim \frac{1}{r} \Rightarrow \nabla \times \mathbf{v} \sim \delta(\mathbf{r})}$$

Collection of vortex filaments $\mathscr{C}$



Vorticity field $\omega_s(\mathbf{x}) = \Gamma \oint_{\mathscr{C}} \delta(\mathbf{x} - \mathbf{s}(\zeta)) \frac{d\mathbf{R}}{d\zeta} d\zeta$

Velocity field

$$\mathbf{v}_s(\mathbf{x}) = -\frac{1}{4\pi} \int \frac{(\mathbf{x} - \mathbf{y})}{|\mathbf{x} - \mathbf{y}|^3} \times \omega(\mathbf{y})\, d^3\mathbf{y} = -\frac{\Gamma}{4\pi} \oint_{\mathscr{C}} \frac{\mathbf{x} - \mathbf{s}(\zeta)}{|\mathbf{x} - \mathbf{s}(\zeta)|^3} \times \frac{d\mathbf{R}}{d\zeta} d\zeta$$

Schwarz vortex filament method

$$\frac{d\mathbf{R}(\zeta, \mathbf{t})}{dt} = \mathbf{v}_s(\mathbf{R}(\zeta, t))$$

\+

- Regularisation of Biot-Savart
- Ad-hoc reconnection rule
- Ad-hoc small-scale dissipation

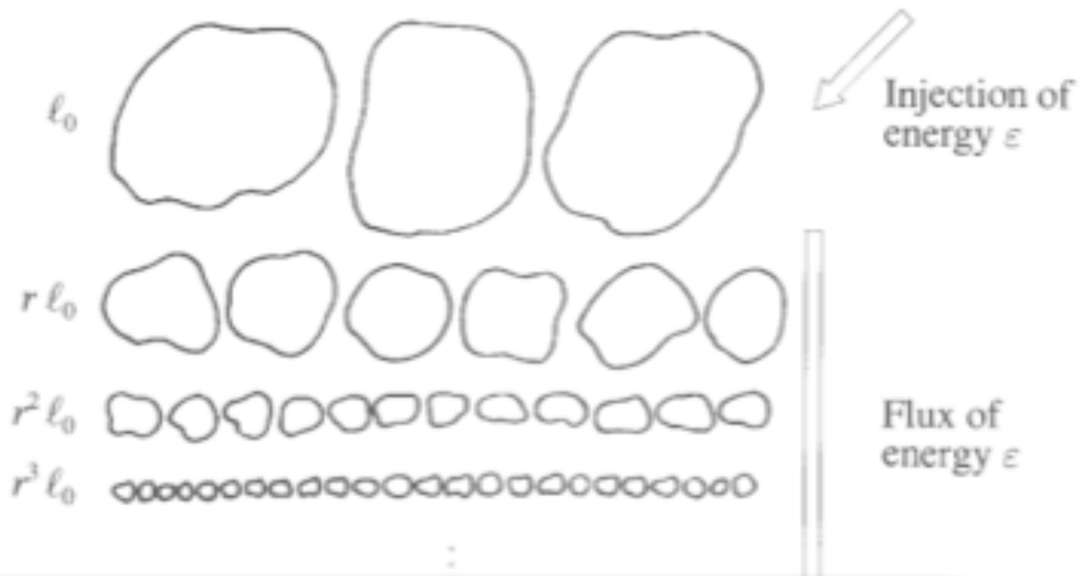
Length scales of ence

energy
injection
$\sim m$

Classical (Kolmogorov)
turbulence

$\ell_{\rm I}$

SHRE



GP and Navier-Stokes turbulence equivalence in 2D and 3D (intermittency)

See J.I. Polanco and
N. P. Müller talks next week

Experiments: Maurer et al. (1998), Salort et al. (2010), Tang et al. (2021), …
Simulations in GP: Nore et al. (1997), Kobayashi et al. (2005), …
Simulations in vortex-filament method: Baggaley et al. (2012), …

Today's talk

inter-vortex
distance
$\sim 10^{-5} m$

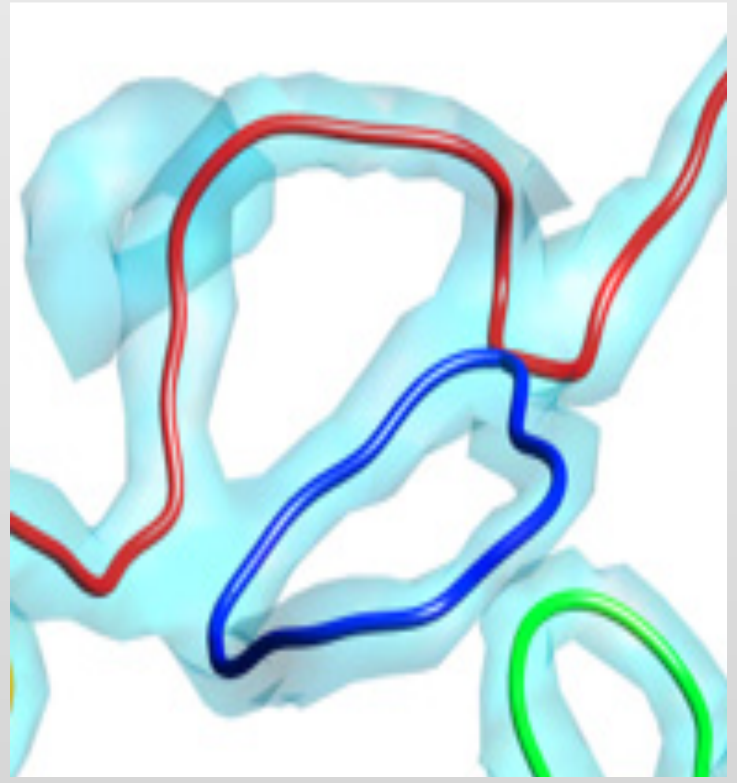
Kelvin wave cascade
&
vortex reconnections

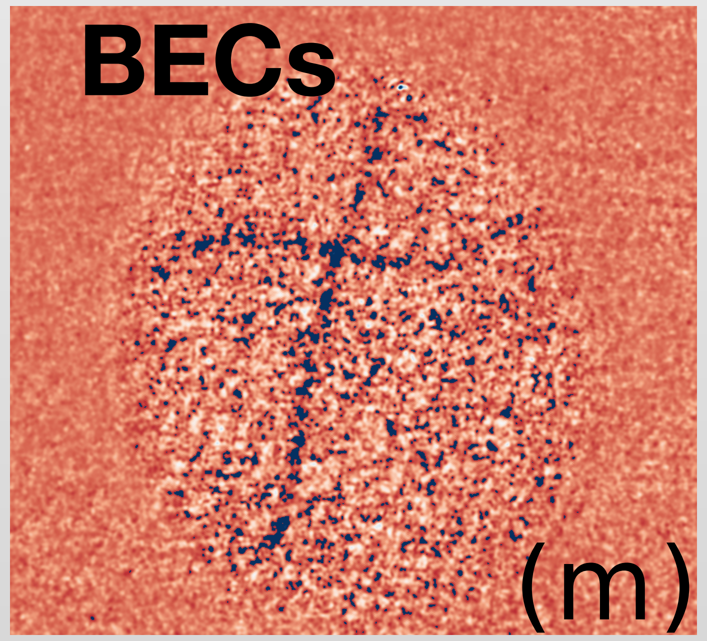
coherence length
vortex core size
$\sim \AA$

sound
emission

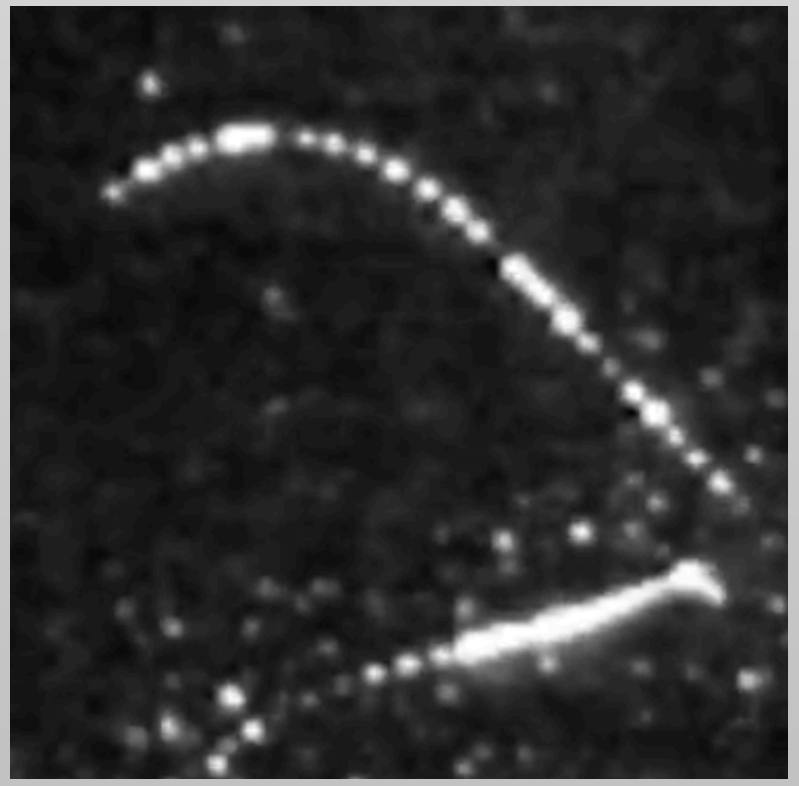
ℓ

ξ

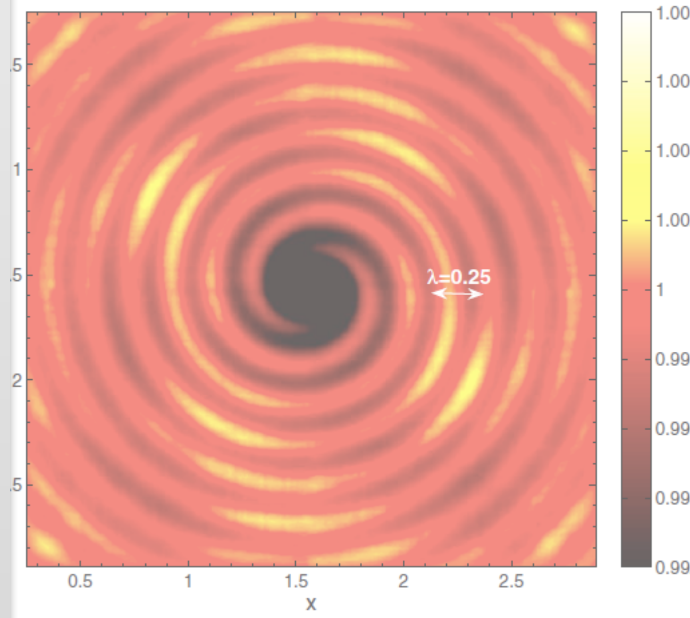




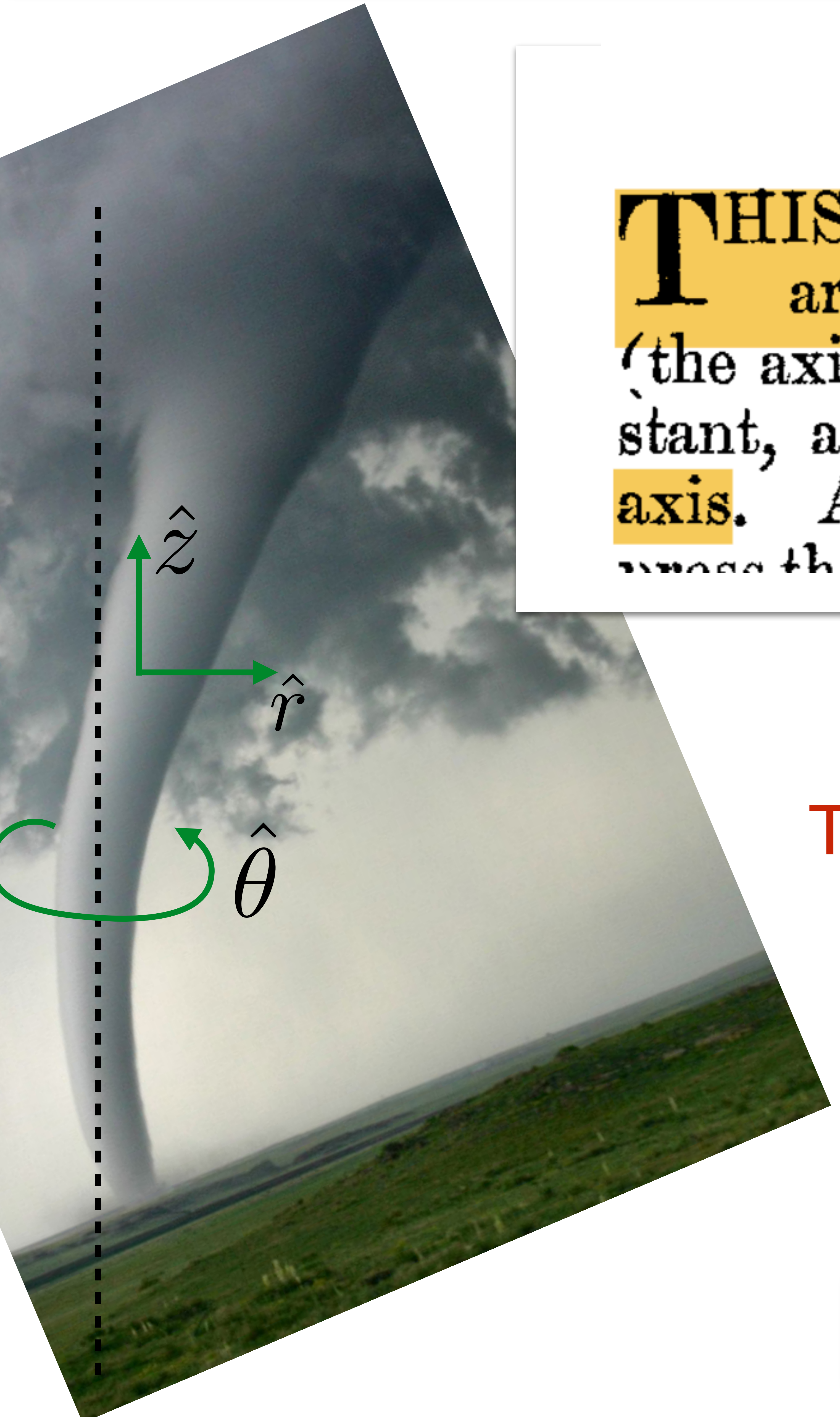
S.Serafini et al. PRL 2015

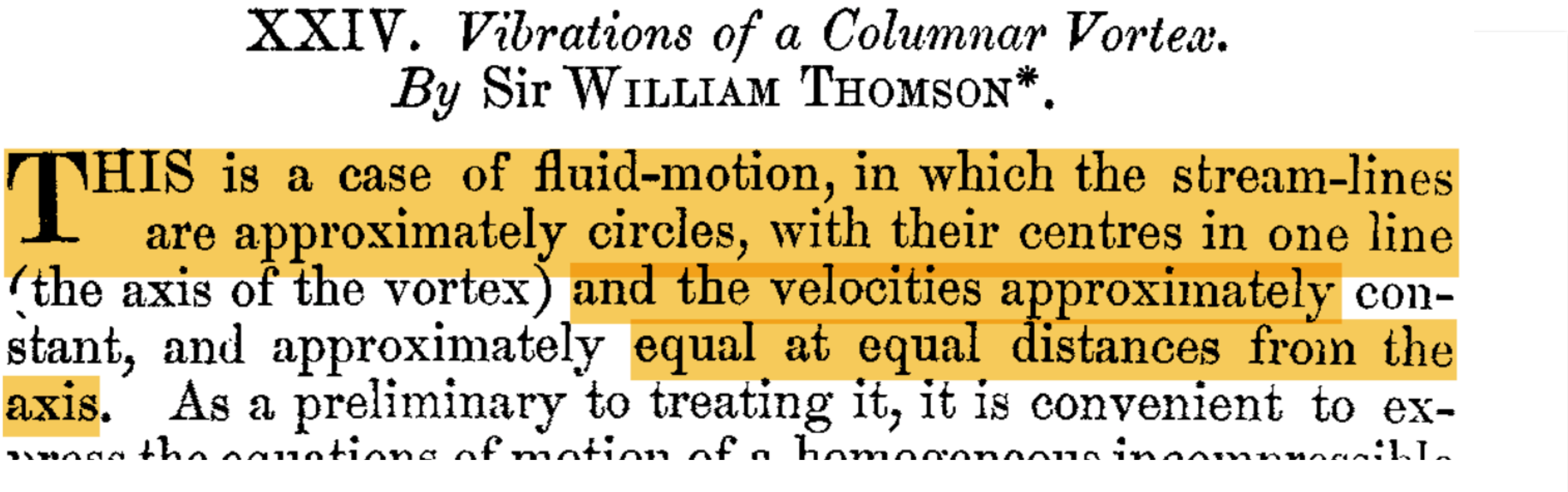


G. Bewley et al. Nature 2006.



Kelvin waves





XXIV. *Vibrations of a Columnar Vortex.*
By Sir WILLIAM THOMSON*.

THIS is a case of fluid-motion, in which the stream-lines are approximately circles, with their centres in one line (the axis of the vortex) and the velocities approximately constant, and approximately equal at equal distances from the axis. As a preliminary to treating it, it is convenient to express the equations of motion of a homogeneous incompressible

Sir William Thomson (1880) XXIV. Philosophical Magazine Series 5, 10:61, 155-168,

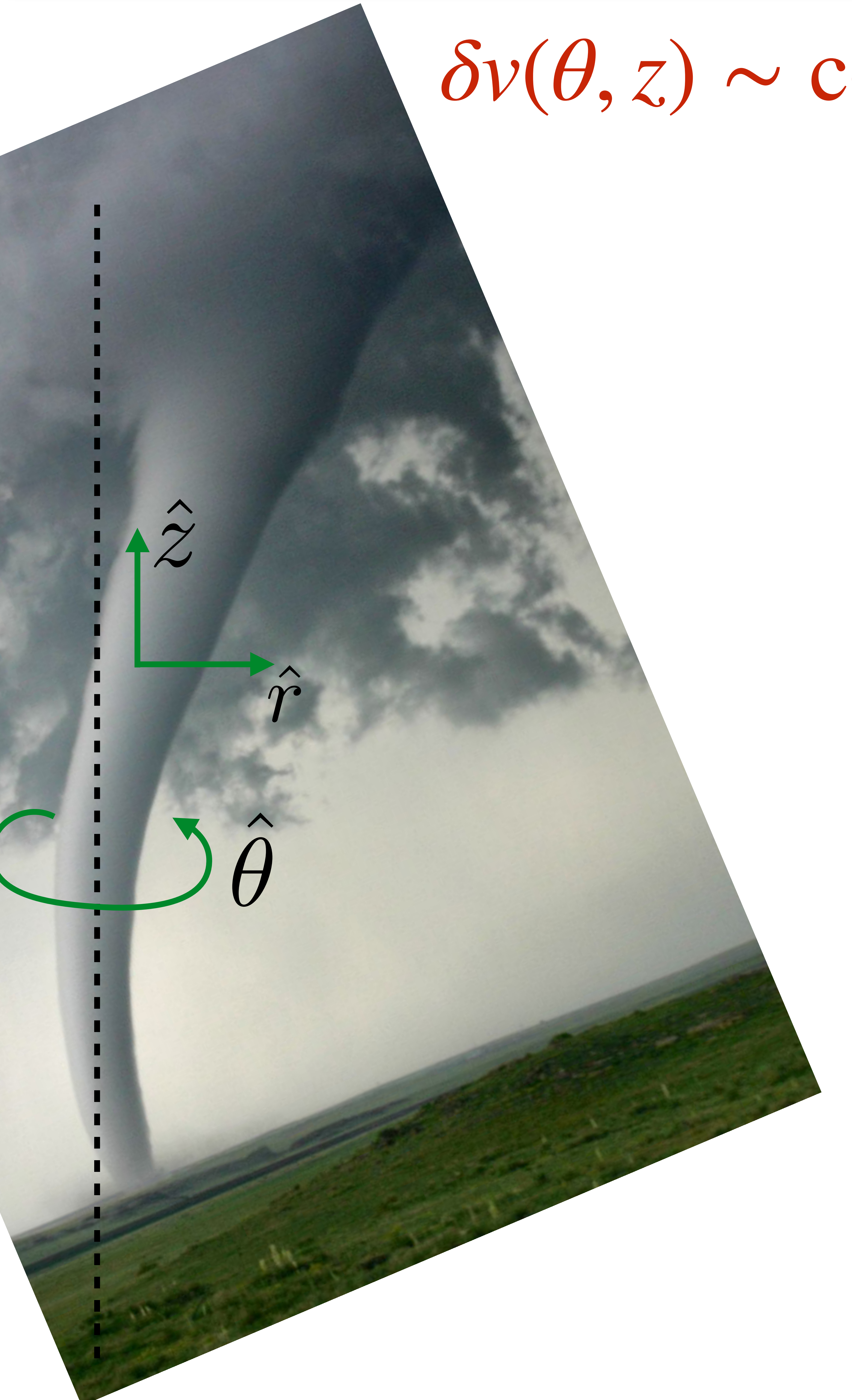
Take the incompressible Euler's equations

$$\mathbf{v}_0(r,\theta,z) = \frac{\alpha(r)}{r}\hat{\theta} \qquad \text{and} \qquad p(r,\theta,z) = p_0(r) = \rho_0 \int_{a_0}^{r} \frac{\alpha(s)^2}{s^3} \mathrm{d}s.$$

$$\Gamma = \oint_C \mathbf{v}\cdot \mathrm{d}\ell = 2\pi\alpha(r)$$

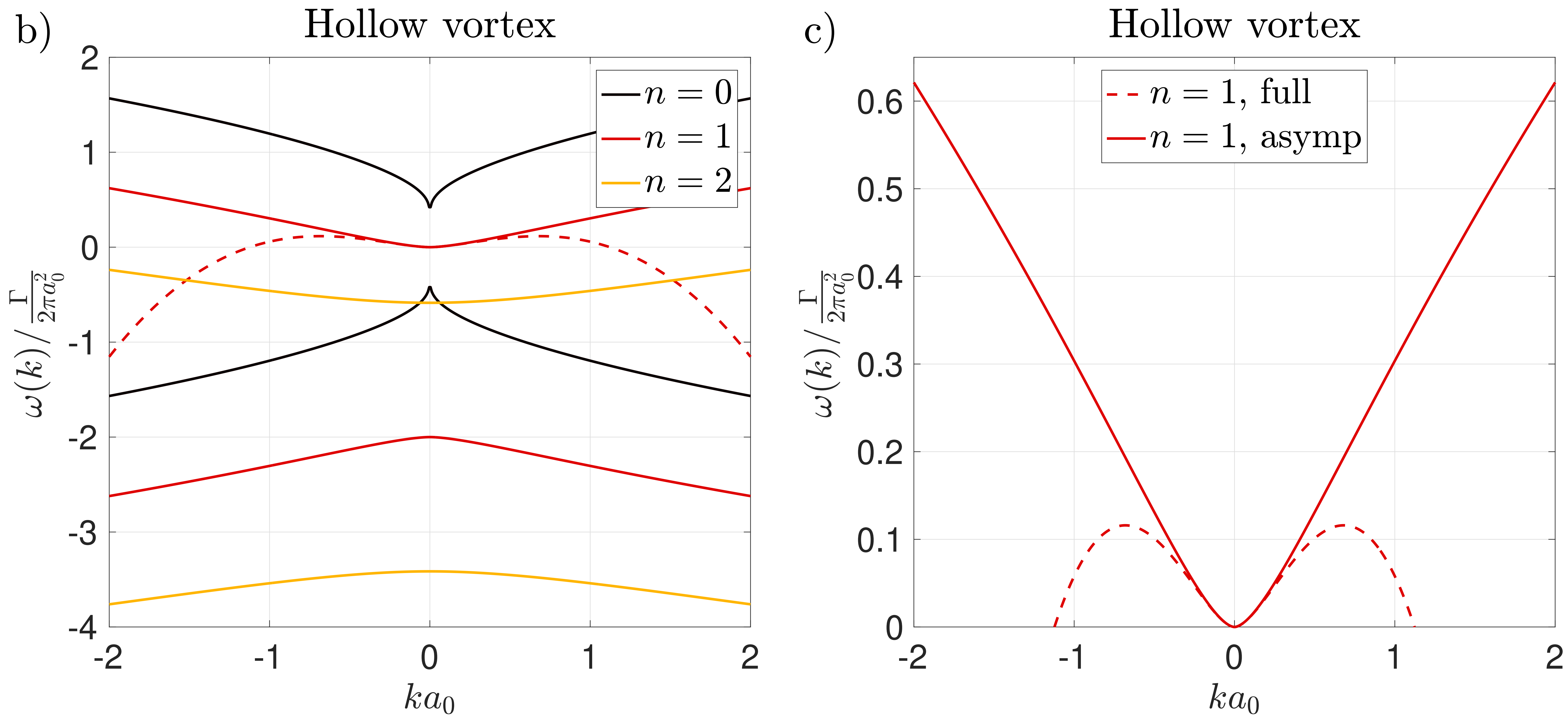
Kelvin Waves : $\mathbf{v} = \mathbf{v}_0 + \delta\mathbf{v} + \ldots$

Kelvin waves



$$\delta v(\theta, z) \sim \cos(kz)\sin(n\theta - \omega t)$$

$$\omega_n^{\pm}(k) = \frac{\Gamma}{2\pi a_0^2}\left(n \pm \sqrt{n + \frac{a_0|k|K_{n-1}(a_0|k|)}{K_n(a_0|k|)}}\right)$$



$n = 1$ and $ka_0 \ll 1$

$$\omega^-(k) = \boxed{-\frac{\Gamma}{8\pi}k^2}\left(\log\frac{1}{a_0|k|} + b\right), \text{ with } b = \log 2 - \gamma_{\mathrm{E}}$$

Vortex excitations in superfluids (GP)

P.H. Roberts. Proc. Royal Society of London A:(2003)

Hydrodynamics

$$k \ll \frac{1}{\xi}$$

$$\Omega_{\mathrm{KW}}(ka_0 \to 0) = -\frac{\Gamma}{4\pi}k^2\left(\ln\frac{2}{a_0|k|} - \gamma_{\mathrm{E}}\right)$$

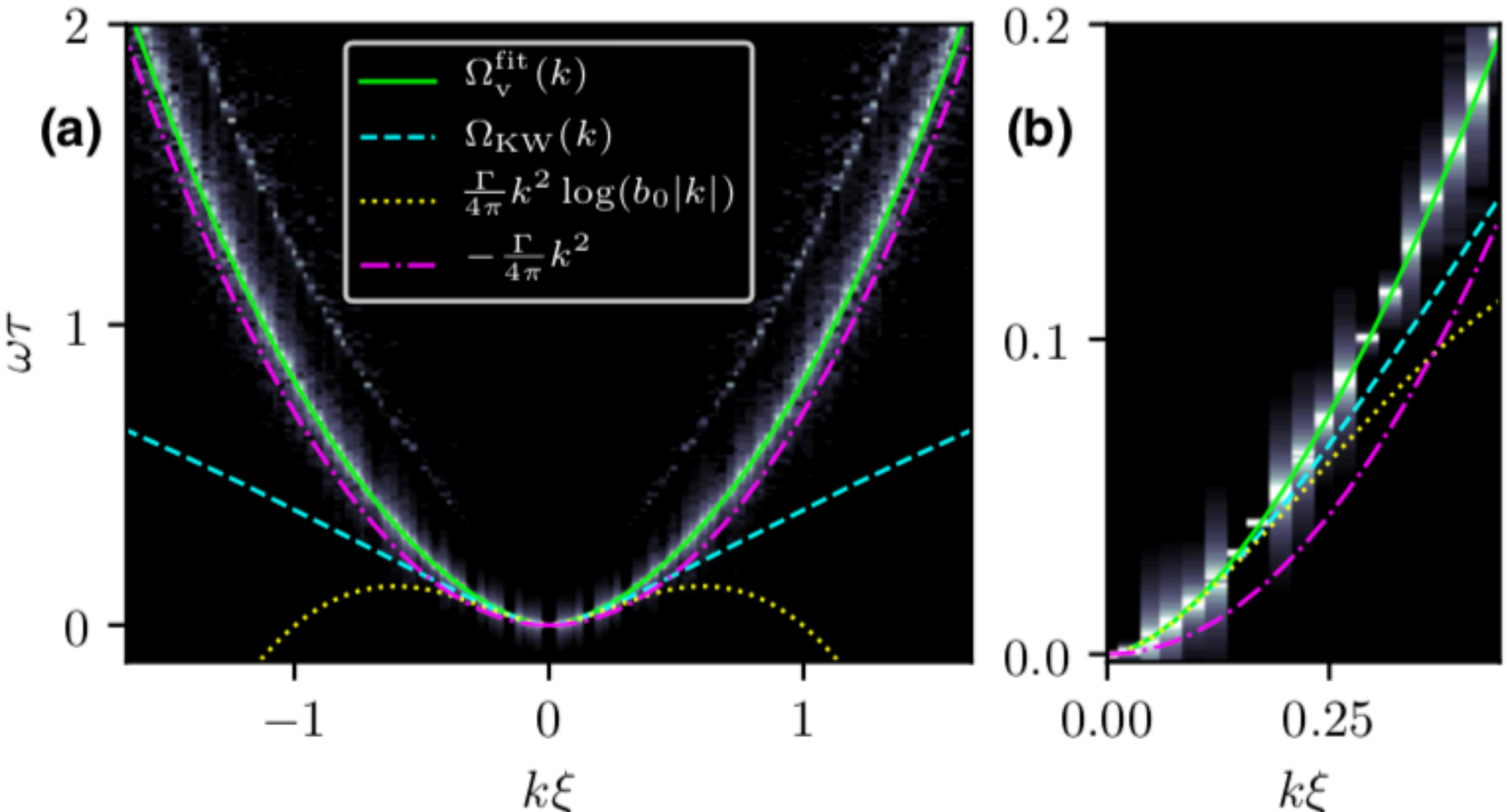
$$a_0 = 1.265\xi$$

Small scales

$$\frac{1}{\xi} \ll k$$

$$\Omega_{\mathrm{v}}(k) \xrightarrow[k\xi\gg1]{} \Omega_B^-(k\xi \to \infty) = -\frac{\Gamma}{4\pi}k^2$$

GP numerical simulations

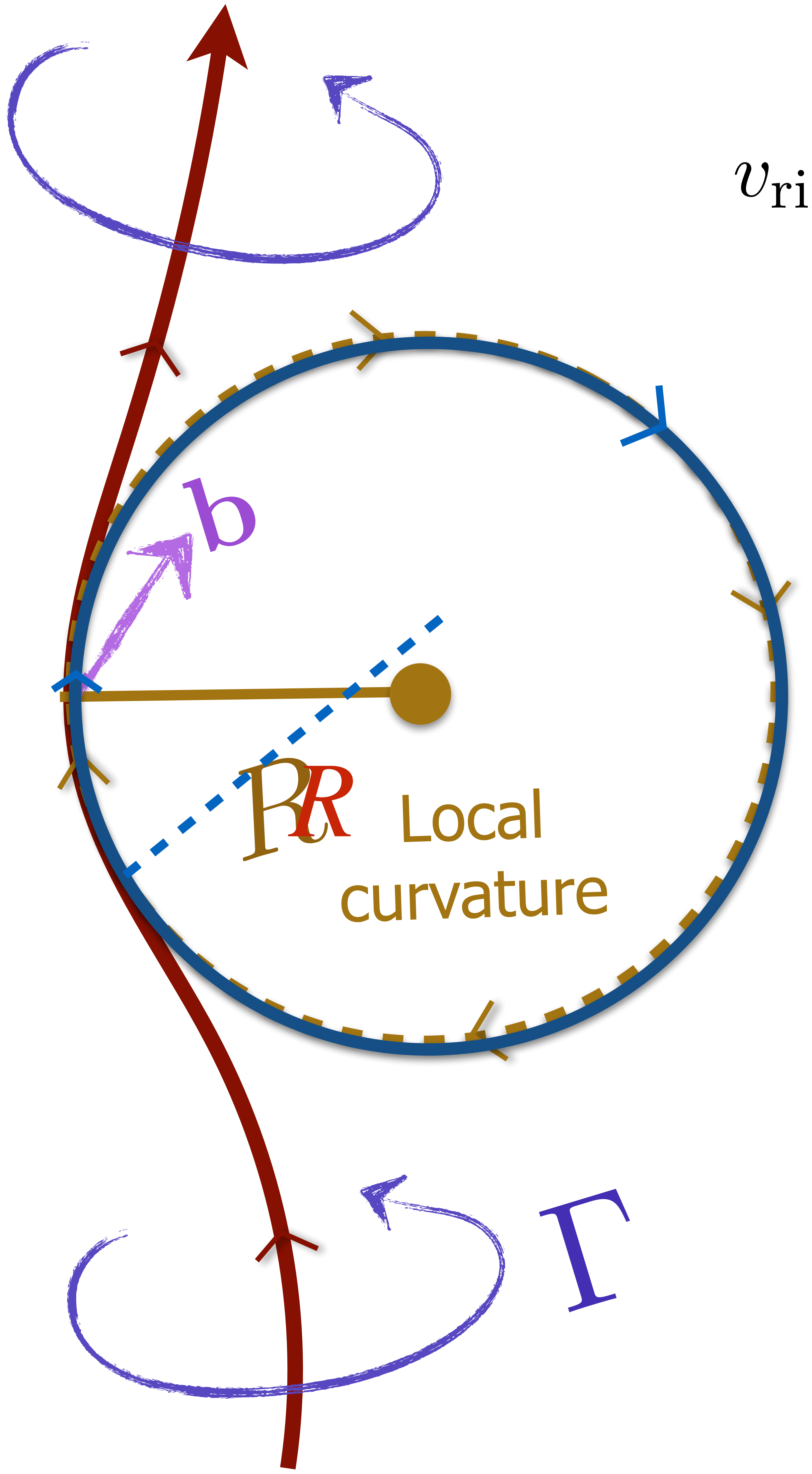


U. Giuriato, G. Krstulovic and S. Nazarenko. Phys. Rev. Research (2020)

Vortex excitations in superfluids (LIA)

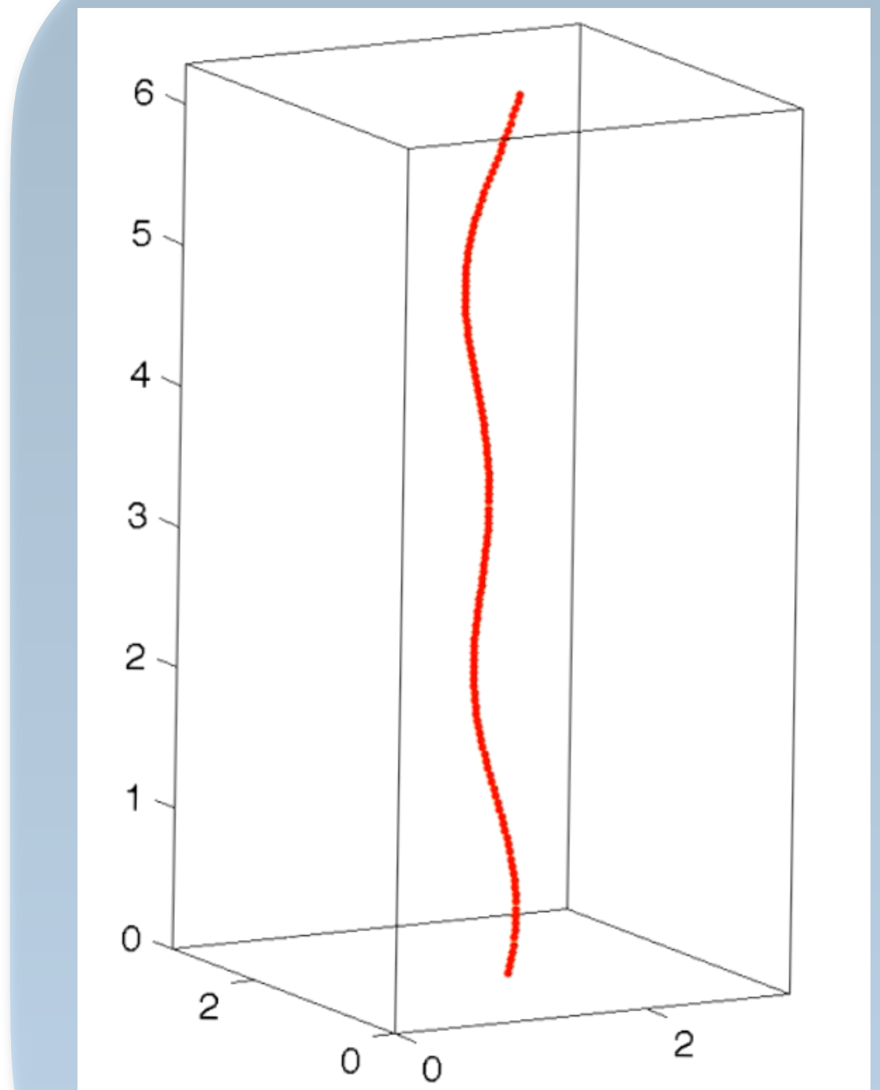
Local Induced Approximation (LIA)

Da Rios, *Rendiconti del Circolo Matematico di Palermo* (1906).



$$v_{\text{ring}} = \frac{\Gamma}{2\pi R}(\log(R/a_0) - d),$$ d a core-dependent constant

$$\dot{\mathbf{s}} = \frac{\Gamma\Lambda}{4\pi R}\widehat{\mathbf{b}} = \frac{\Gamma\Lambda}{4\pi}\mathbf{s}' \times \mathbf{s}''$$



Small amplitudes Kelvin waves

$$s(z,t) = X(z,t) + iY(z,t)$$

$$i\Gamma\dot{s} = \frac{\delta H_{\text{LIA}}}{\delta s^*} = -\frac{\Gamma^2\Lambda}{4\pi}\frac{\partial^2 s}{\partial z^2}, \quad \text{with} \quad H_{\text{LIA}} = \frac{\Gamma^2\Lambda}{4\pi}\int |\frac{\partial s}{\partial z}|^2 \mathrm{d}z$$

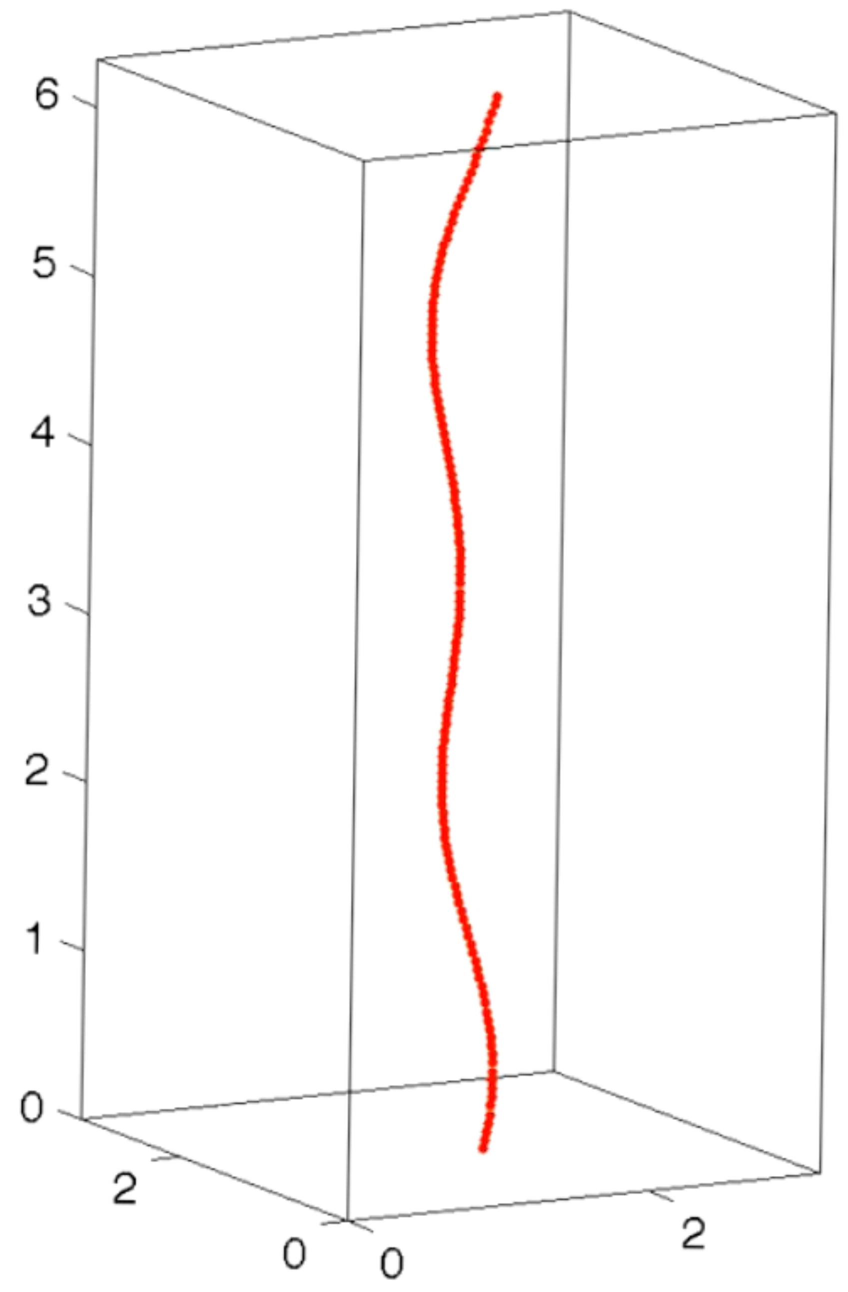
$$\omega_{\text{LIA}}(k) = -\frac{\Gamma\Lambda}{4\pi}k^2$$

Kelvin-wave cascade

Vortex filament model

Biot-Savart description of a perturbed straight vortex

[Sonin 87 - Svistunov 95]



$$s(z,t) = X(z,t) + iY(z,t)$$

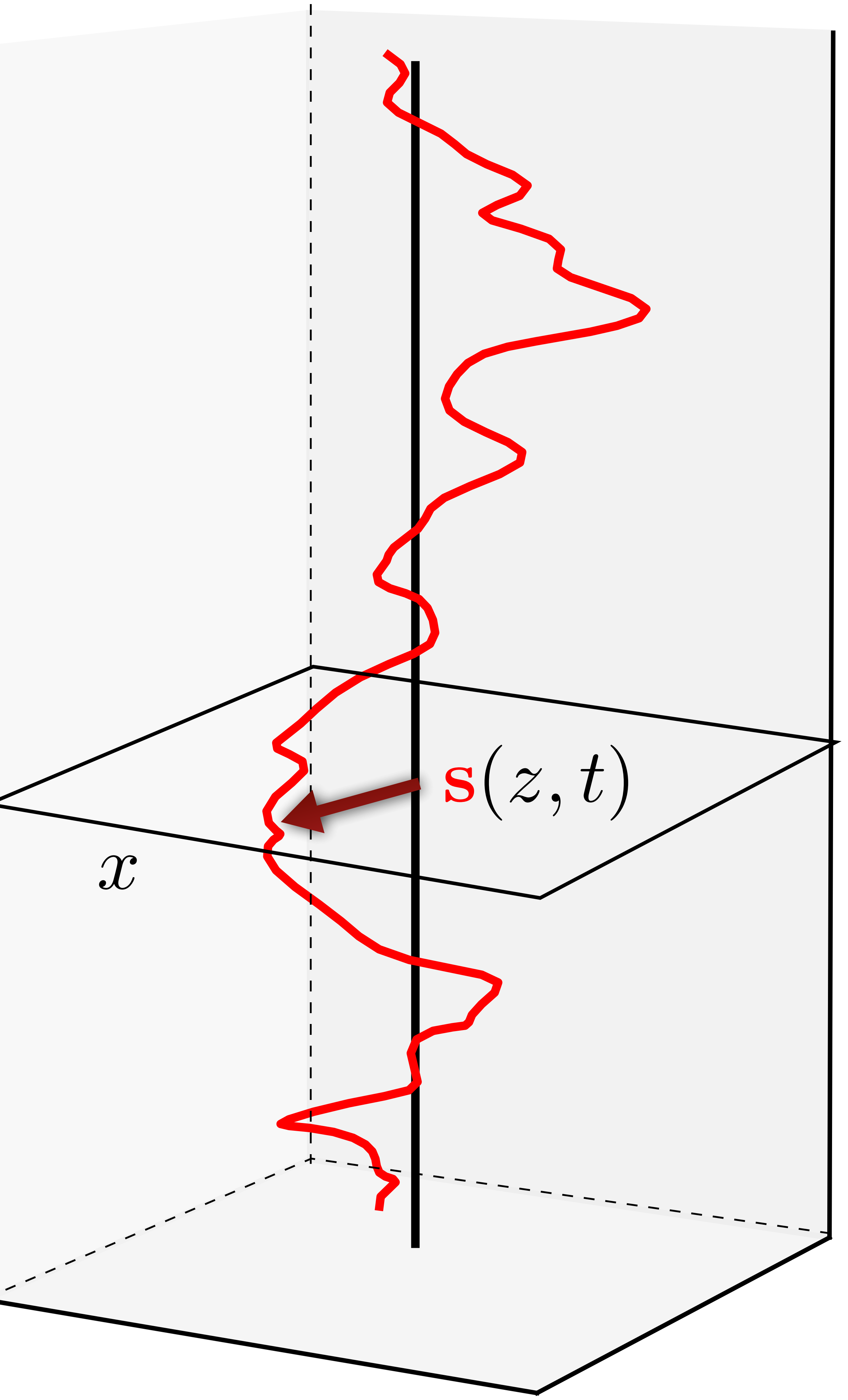
$$i\Gamma\dot{s}(z) = \frac{\delta H_{\mathrm{NL}}}{\delta s^*(z)}, \qquad H_{\mathrm{NL}} = \frac{\Gamma^2}{4\pi}\int \frac{1+\mathcal{R}e[s'^*(z_1)s'(z_2)]}{\sqrt{(z_1-z_2)^2+|s(z_1)-s(z_2)|^2}}\mathrm{d}z_1\mathrm{d}z_2$$

Small amplitude waves:

$$H_{\mathrm{NL}} = \sum_k \omega_k |s_k|^2 + H_4 + H_6 + \dots$$

$$\omega_k = -\frac{\Gamma}{4\pi}k^2(\log{(k\ell)} - \Lambda)$$

Vortex motion and Kelvin wave cascade



Kelvin waves

Helicoidal displacement of vortex filament

$$s(z,t) = X(z,t) + iY(z,t)$$

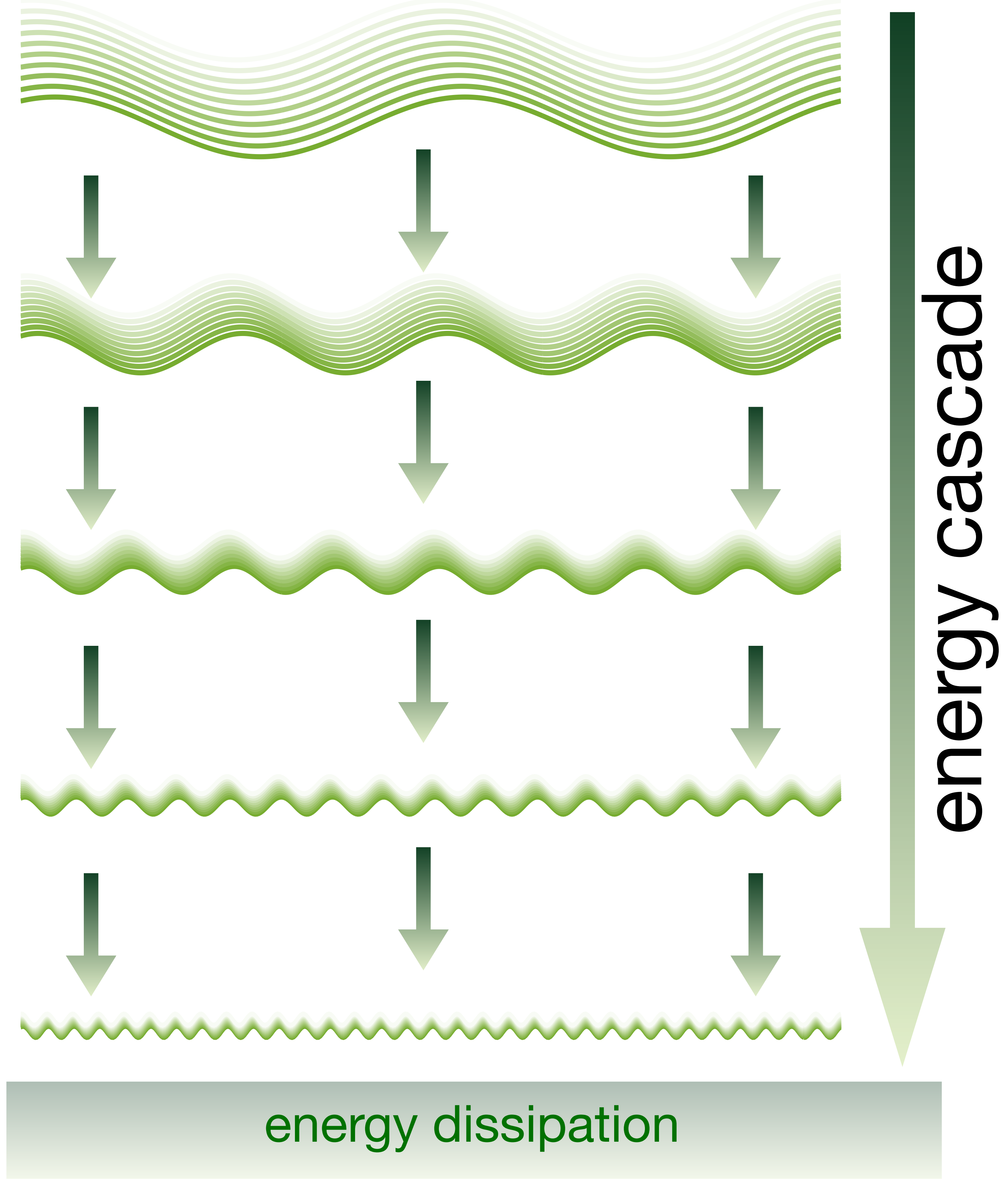
1D dispersive non-linear wave dynamics

$$i\frac{\partial \hat{s}_k}{\partial t} = \omega_k \hat{s}_k + \mathrm{NL}[\hat{s}_k]$$

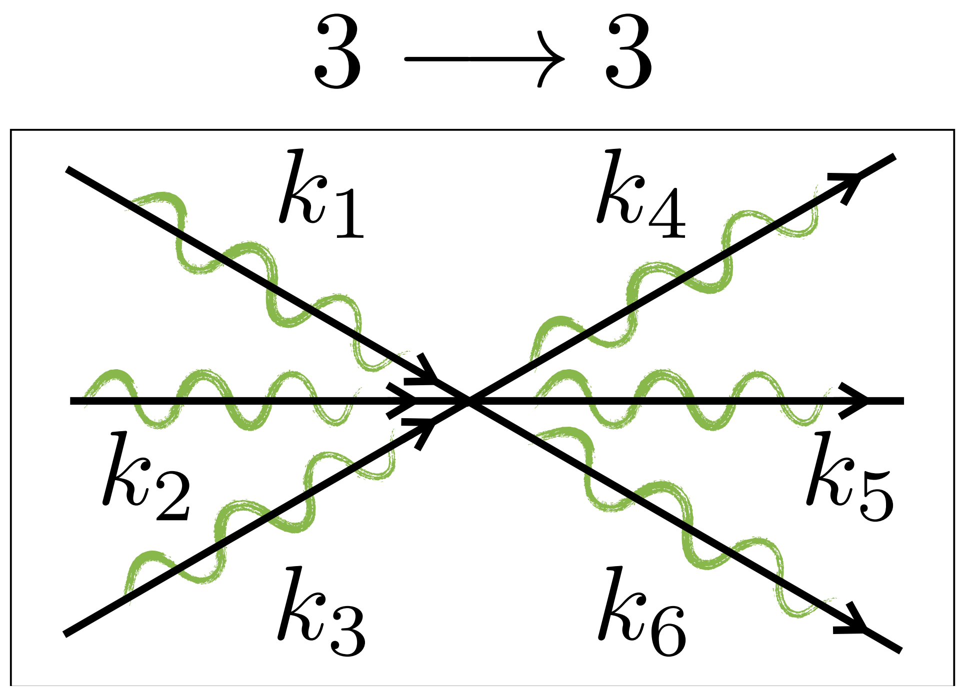
$$\omega_k \underset{k\xi \ll 1}{\sim} \frac{\Gamma}{4\pi} k^2 \left(\log k\xi + \mathrm{c}\right)$$

$$n_k = |s_k|^2$$

$$E(k) = \omega_k n_k \propto \epsilon^\alpha k^{-\gamma} \ ?$$

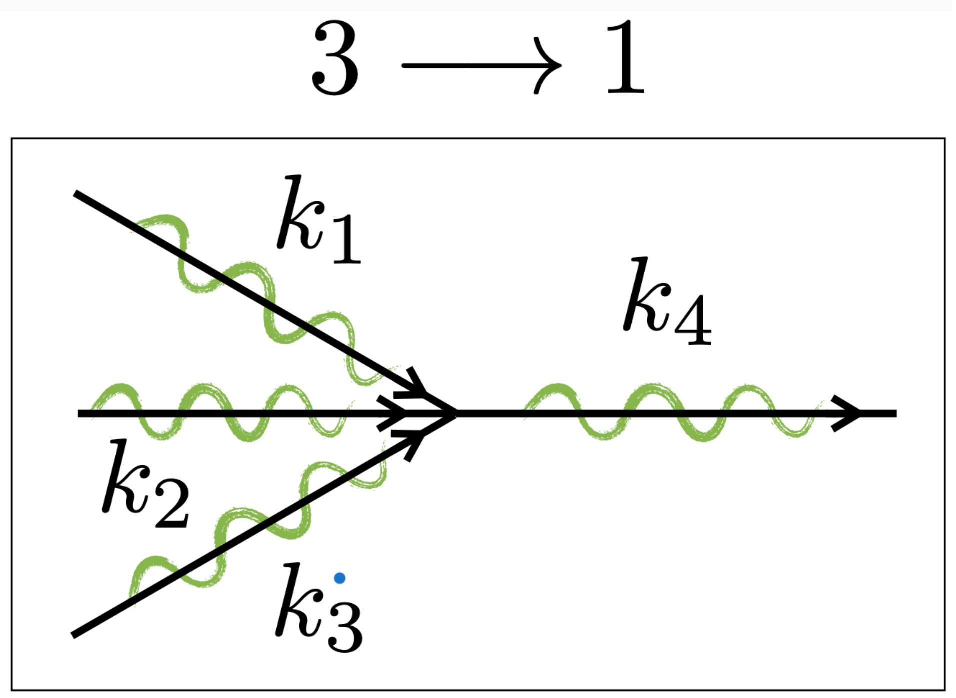


Wave turbulence predictions



$$H_{\mathrm{NL}} = \sum_k \omega_k |s_k|^2 + \cancel{H_4} + H_6 + \ldots$$

non-resonant



Kozik-Svistunov (2004) :

(6 waves)

$$E_{\mathrm{KS}}(k) = C_{\mathrm{KS}} \frac{\Lambda\, \kappa^{7/5}\, \epsilon^{1/5}}{k^{7/5}}$$

$$\Lambda = \ln(\ell/a)$$

a (wave-turbulence) controversy!!

L'vov-Nazarenko (2010):

(effective 4 wave theory)

$$E_{\mathrm{LN}}(k) = C_{\mathrm{LN}} \frac{\Lambda\, \kappa\, \epsilon^{1/3}}{\Psi^{2/3}\, k^{5/3}}\,, \quad \Psi \equiv \frac{8\,\pi\, E}{\Lambda\, \kappa^2}$$

$$C_{\mathrm{LN}} = 0.304$$

- Kivotides, Vassilicos, Samuel, Barenghi PRL 2001
- E. Kozik & B. Svistunov. PRL 2004
- L'vov & Nazarenko JETP 2010
- Boué et al PRB 2011
- Laurie and Baggaley PRE 2014
- many others works....

Vinen et al. :

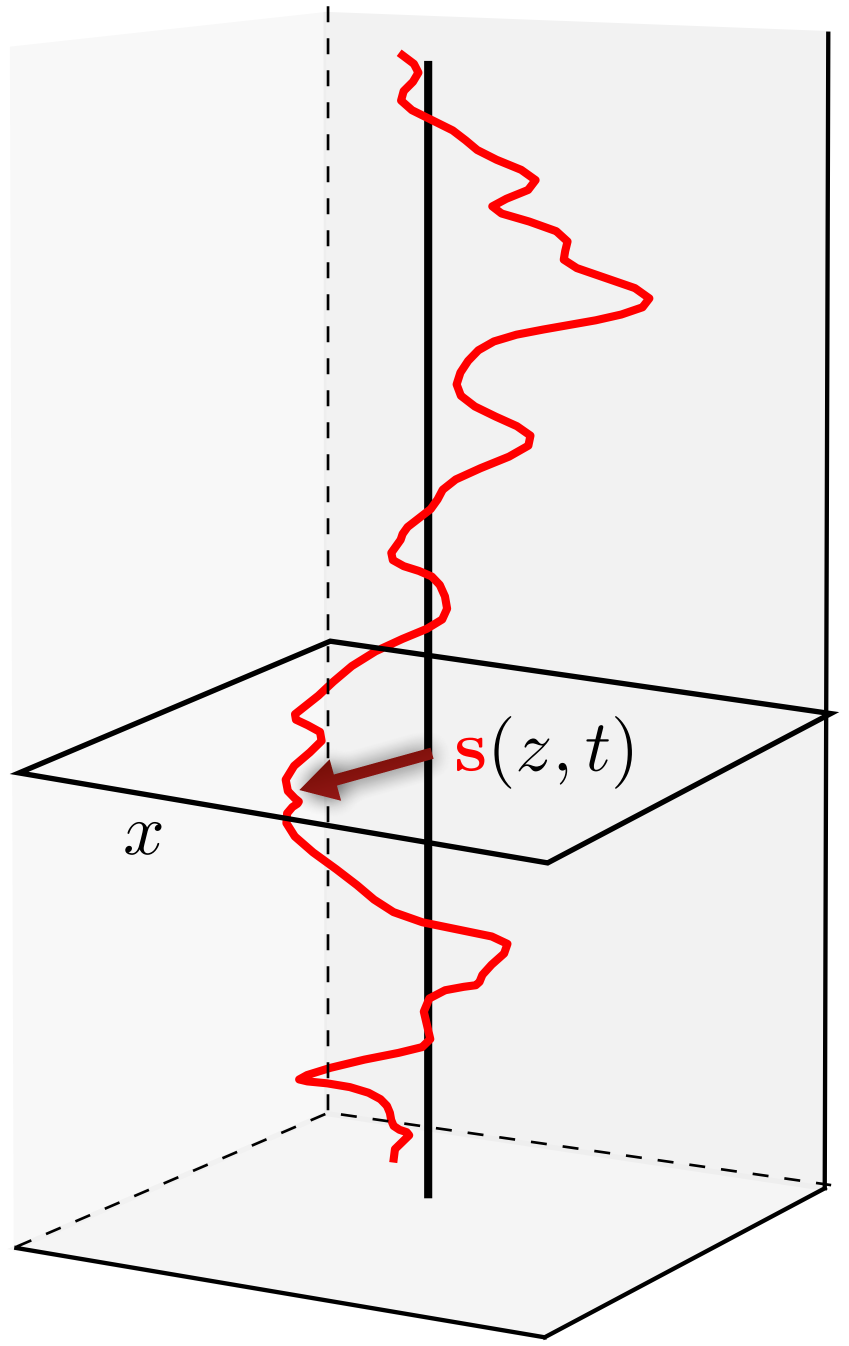
(PRL 2003, J. Phys.: Condens. Matt. 2005)

$$E_{\mathrm{C.B.}}(k) \sim k^{-1}$$

Kelvin-wave cascade

Numerical simulations

We consider a perturbed straight vortex:



Biot-Savart dynamics:

$$i\Gamma\dot{s}(z) = \frac{\delta H_{\rm NL}}{\delta s^*(z)}, \qquad H_{\rm NL} = \frac{\Gamma^2}{4\pi}\int \frac{1+\mathcal{R}e[s'^*(z_1)s'(z_2)]}{\sqrt{(z_1-z_2)^2+|s(z_1)-s(z_2)|^2}}\mathrm{d}z_1\mathrm{d}z_2$$

$$\dot{\mathbf{s}}(\zeta) = \frac{\Gamma}{4\pi}\oint \frac{\mathrm{d}\mathbf{s}(\zeta')\times(\mathbf{s}(\zeta)-\mathbf{s}(\zeta'))}{|\mathbf{s}(\zeta)-s(\zeta'|^3}$$

Non-local equation, needs to be regularised, dissipation is added in an ad-hoc manner

Gross-Pitaevskii dynamics:

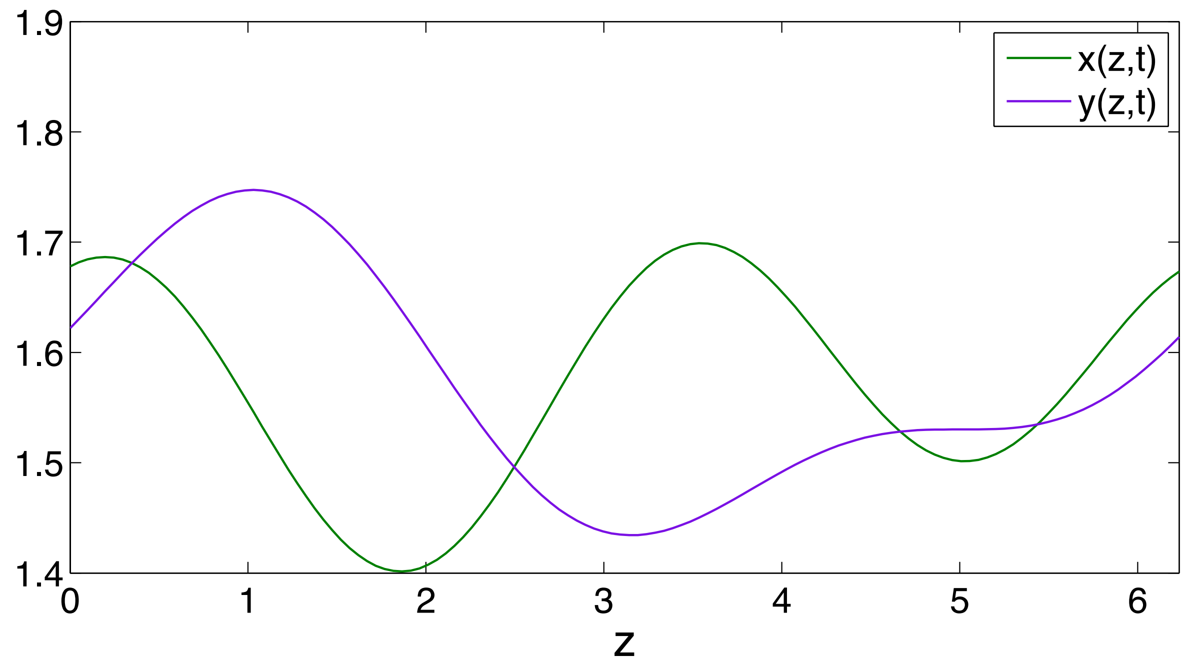
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g|\psi|^2\psi,$$

3D PDE but everything is regular. Effective dissipation is provided by acoustic emission.

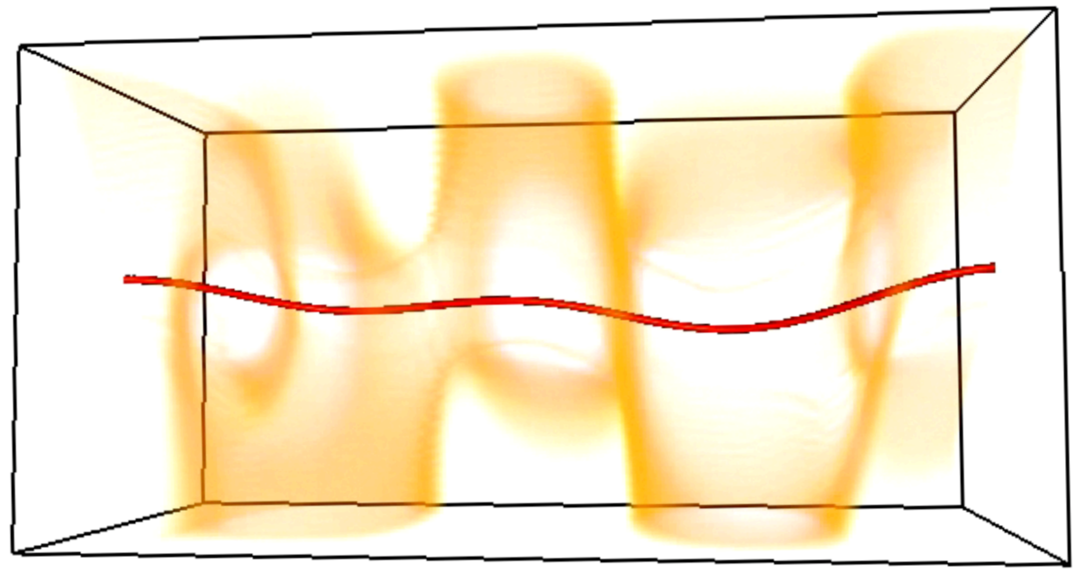
One gets $\psi(x,y,z,t)$ but we need a filament $\mathrm{s}(z,t)$!

Tracking vortices

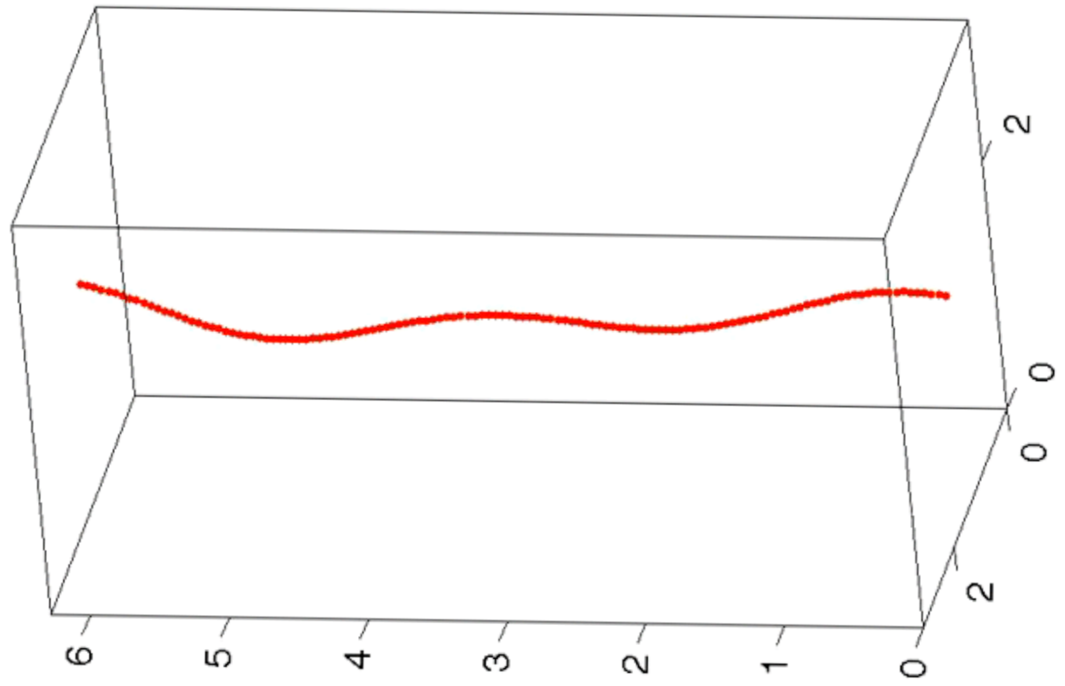
$$\psi(x,y,z,t) = 0 \xrightarrow[\text{Newton method + Fourier interpolation}]{} \begin{matrix} x(z,t) \\ y(z,t) \end{matrix}$$



G. Krstulovic
PRE 86, 055301(R), (2012)

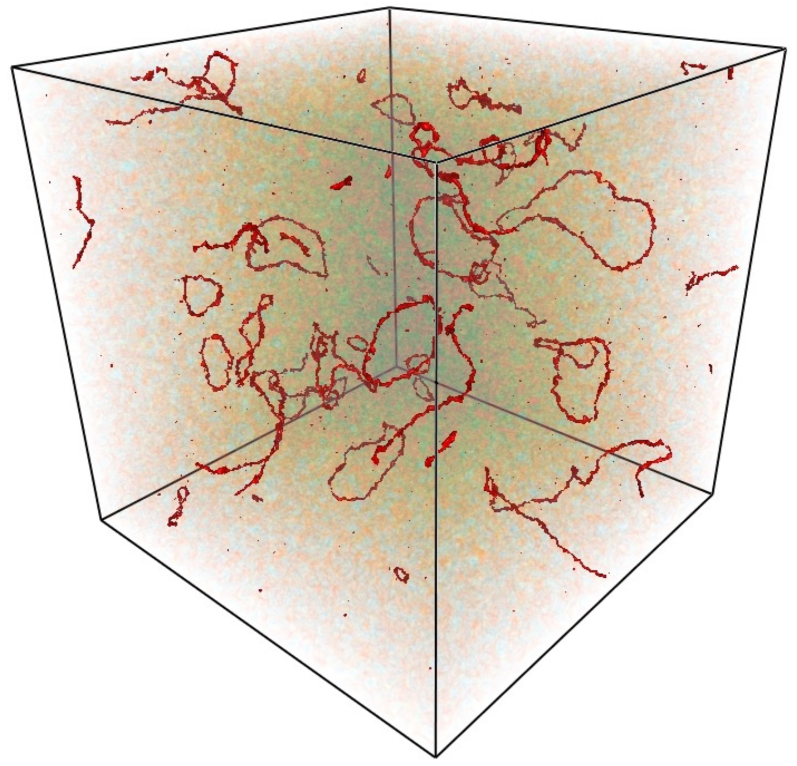


tracking



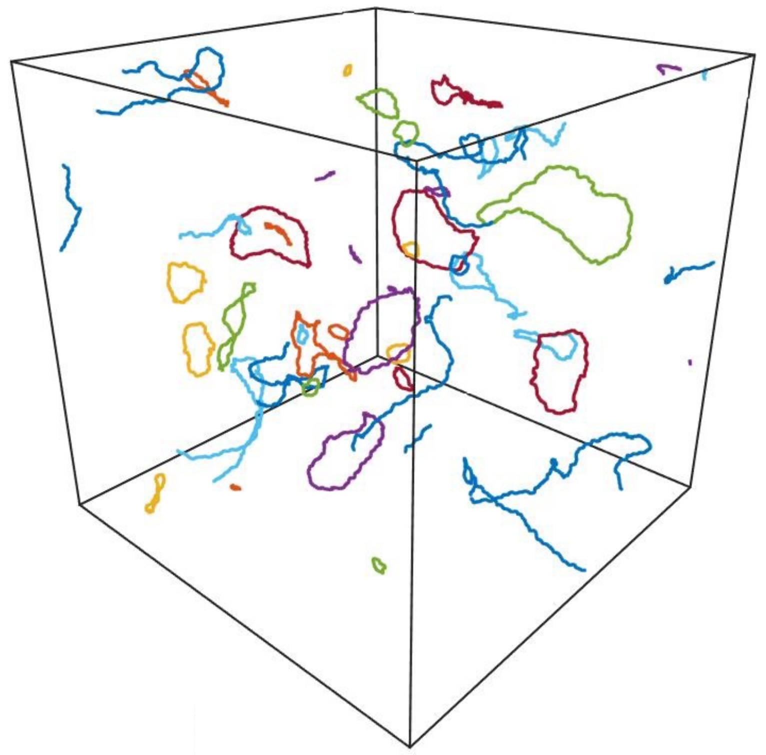
Isosurface

Tracked lines



$|\psi(\mathbf{x})|^2 \approx 0$

tracking



$\{\mathbf{R_1}(\mathrm{s}), \mathbf{R_2}(\mathrm{s}), \ldots\}$

- Highly accurate (spectral precision)
- Geometry independent
- Arbitrary number of objects

A. Villois, G. Krstulovic, D. Proment and H. Salman. J. Phys. A (2016)

Superfluid turbulence

energy injection

Classical (Kolmogorov) turbulence

inter-vortex distance

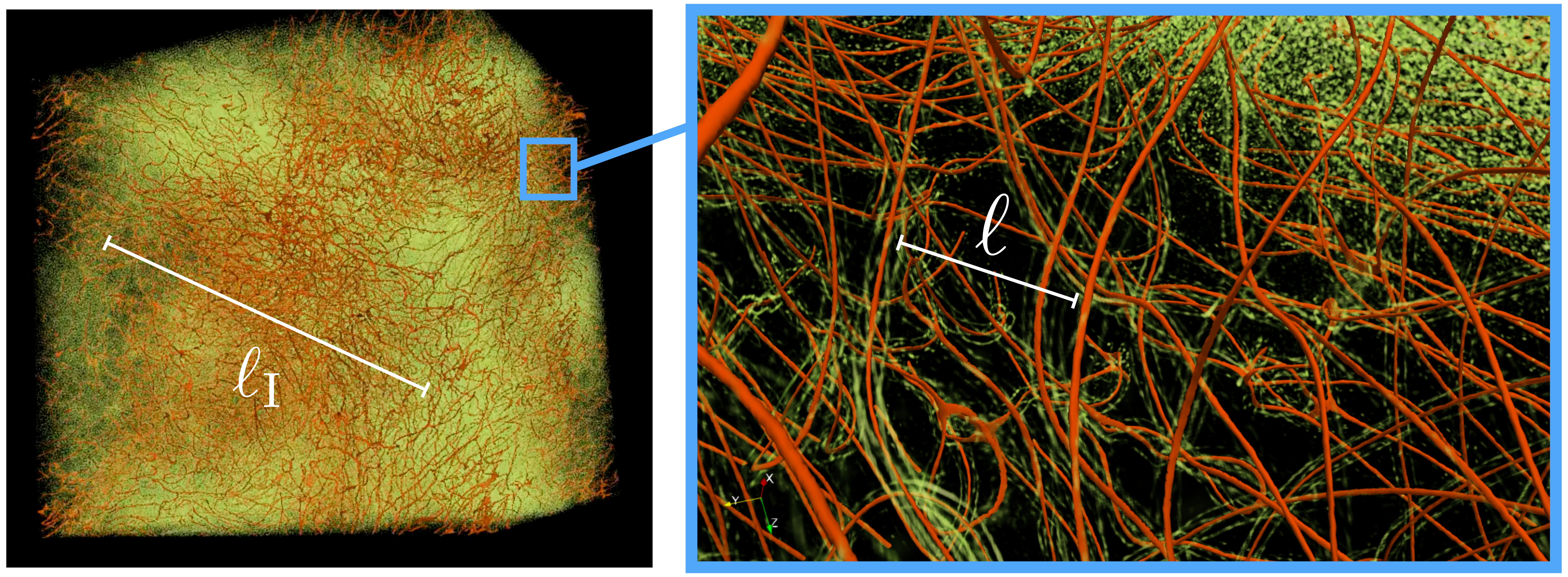
Kelvin wave cascade & vortex reconnections

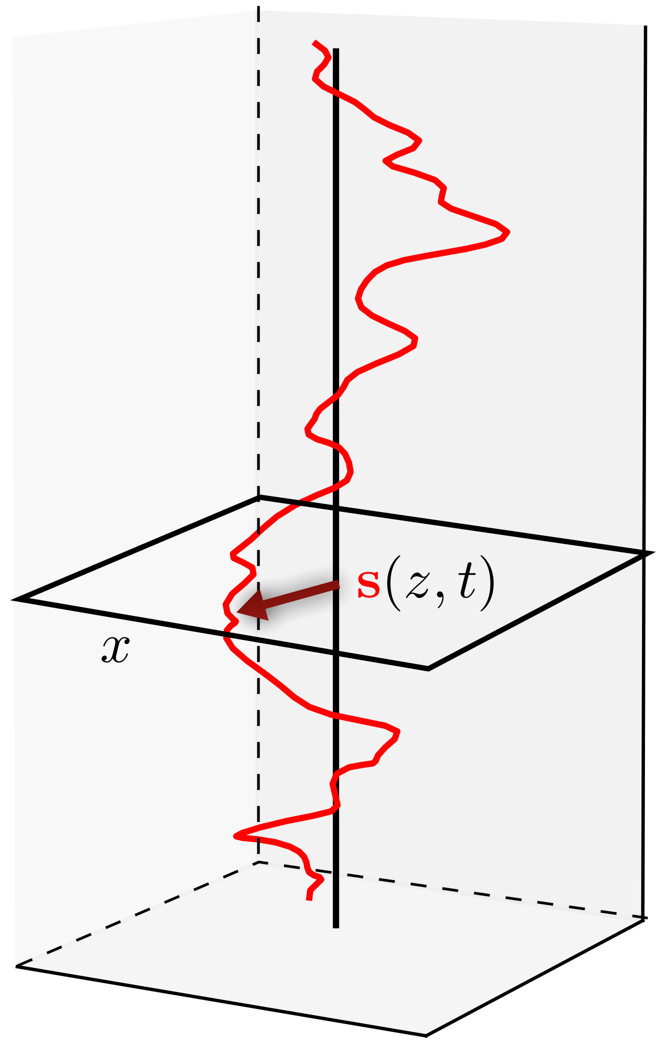
vortex core size

sound emission

$$\cdots\rightarrow \ell_{\mathrm{I}} \longrightarrow \ell \longrightarrow \xi \cdots\rightarrow$$

Is the Kelvin wave cascade relevant for a turbulent tangle?

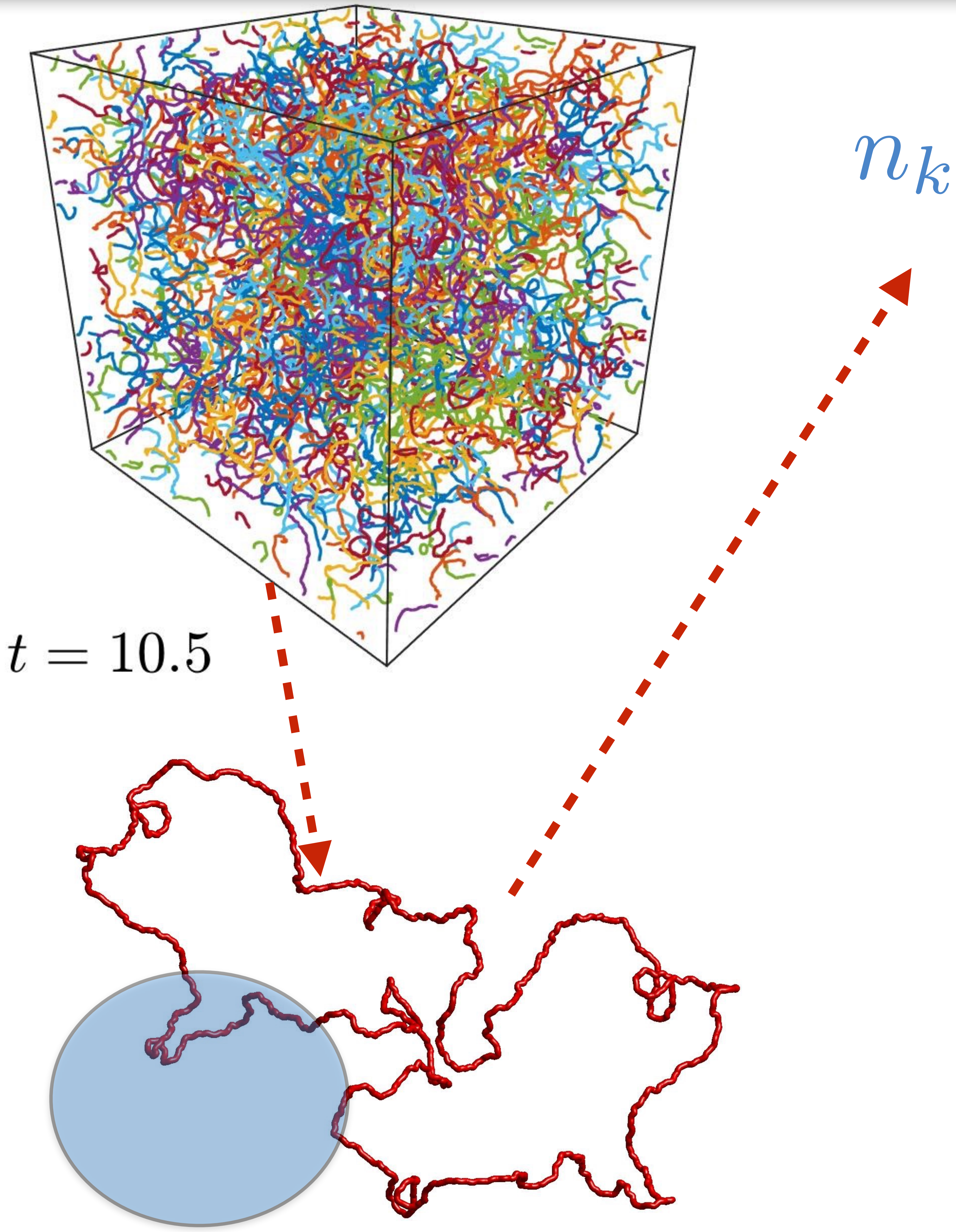




Quantum turbulence

Kelvin waves in a turbulent tangle

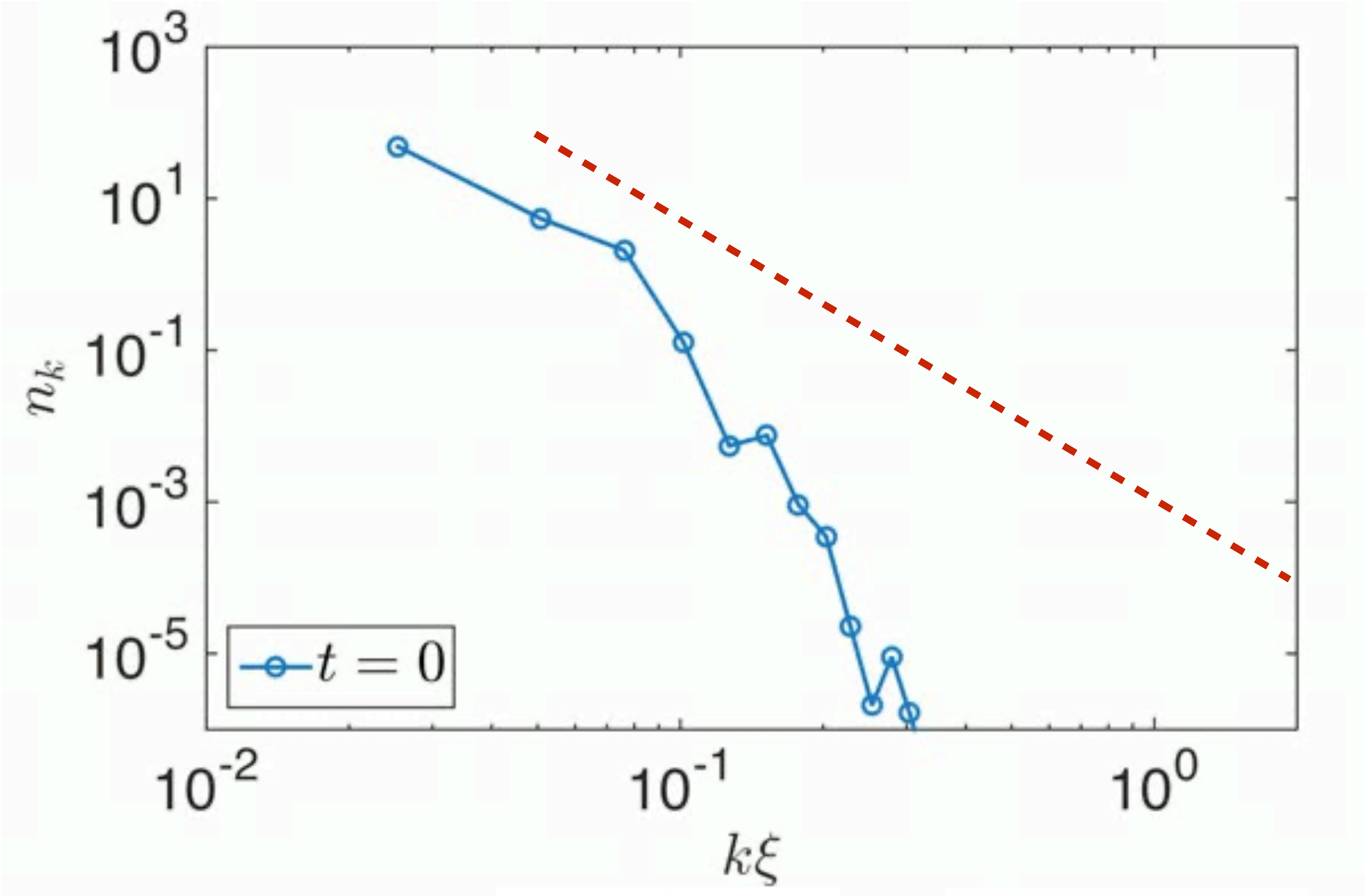
A. Villois, D. Proment and G. Krstulovic. PRE 2016

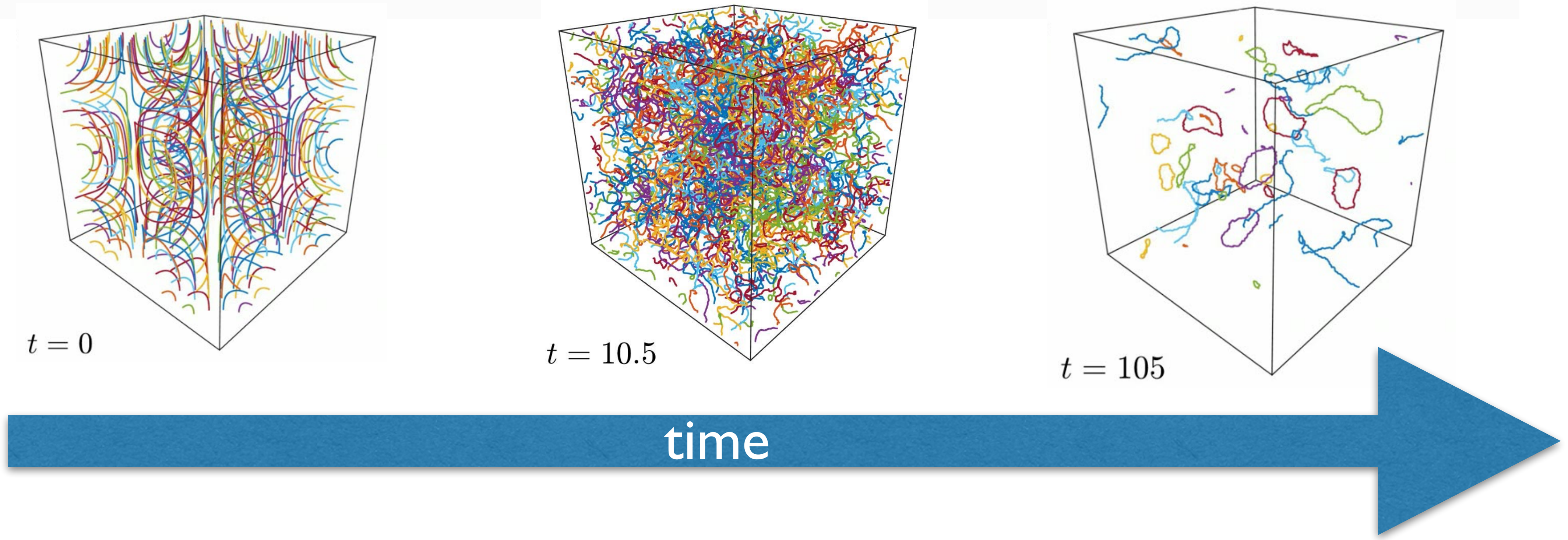


$$n_k = |\hat{\mathbf{R}}_{\mathrm{KW}}(k)|^2$$

Kelvin waves

A. Villois, G. Krstulovic, D. Proment and H. Salman. J. Phys. A (2016)



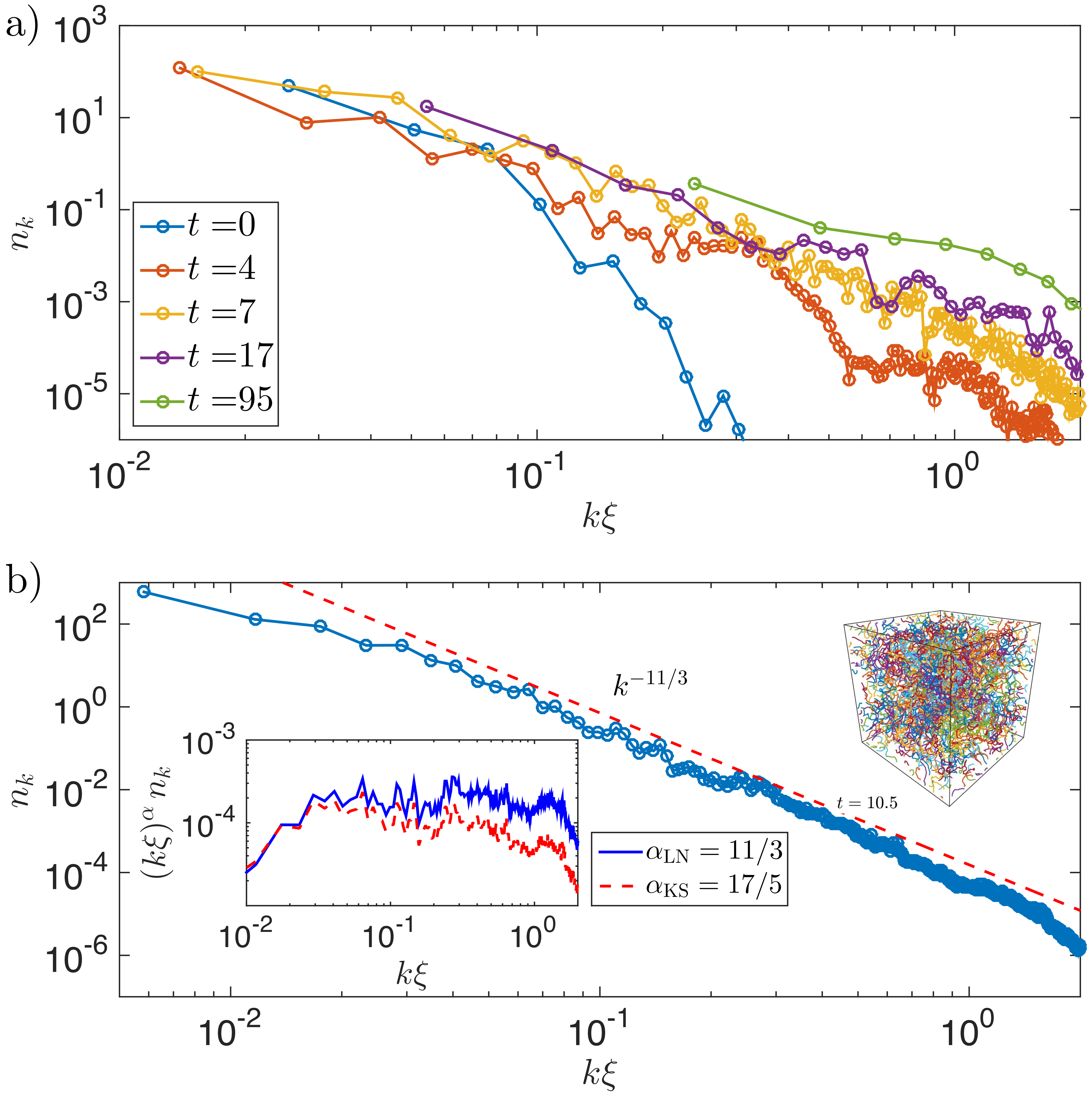


Quantum turbulence

Kelvin wave cascade

A. Villois, D. Proment and G. Krstulovic. PRE 2016

$$n_k = |\hat{\mathbf{R}}_{\mathrm{KW}}(k)|^2$$



Take away message

QT is the result of the collective effect of many vortex lines each of them inducing a weak wave turbulent cascade, the whole leading to Kolmogorov turbulence.

L'vov & Nazarenko (JETP 2010):
(non-locality of energy transfer, 4 waves)

$$E_k \sim k^{-5/3} \Longleftrightarrow n_k \sim k^{-11/3}$$

Quantum turbulence

Strong turbulence

Kolmogorov scaling for the energy spectrum (K41)

$$E(k) = C_K \epsilon^{2/3} k^{-5/3}$$

$$k_0 \ll k \ll k_\ell$$

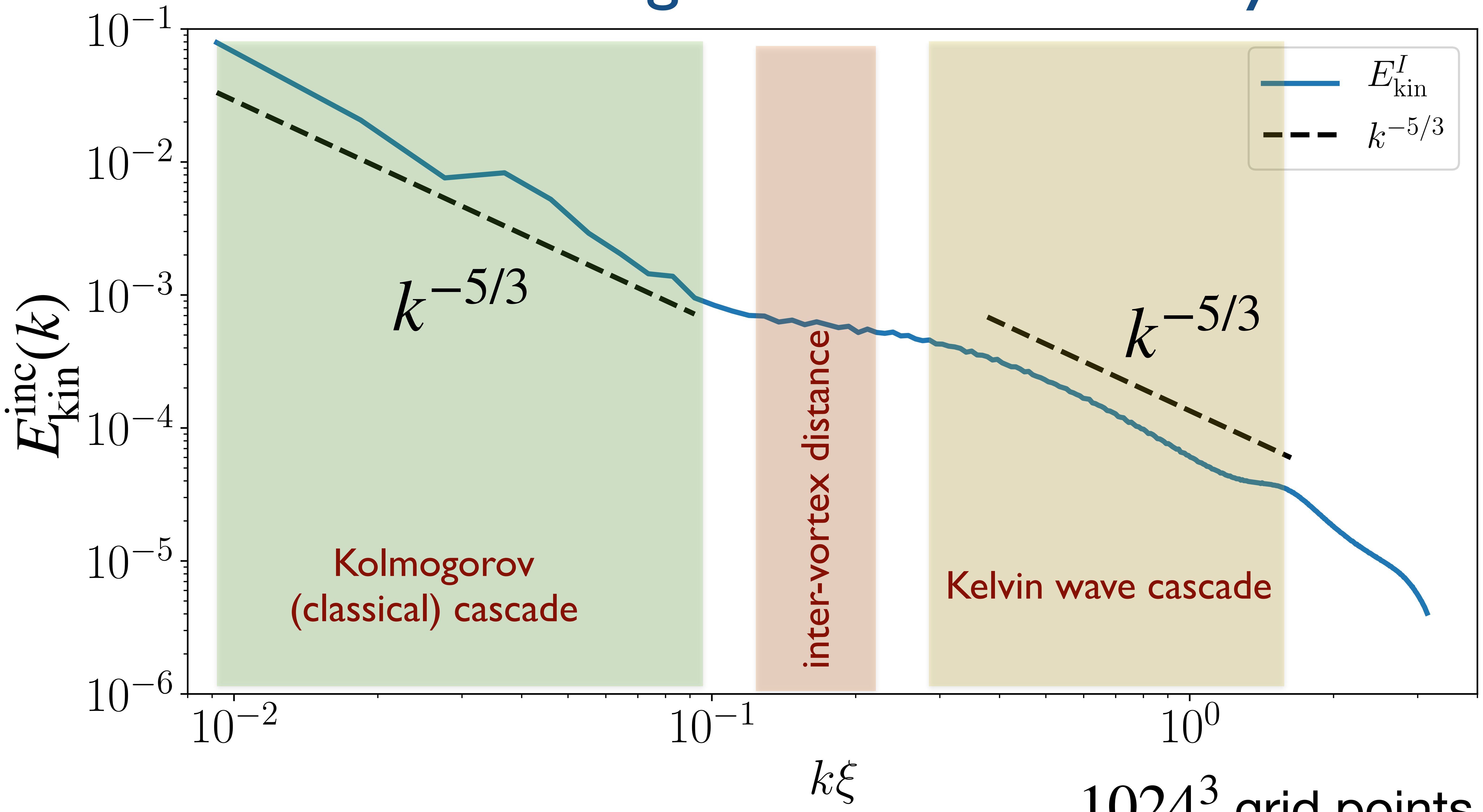
Weak wave turbulence

Kelvin wave scaling for the energy spectrum

$$E(k) \sim \kappa \epsilon^{1/3} \ell^{-4/3} k^{-5/3}$$

$$k_\ell \ll k \ll k_\xi$$

Non-local high-order nonlinearity GP



1024^3 grid points

Simultaneous observation of two cascades

Experiments: Maurer et al. (1998), Salort et al. (2010), Tang et al. (2021), …
Simulations in GP: Nore et al. (1997), Kobayashi et al. (2005), Clark di Leoni et al. (2017), …
Simulations in vortex-filament method: Baggaley et al. (2012), …

Müller & Krstulovic - *PRB* (2020)

Quantum turbulence

	Initial condition		Turbulence	
	k_0	L/ξ	ϵ	ℓ/L
╍╍╍╍	2	341	0.01	0.412
━━━━	2	171	0.01	0.494
━━━━	2	341	0.01	0.255
━━━━	3	341	0.02	0.235
━━━━	4	341	0.03	0.227
━━━━	2	683	0.01	0.139

→ **Local**

Rotons

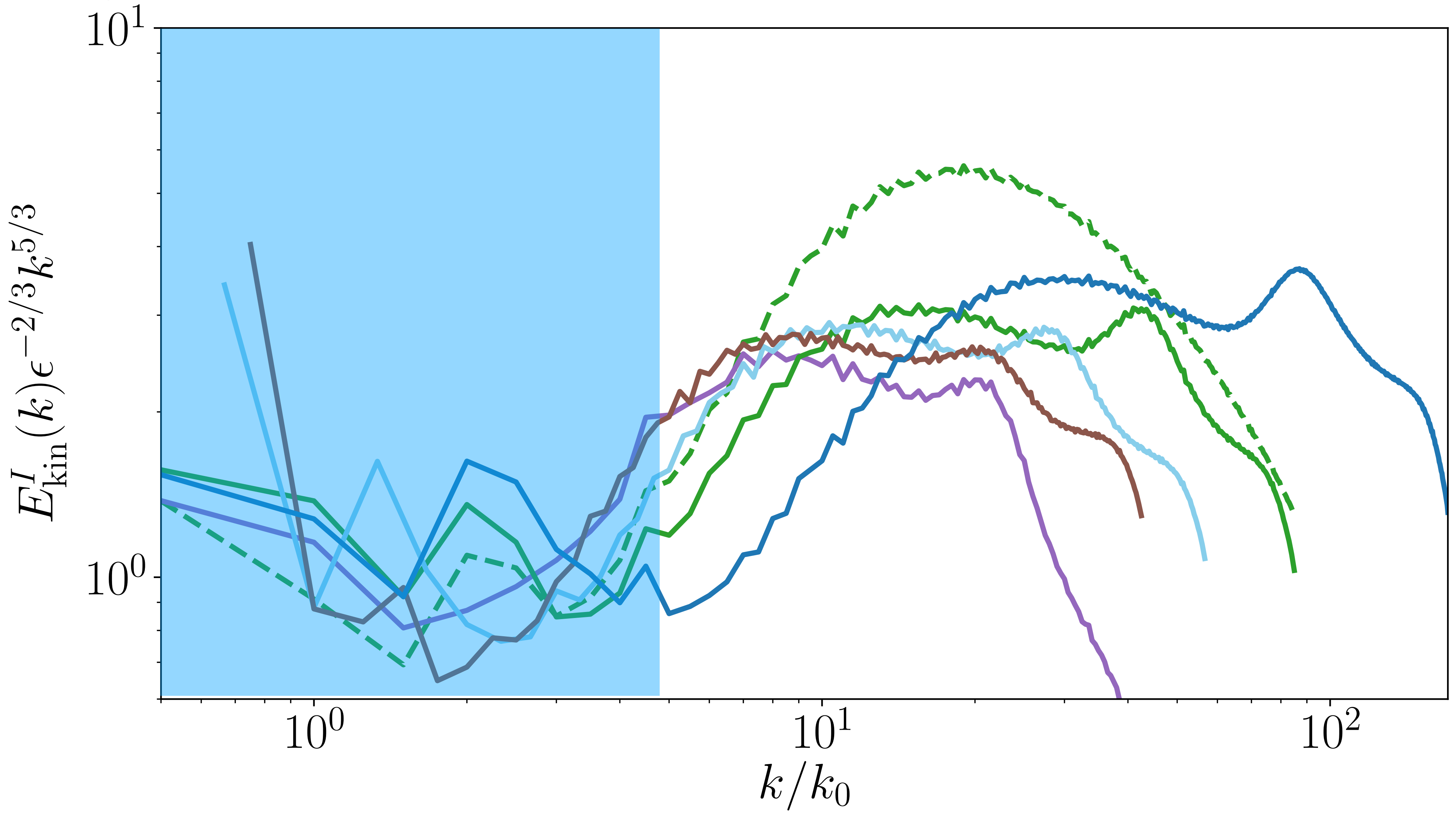
$$E(k) = C_K \epsilon^{2/3} k^{-5/3}$$

Kolmogorov spectrum

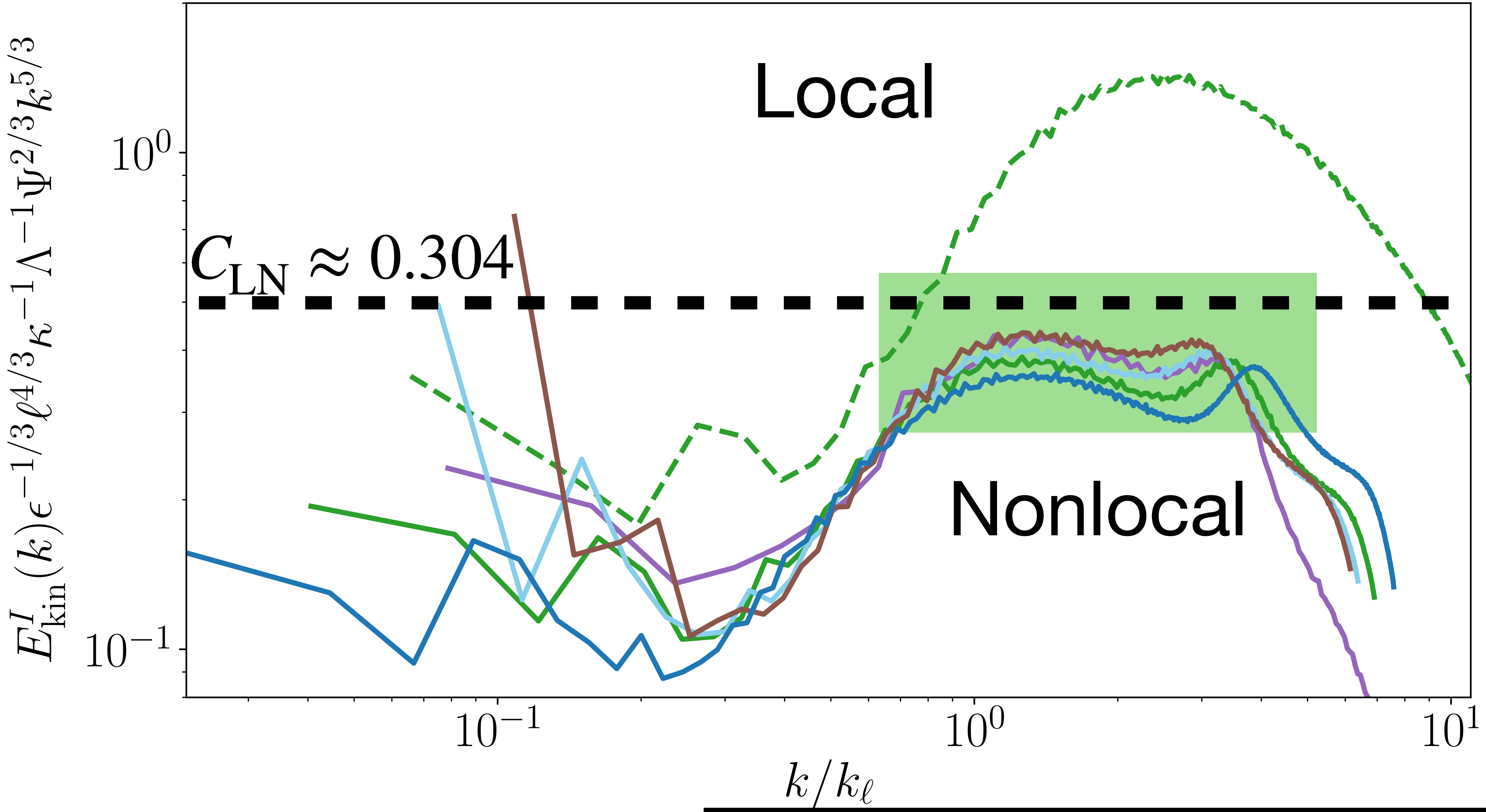
$$E_{\mathrm{KW}}(k) = C_{\mathrm{LN}}^{3/5} \frac{\kappa \Lambda \epsilon^{1/3} \ell^{-4/3}}{\tilde{\Psi}^{2/3} k^{5/3}}$$

Kelvin wave spectrum

Energy spectrum compensated by **Kolmogorov spectrum**



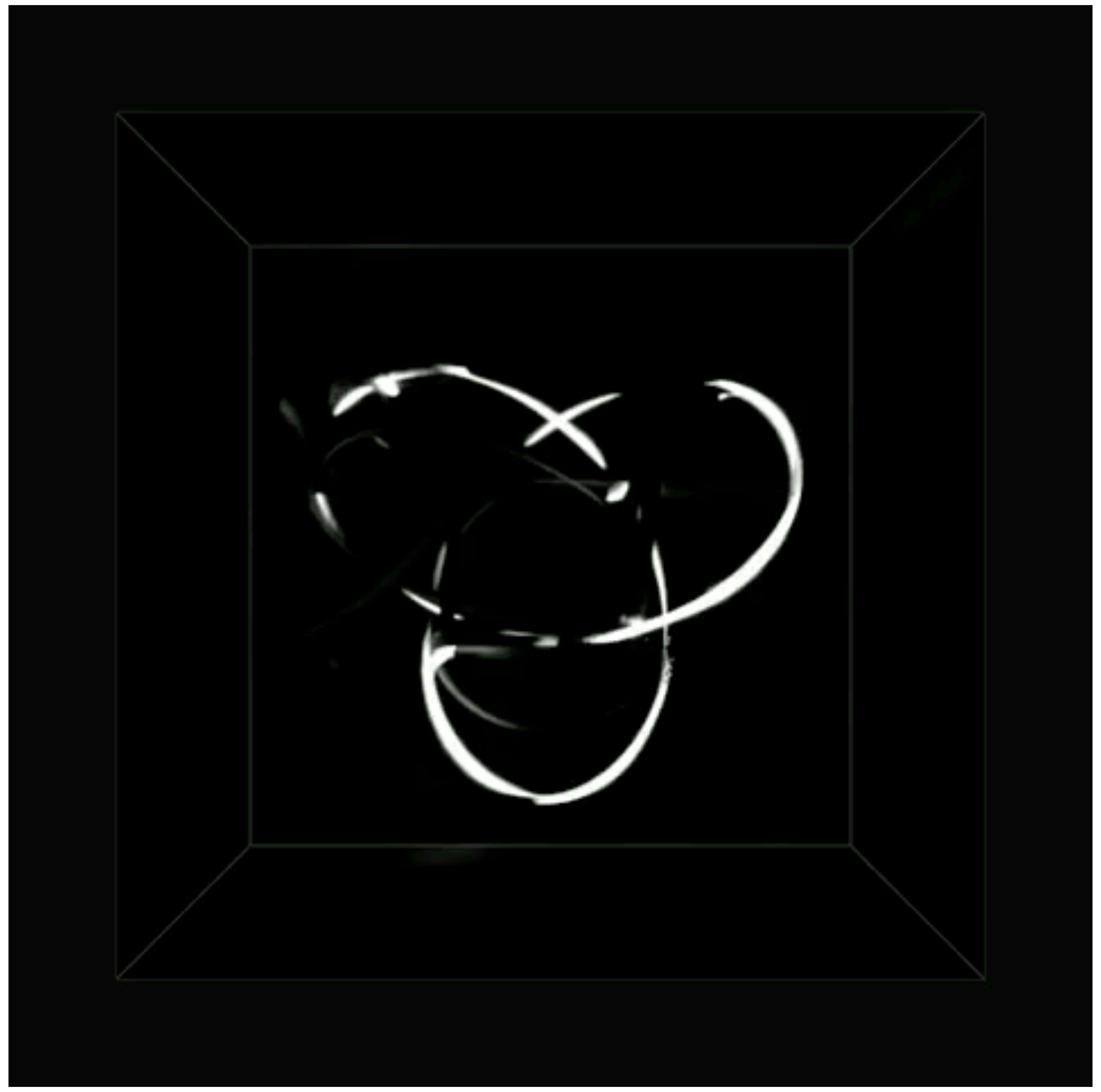
Energy spectrum compensated by **Kelvin wave spectrum**



Müller & Krstulovic - *PRB* (2020)

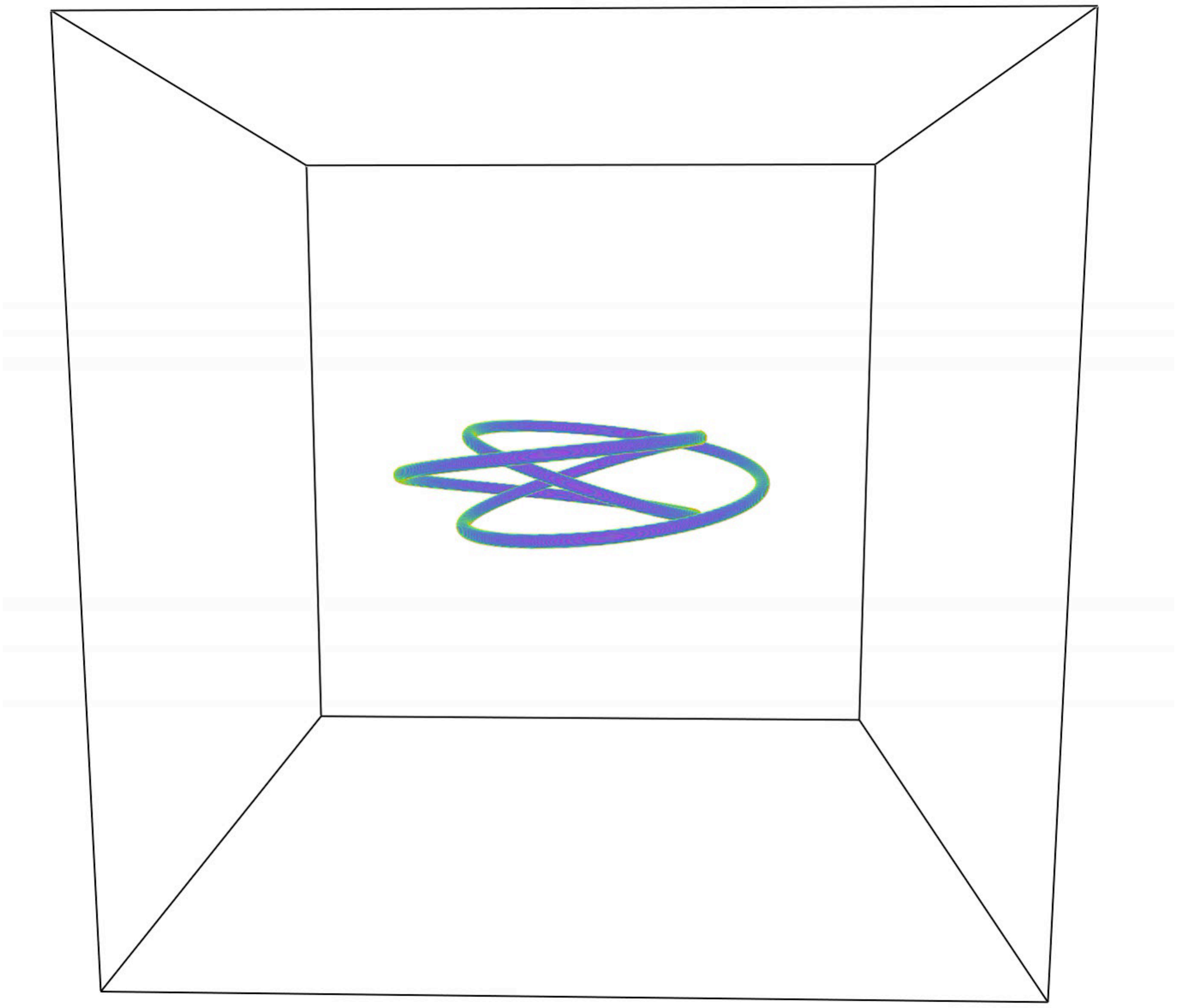
Vortex reconnections

Experiments in water



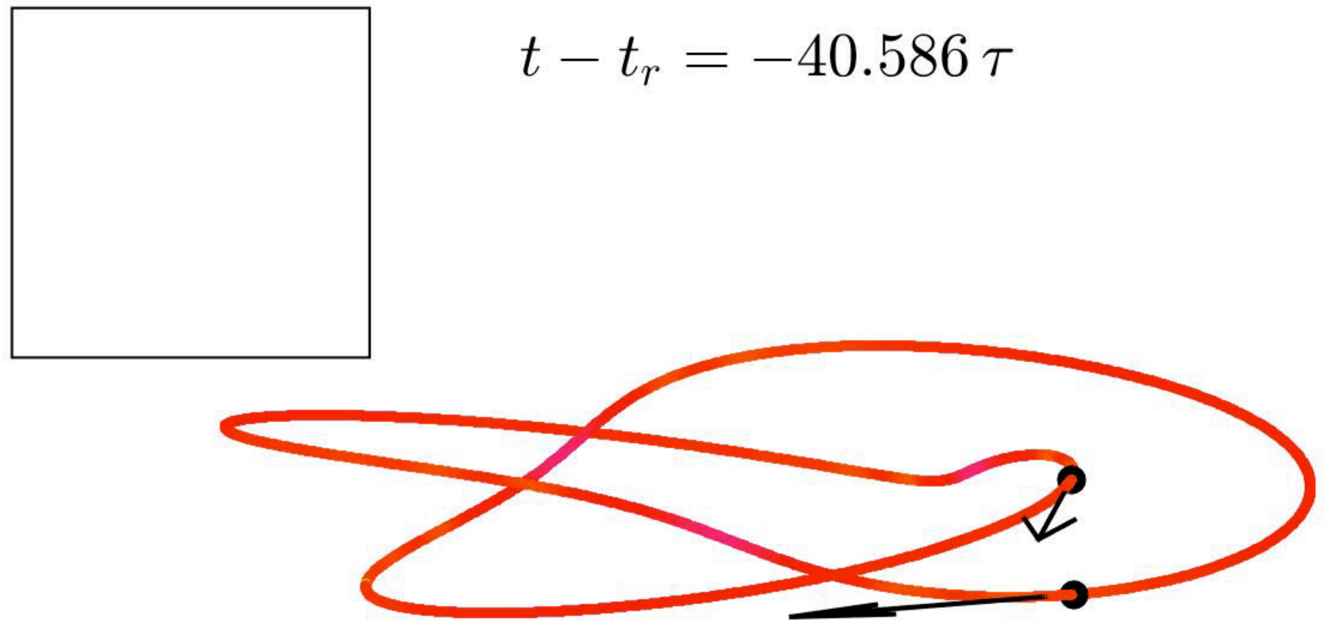
Kleckner & Irvine. Nature Phys. 2013

Numerical simulations of classical fluids



Navier-Stokes equations

Numerical simulations of superfluids



Gross-Pitaevskii model

Ideal for a theoretical description! $\xi \ll R$

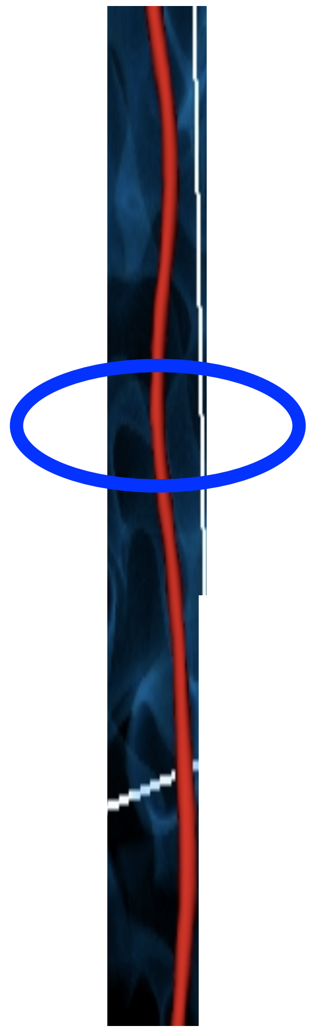
Minimal vortex distance

Dimensional analisys:

ξ : vortex core size

δ : minimal distance between vortices

R : system size

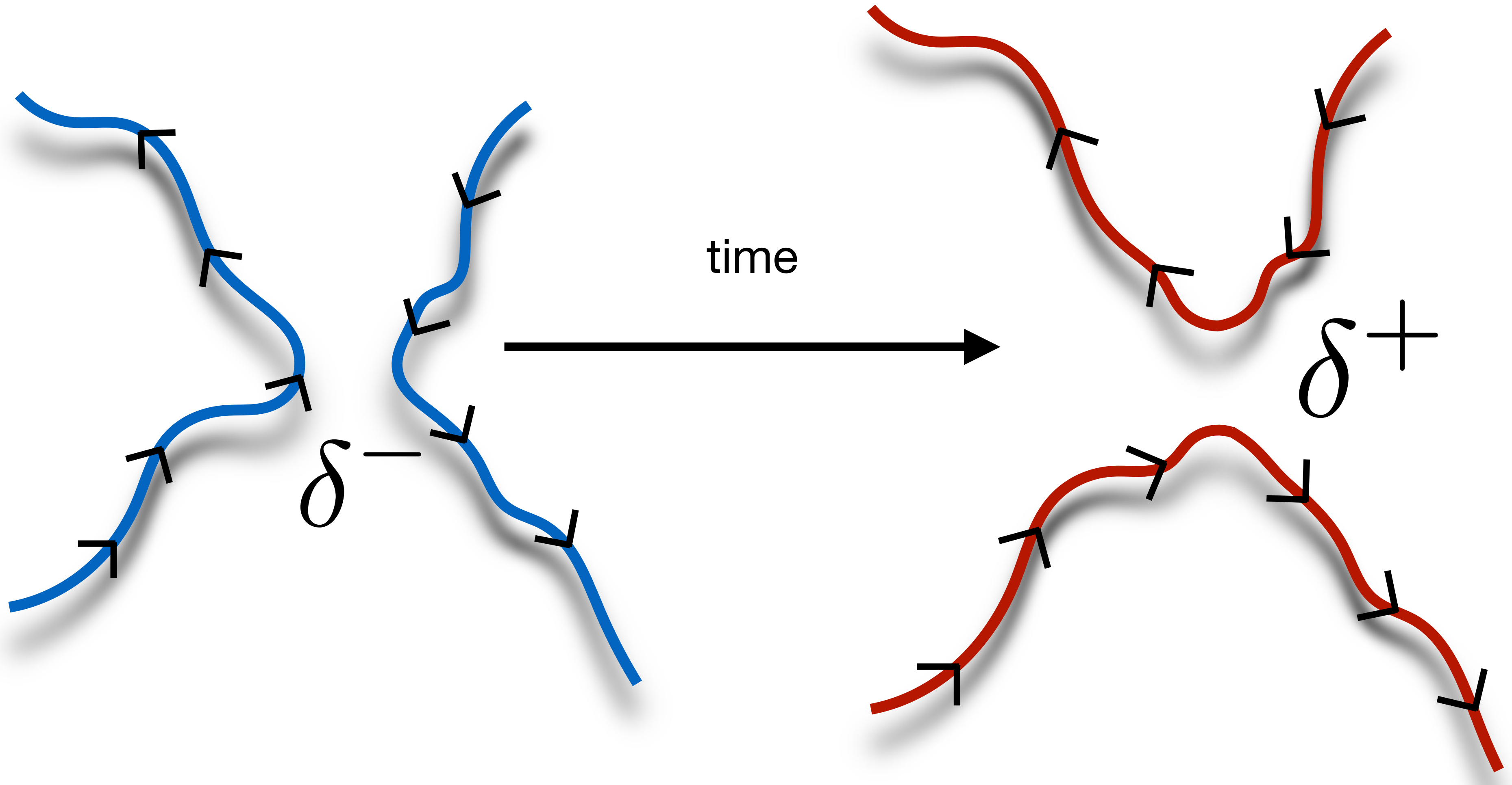


$$\Gamma = \oint \mathbf{v} \cdot \mathrm{d}\ell$$

$$[\Gamma] = \frac{L^2}{T}$$

$$\xi \ll \delta \ll R$$

$$\boxed{\delta^{\pm}(t) = A^{\pm}|\Gamma(t - t_r)|^{1/2}}$$



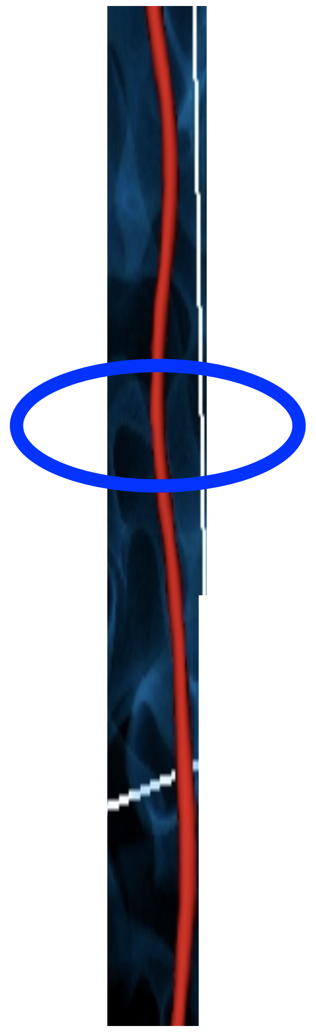
Minimal vortex distance

Analytic calculations

ξ : vortex core size

δ : minimal distance between vortices

R : system size



$$\Gamma = \oint \mathbf{v}\cdot \mathrm{d}\ell$$

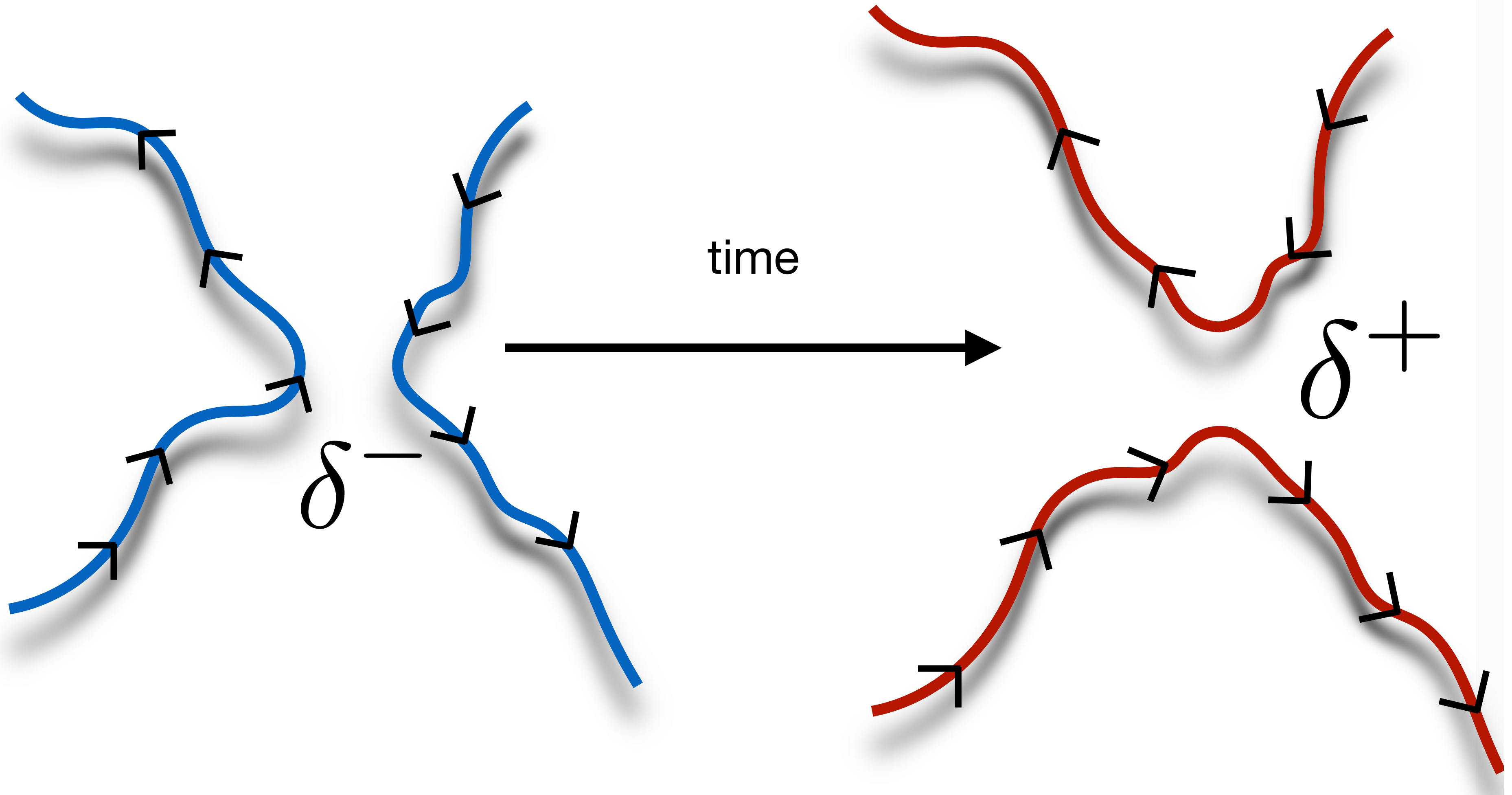
$$[\Gamma] = \frac{L^2}{T}$$

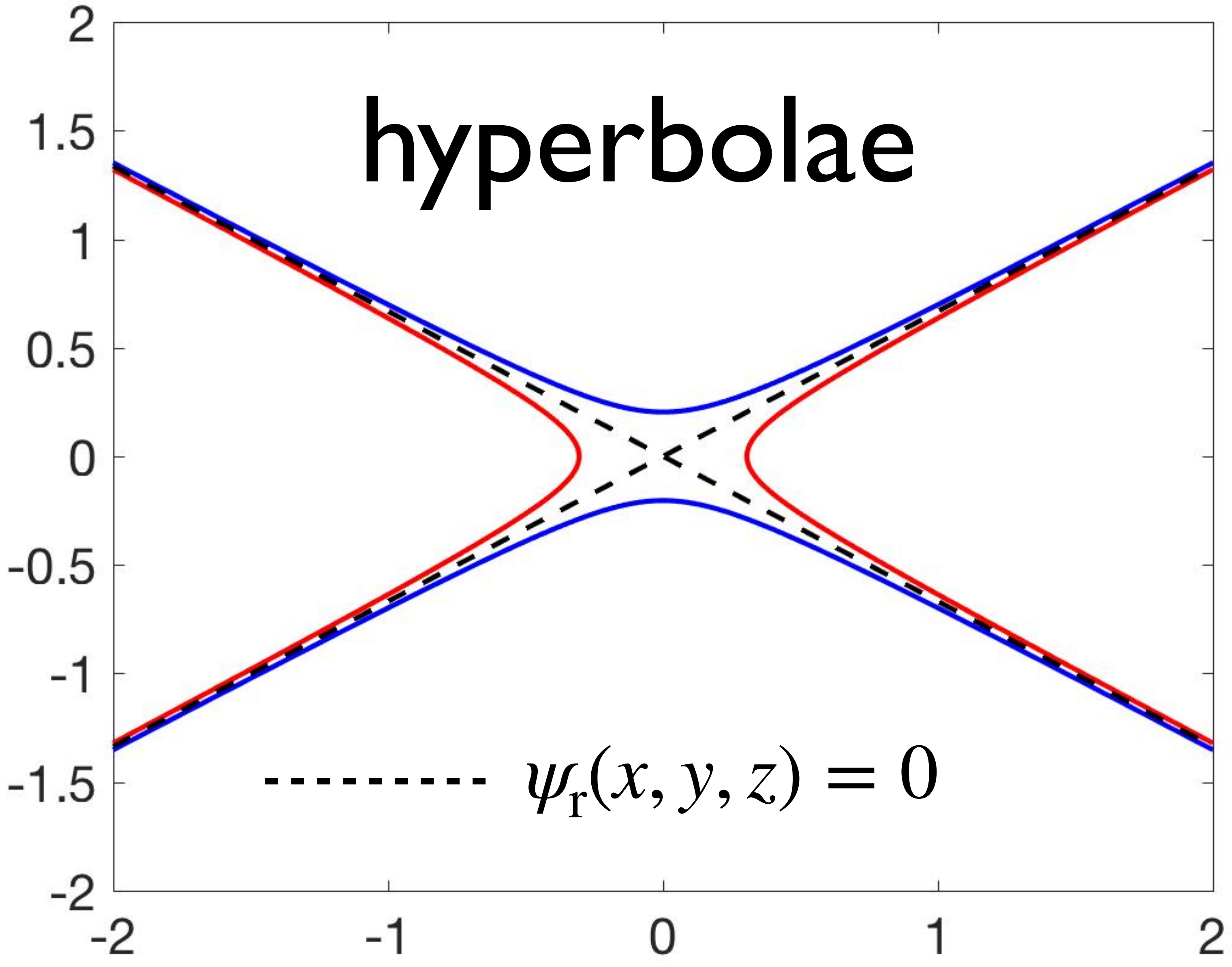
Nazarenko & West, JLTP (2003):

Very close to reconnecting vortices:

$$\delta(t) \ll \xi \quad \psi(\mathbf{x}) \approx 0$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \cancel{g|\psi|^2}\psi,$$





$$\psi_\mathrm{r}(x,y,z) = z + i(az + \beta x^2 - y^2)$$

$$\psi(\mathbf{x},t) = e^{i\frac{\hbar}{2m}(t-t_\mathrm{r})\nabla^2}\psi_\mathrm{r}(\mathbf{x})$$

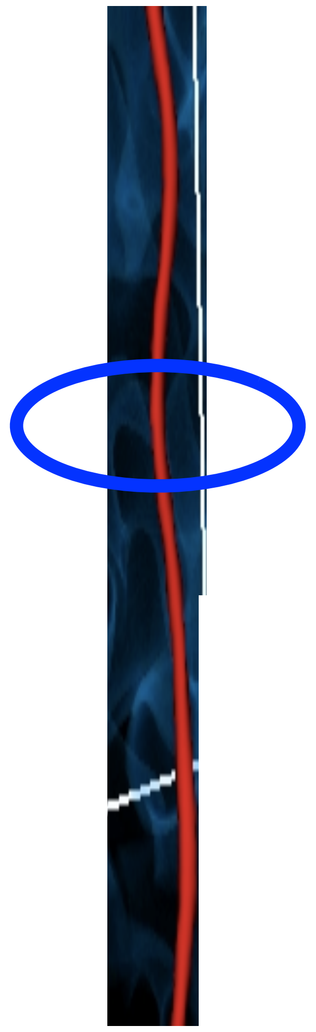
Minimal vortex distance

Analytic calculations

ξ : vortex core size

δ : minimal distance between vortices

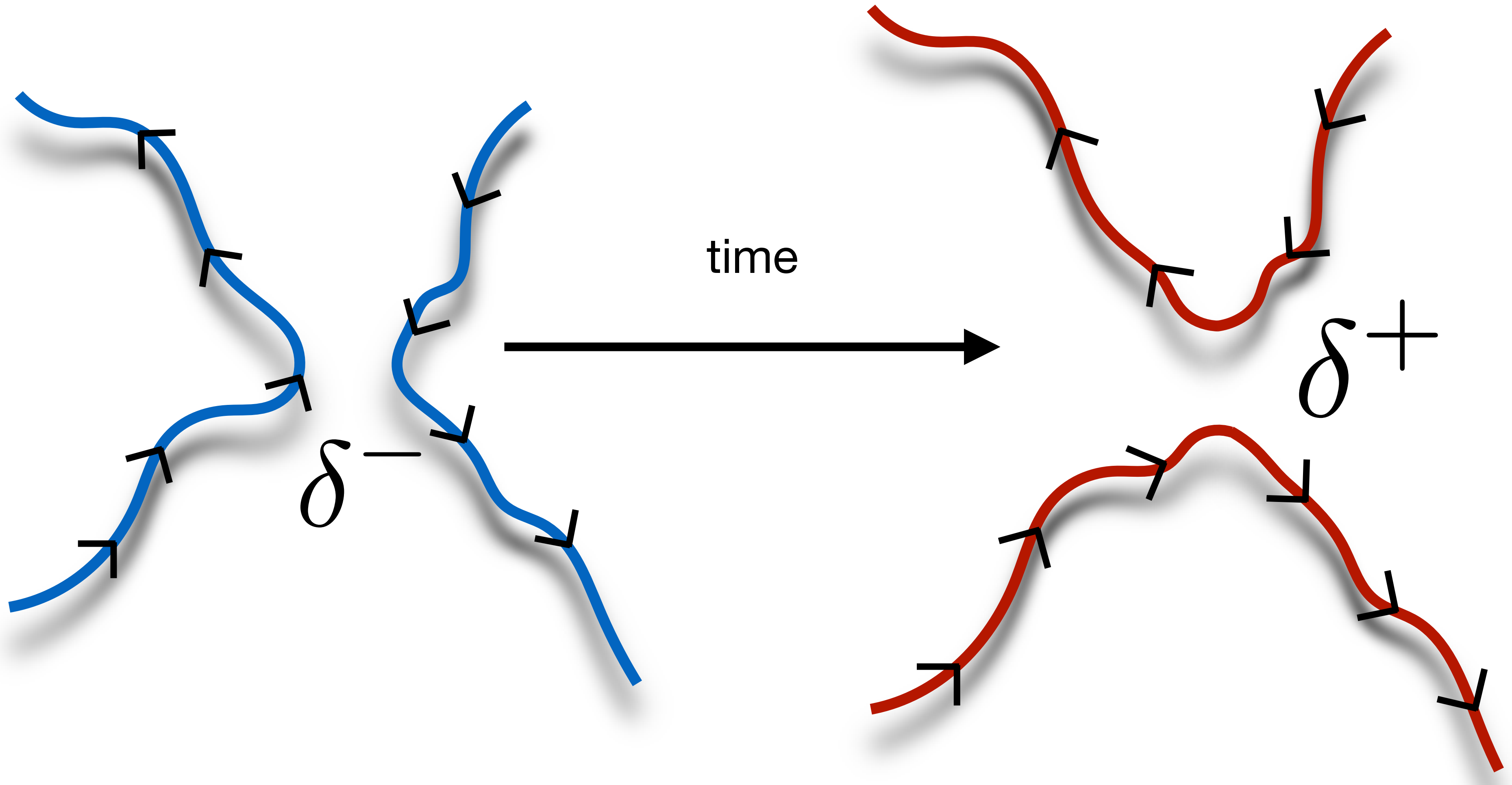
R : system size



$$\Gamma = \oint \mathbf{v} \cdot \mathrm{d}\ell$$

$$[\Gamma] = \frac{L^2}{T}$$

$$\left.\begin{array}{l} \xi \ll \delta \ll L \\ \\ \delta \ll \xi \end{array}\right\} \longrightarrow \boxed{\delta^{\pm}(t) = A^{\pm}|\Gamma(t - t_r)|^{1/2}}$$



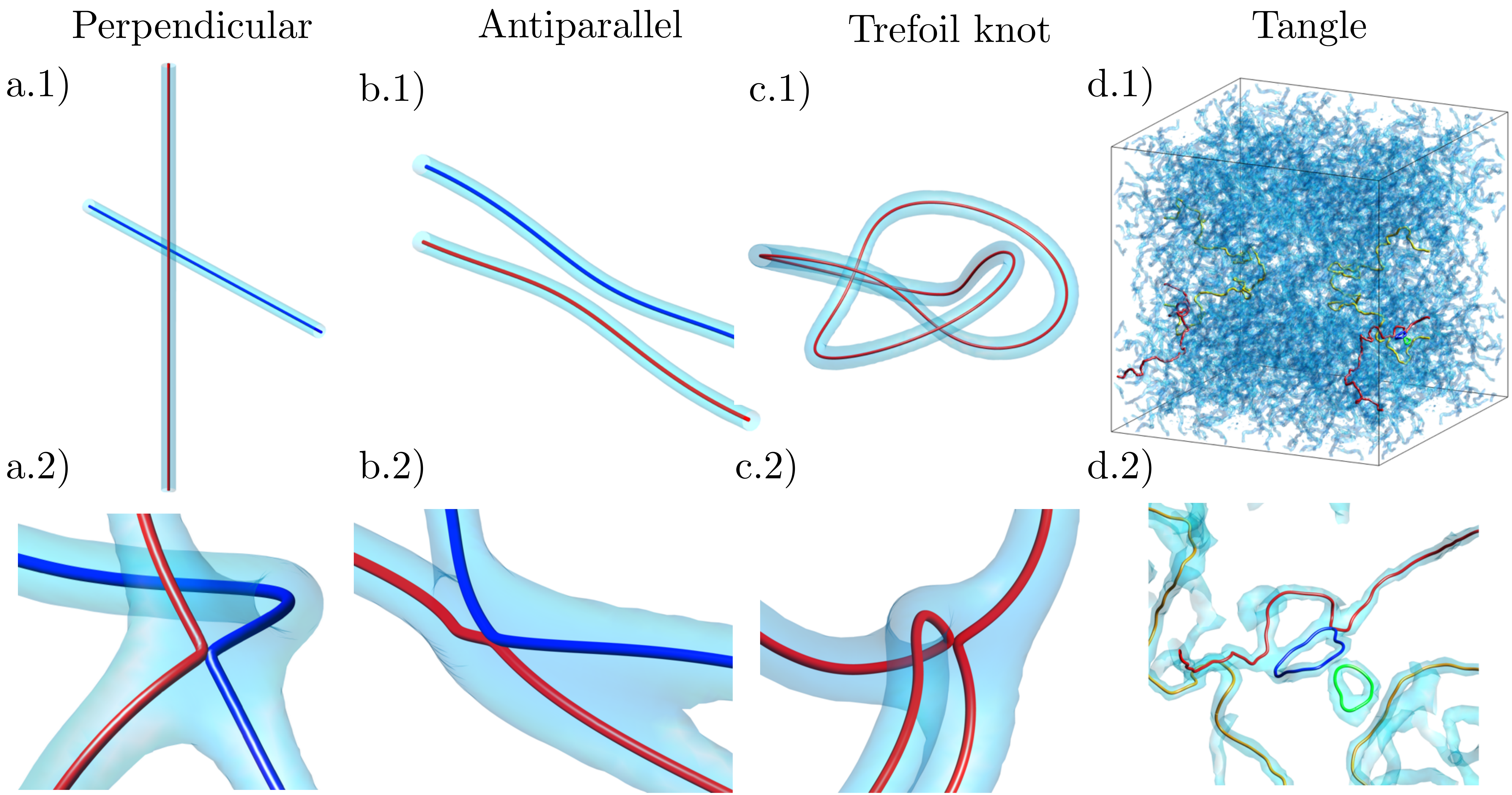
Previous works reported different exponents:

Zuccher et al Phys Fluids (2013)
Allen et al. PRA (2014)
Rorai et al. JFM (2016)

Quantum vortex filaments

4 study cases

A. Villois, GK and D. Proment
Phys. Rev. Fluids **2**, 044701 (2017)



Perpendicular

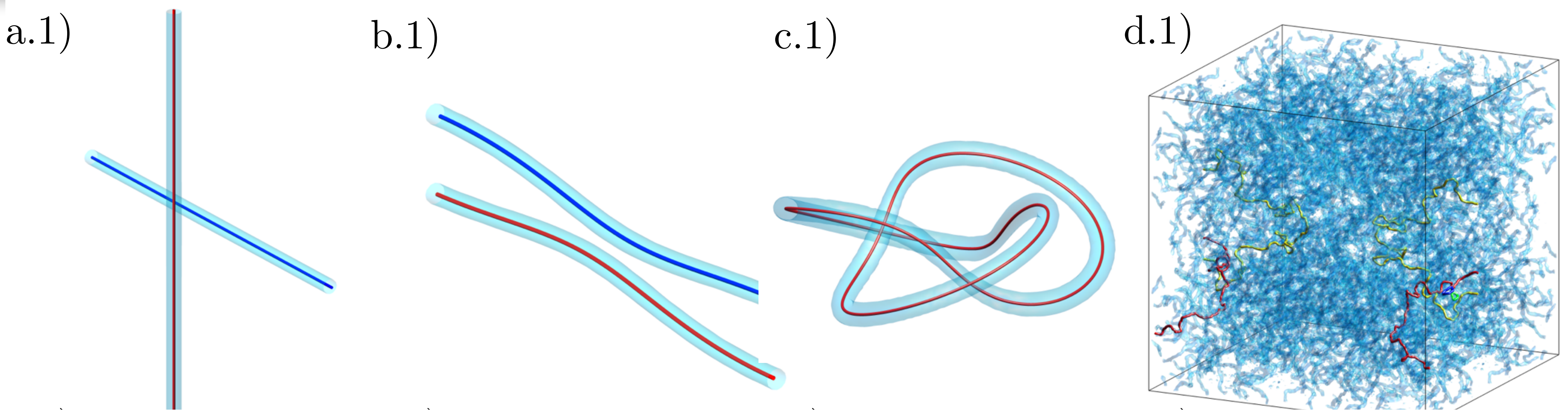
Antiparallel

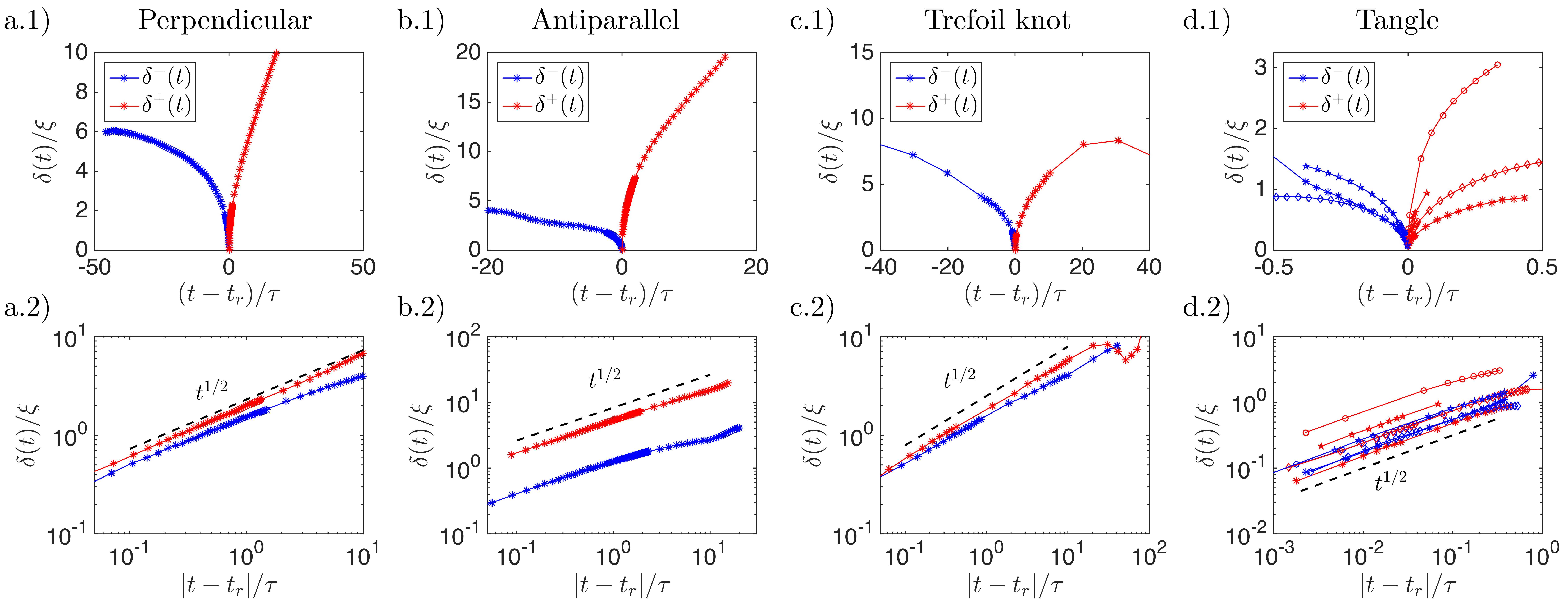
Trefoil knot

Tangle

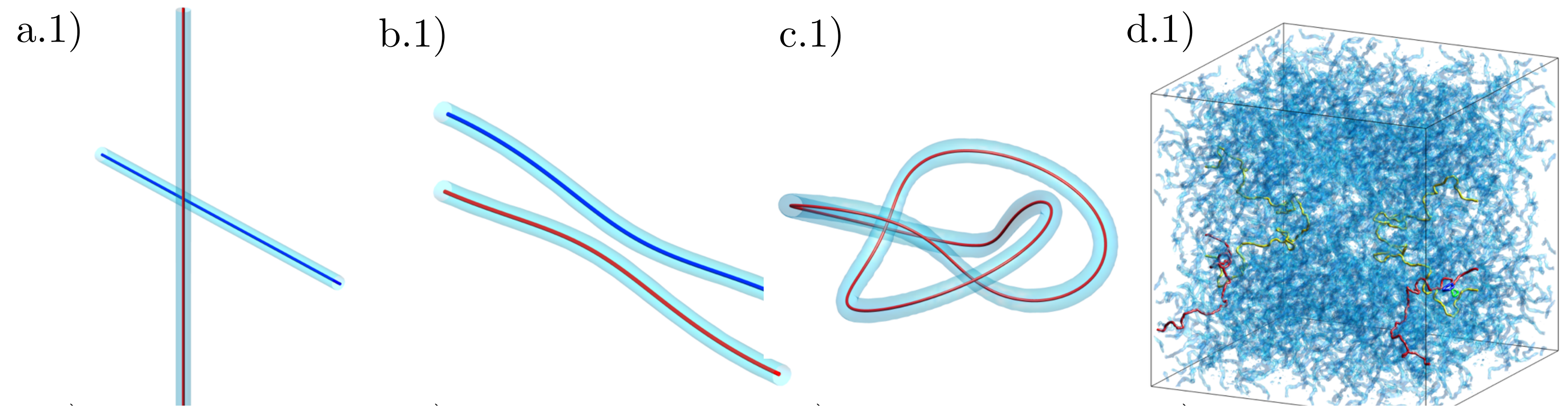
Separation rates

A. Villois, GK and D. Proment
Phys. Rev. Fluids **2**, 044701 (2017)

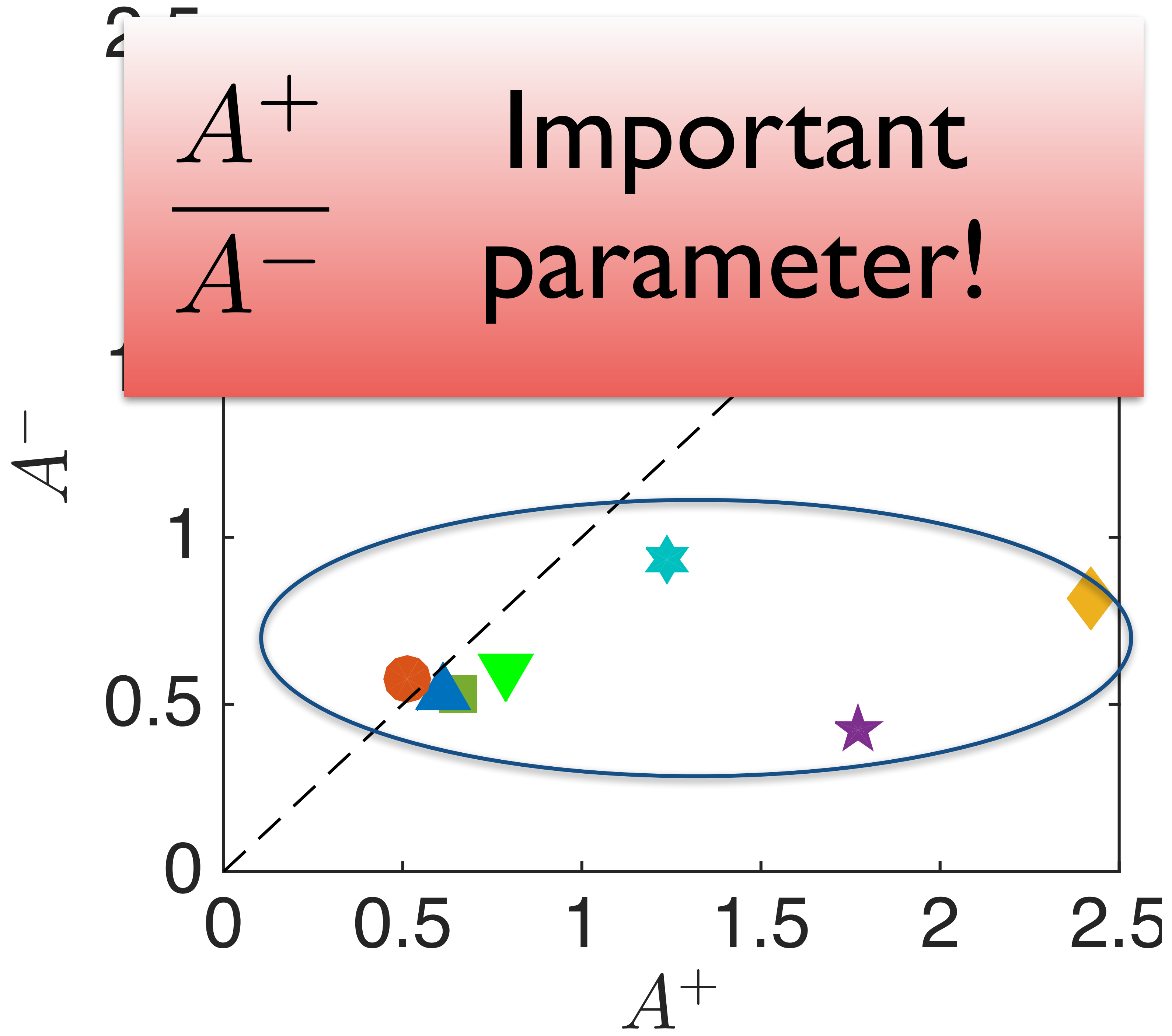




Geometry of reconnections



- $A^+ \geq A^-$: vortices separate faster than they approach



$\frac{A^+}{A^-}$ Important parameter!

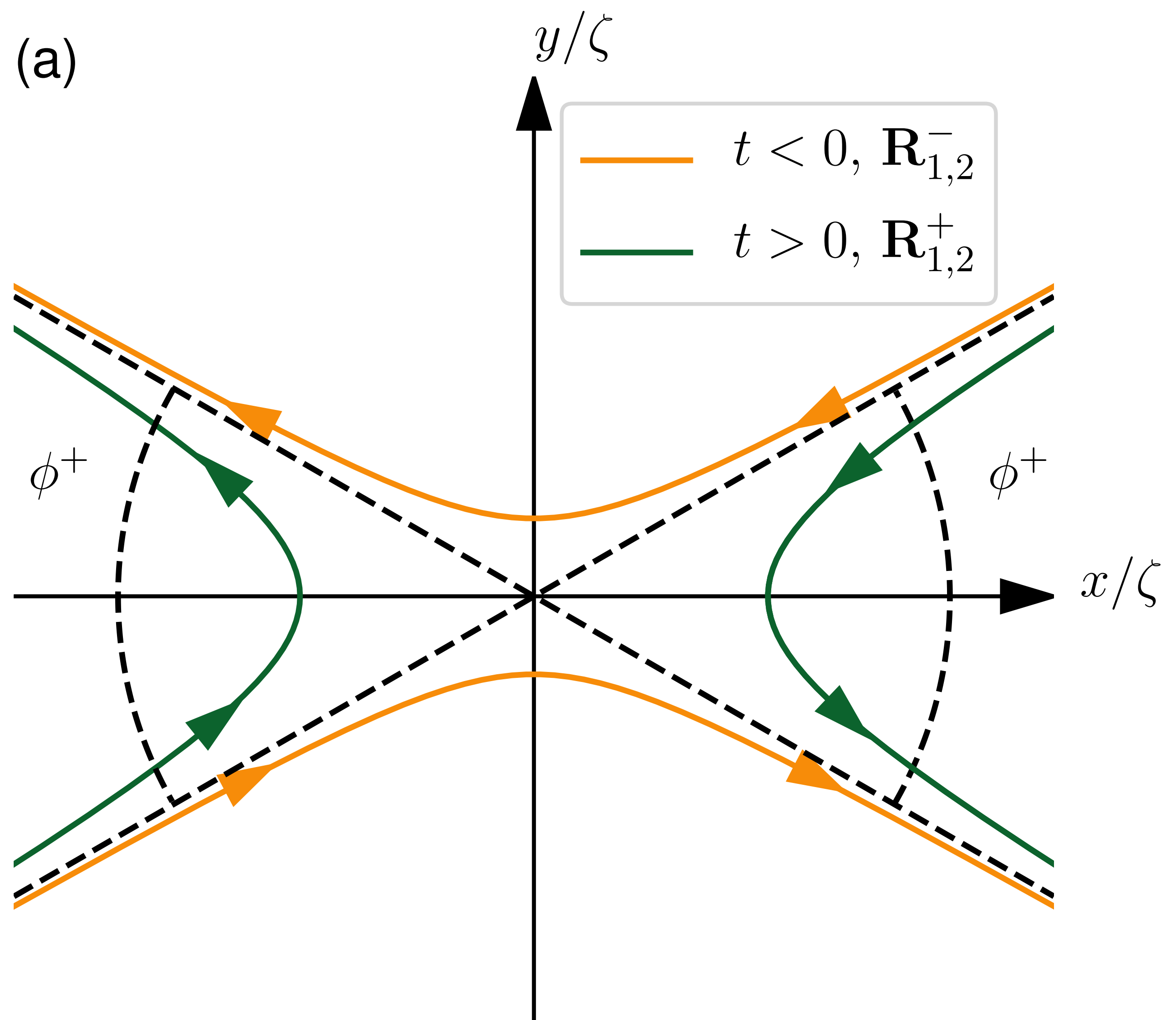
$$\boxed{\delta^\pm(t) = A^\pm|\Gamma(t - t_r)|^{1/2}}$$

Linear (Schrodinger equation) theory:

- A^+/A^- controls curvature, torsion and approach angle:

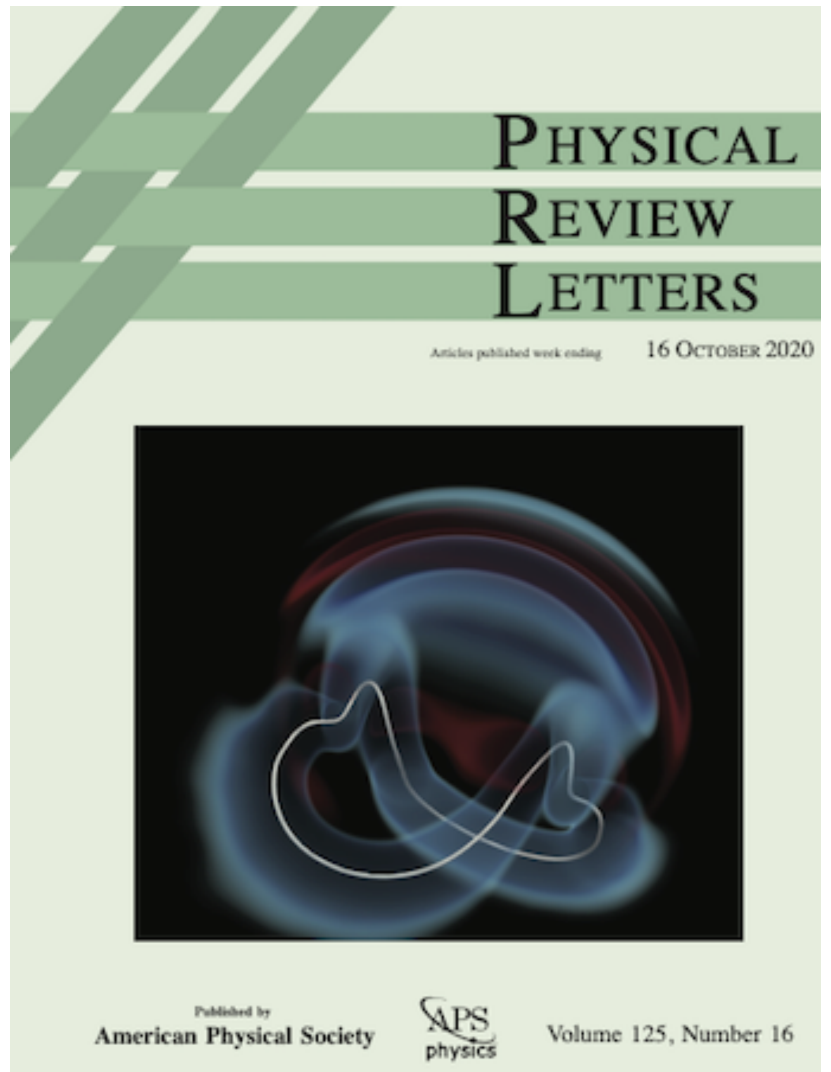
$$\frac{A^+}{A^-} = \cot\left(\frac{\phi^+}{2}\right)$$

$$\frac{A^+}{A^-} \underline{\gtrless} 1 \iff \phi^+ \lessgtr \frac{\pi}{2}$$



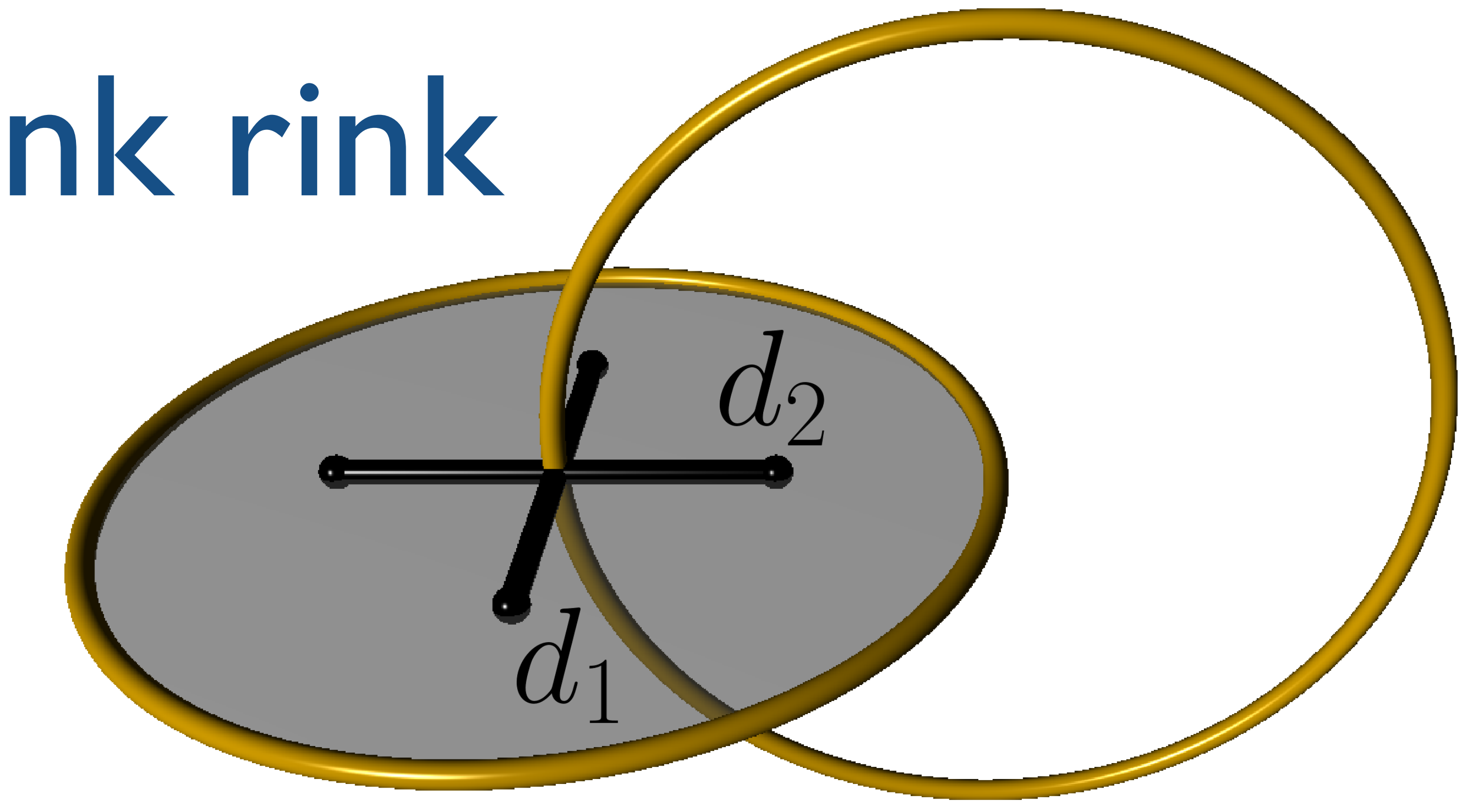
A. Villois, GK and D. Proment
Phys. Rev. Fluids **2**, 044701 (2017)

Quantum vortex reconnections

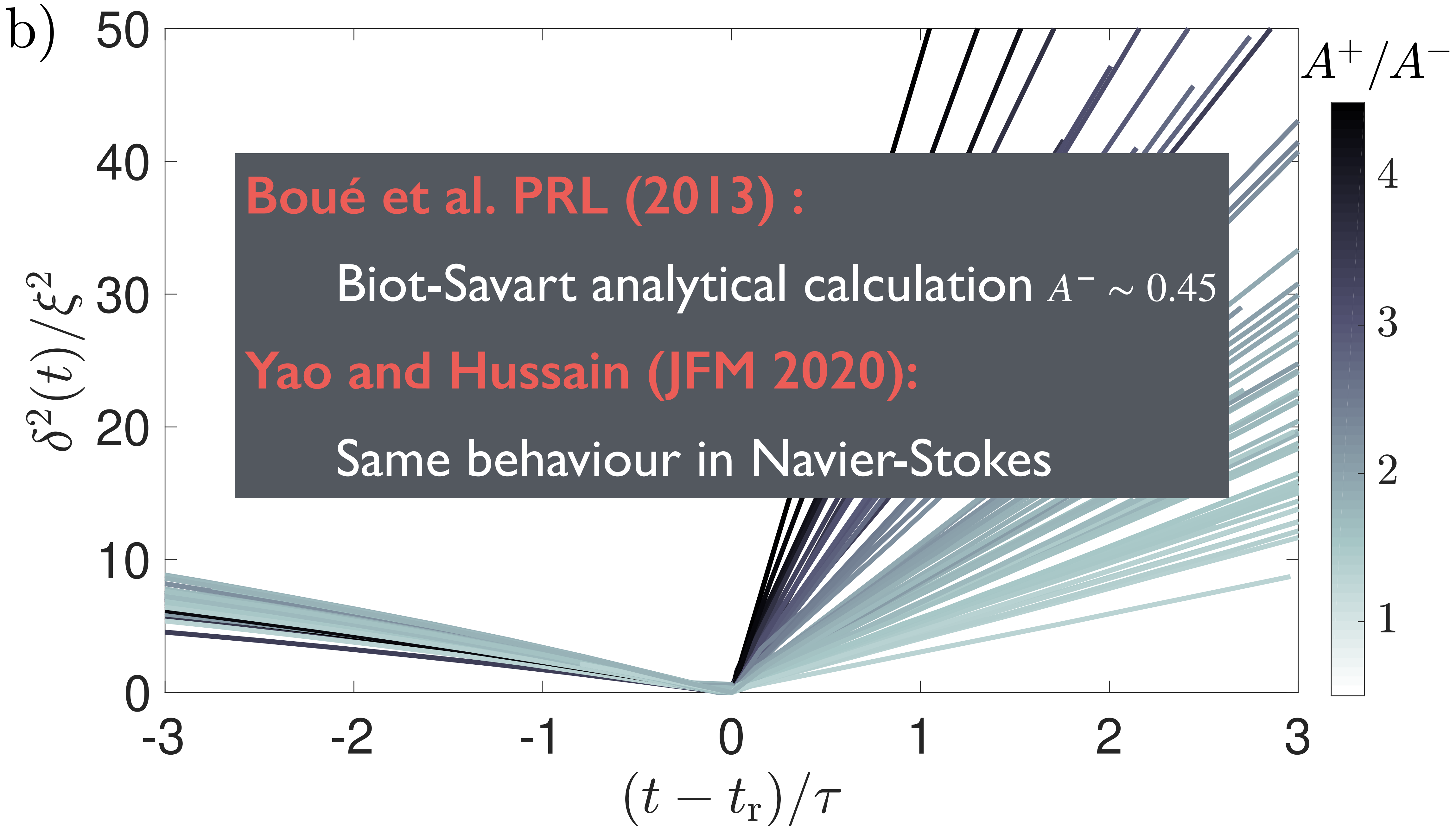


Villois, Proment and Krstulovic
PRL **128**,164501 (2020)

Hopf-link rink



More than 40 reconnections

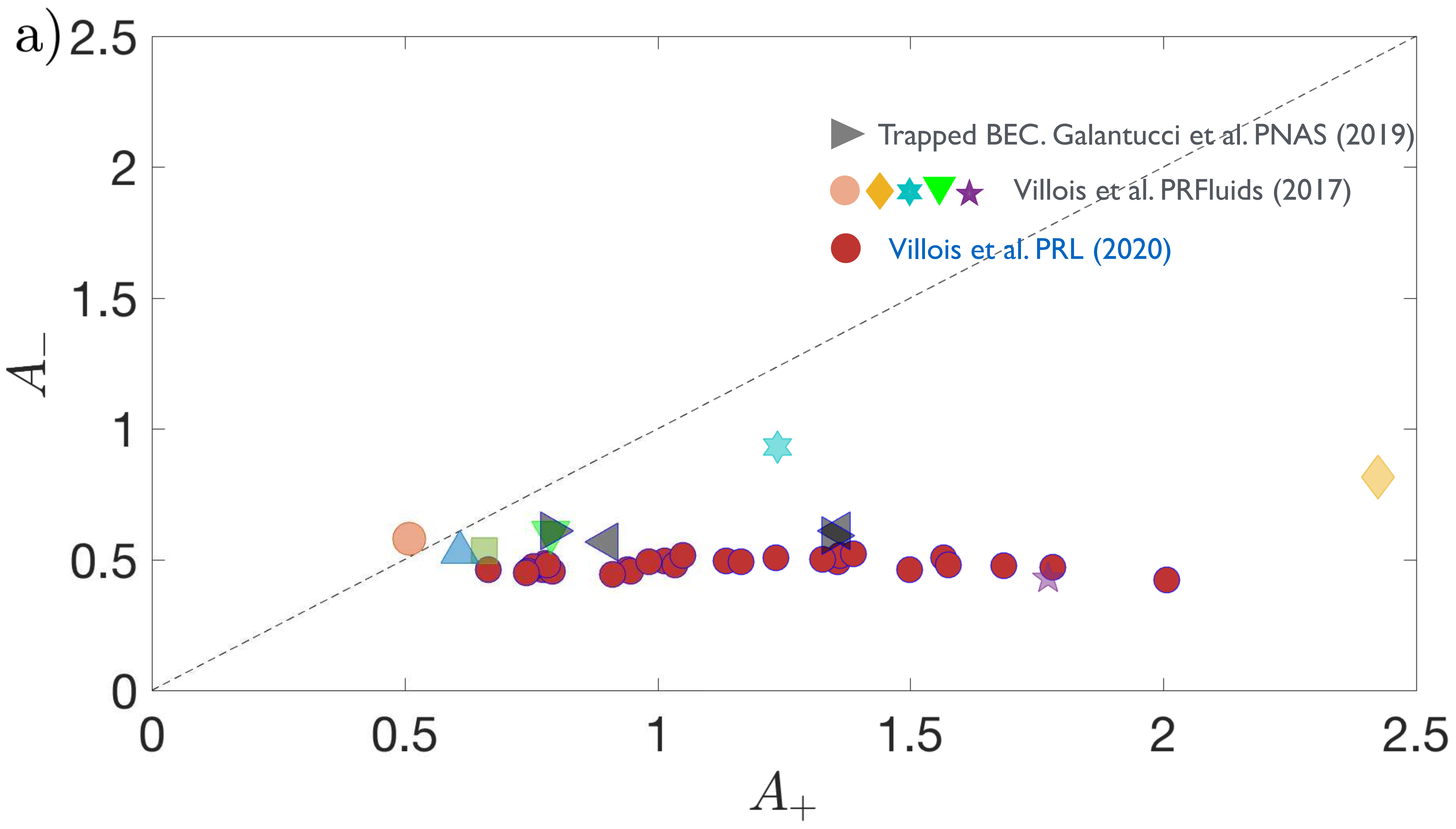


Boué et al. PRL (2013) :

Biot-Savart analytical calculation $A^- \sim 0.45$

Yao and Hussain (JFM 2020):

Same behaviour in Navier-Stokes



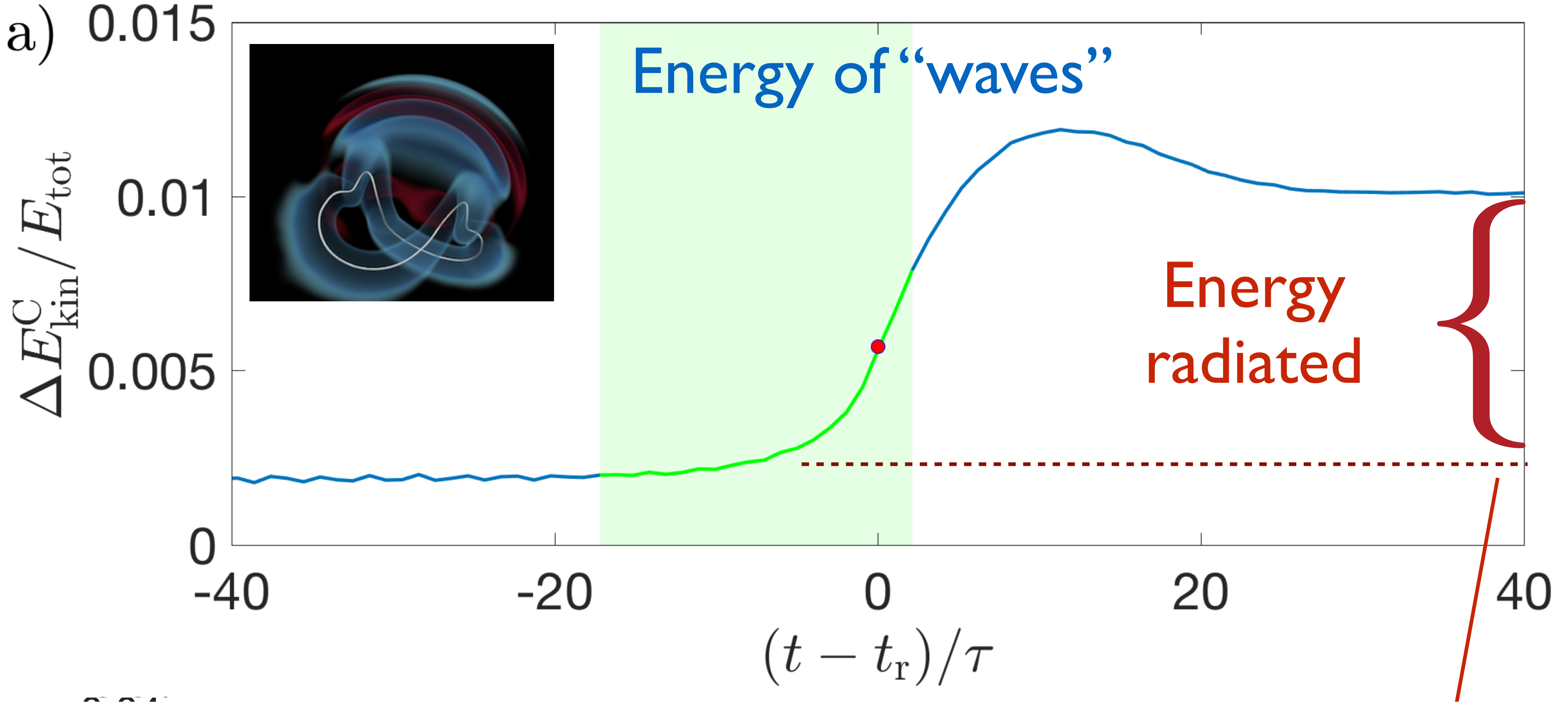
QUANTUM VORTEX RECONNECTIONS

Sound emission and irreversibility

Quantum vortex reconnections. Irreversibility and sound emission.
Krstulovic & Proment YouTube video : https://youtu.be/OhKUOV5irGI

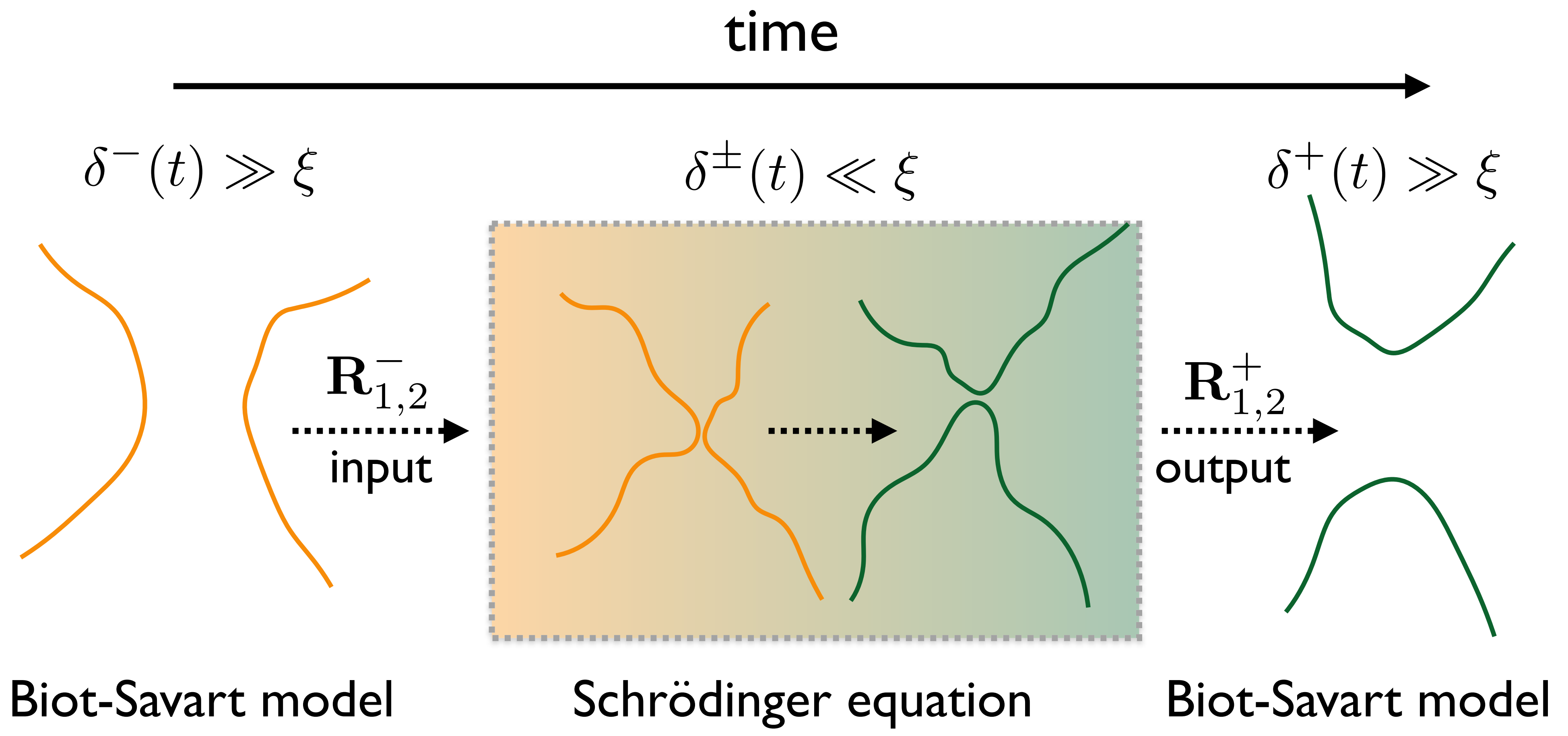
Quantum vortex reconnections

Numerical measurements



Vortex separate faster than they approach ↔ Energy is sent away irreversibly

Matching theory



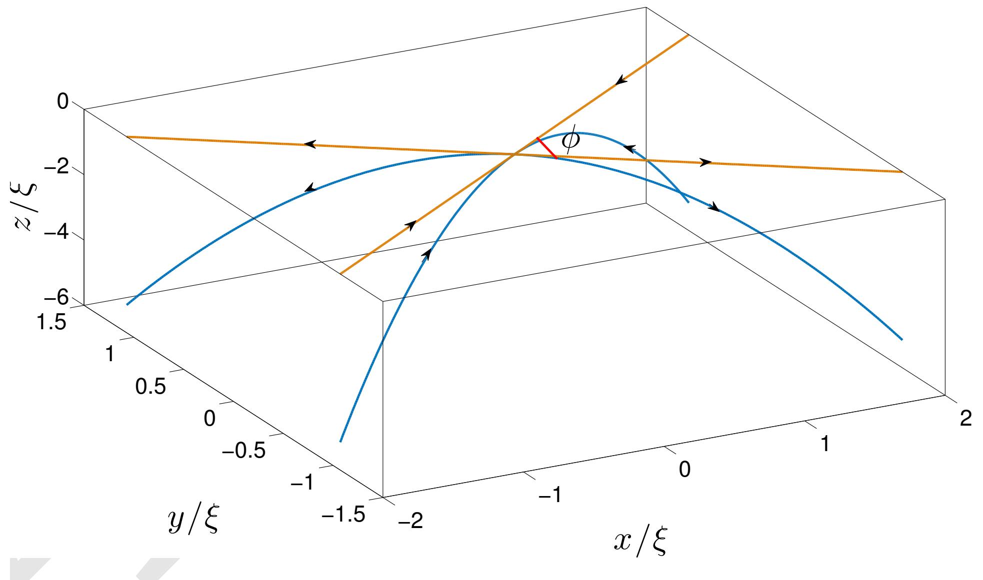
Analytical theory based on conservation of momentum and energy

Proment and Krstulovic
PRFluids **5**,704701 (2020)

Quantum vortex reconnections

A more general ansatz

$$\psi_r(x,y,z) = z + \underbrace{\frac{\gamma}{a}(x^2+y^2)}_{\text{torsion}} + \underbrace{i(az+\beta x^2-y^2)}_{\text{curvature}}$$



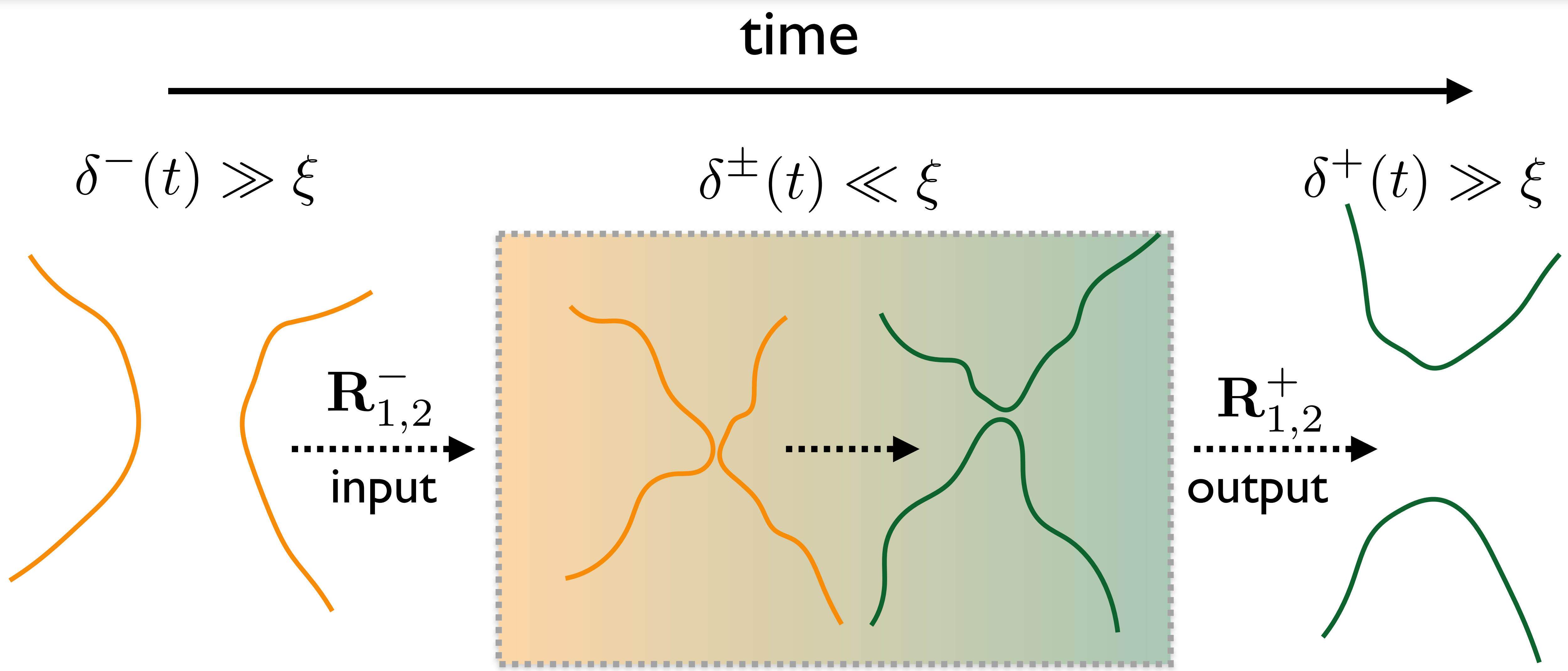
$$\psi(\mathbf{x},t) = e^{i\frac{1}{2}(t-t_r)\nabla^2}\psi_r(\mathbf{x})$$

$$\psi(\mathbf{x},t) = 0$$

$$\mathbf{R}^-_{1,2}(s,t) =\Bigg(s,\ \pm\sqrt{\frac{(t_r-t)(a^2(1-\beta)-2\gamma)+as^2(\beta-\gamma)}{a(\gamma+1)}},\ \frac{(t-t_r)(a^2(\beta-1)-2\gamma^2)-a\gamma(\beta+1)s^2}{(\gamma+1)a^2}\Bigg)$$

$$\boxed{\delta^\pm(t) = |\mathbf{R}^\pm_1(0,t)-\mathbf{R}^\pm_2(0,t)| = \sqrt{2\pi}A^\pm|t-t_r|^{1/2}}$$

$$\boxed{\frac{A^+}{A^-} = \sqrt{\frac{1+\gamma}{\beta-\gamma}}}$$



Biot-Savart model

Schrödinger equation

Biot-Savart model

$$\mathbf{P}^- = \frac{\Gamma}{2}\oint \mathbf{R}^-\times \mathrm{d}\mathbf{R}^-$$

Momentum

$$\mathbf{P}^+ = \frac{\Gamma}{2}\oint \mathbf{R}^+\times \mathrm{d}\mathbf{R}^+$$

$$E^-_{\mathrm{kin}} \propto \Gamma^2\oint|\mathrm{d}\mathbf{R}^-|$$

Energy

$$E^+_{\mathrm{kin}} \propto \Gamma^2\oint|\mathrm{d}\mathbf{R}^+|$$

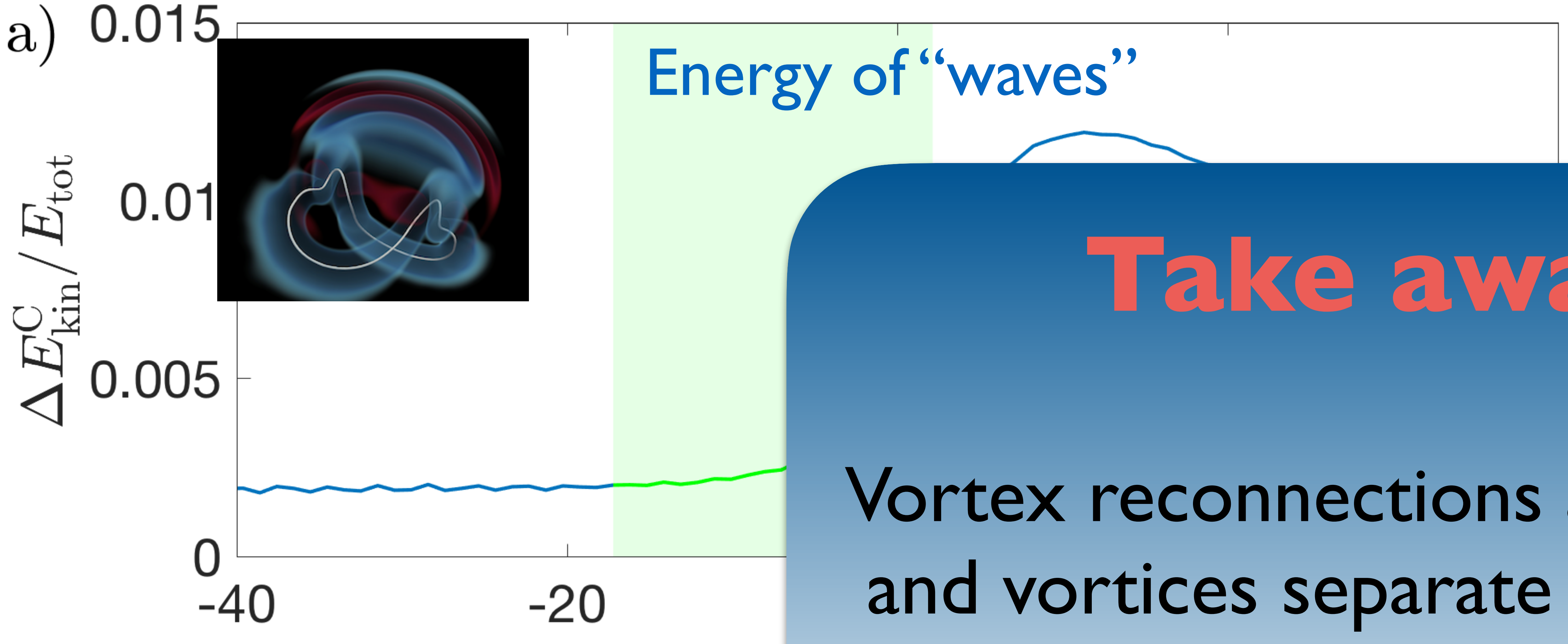
$$\Delta\mathbf{P} = \mathbf{P}^+ - \mathbf{P}^-$$

$$\Delta E_{\mathrm{kin}} = E^+_{\mathrm{kin}} - E^-_{\mathrm{kin}}$$

Proment and Krstulovic
PRFluids **5**,704701 (2020)

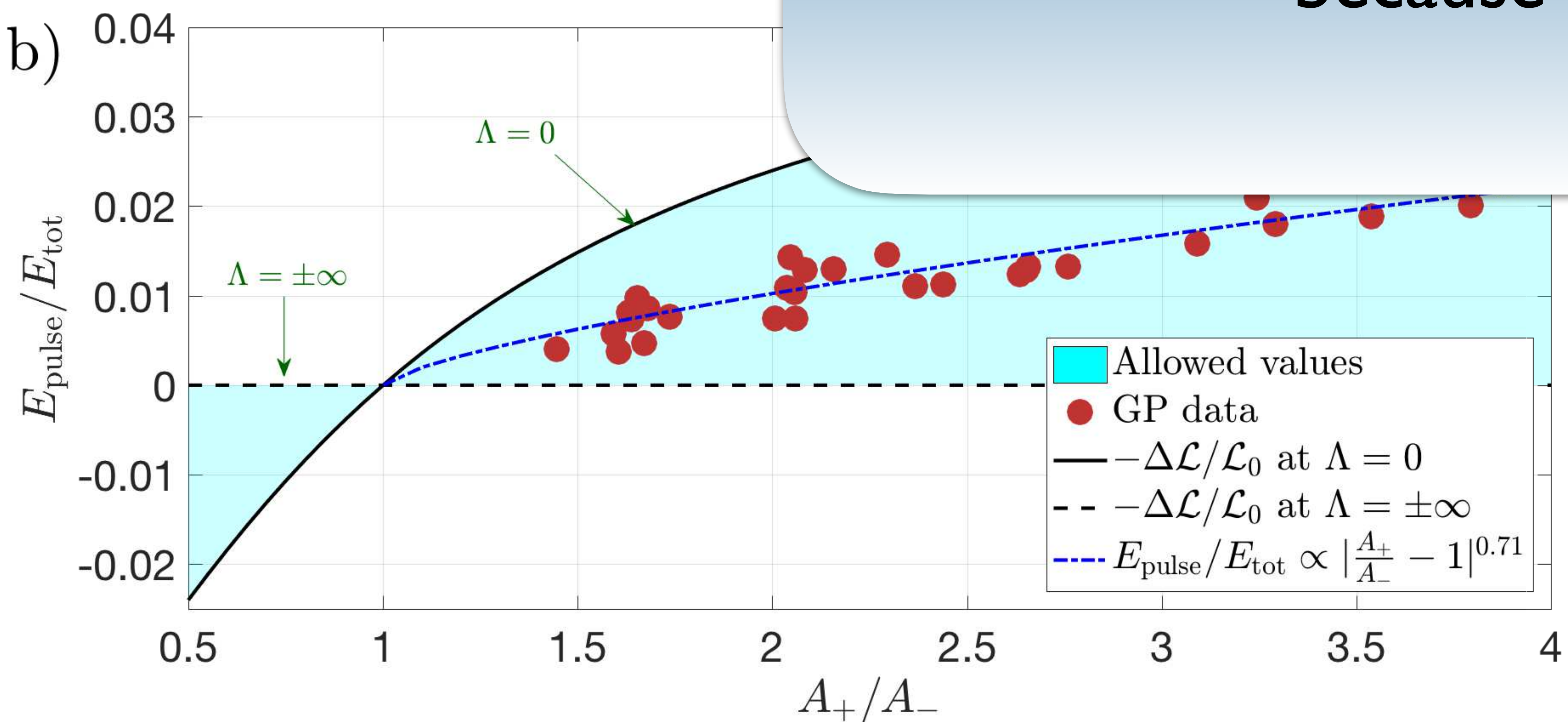
Quantum vortex reconnections

Numerical measurements

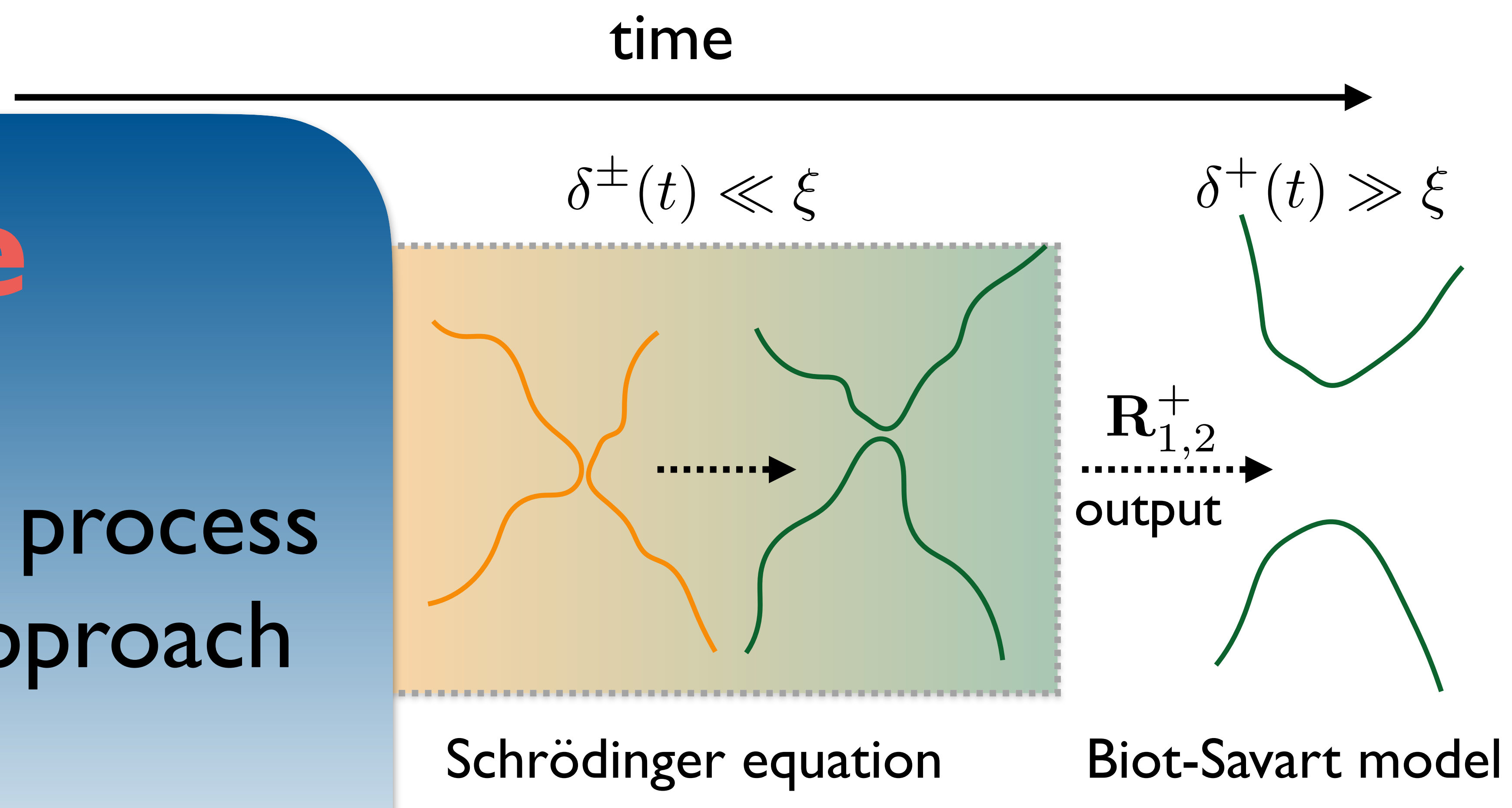


Take away message

Vortex reconnections are an irreversible process and vortices separate faster than they approach because is "cheaper"



Matching theory



Analytical theory based on conservation of momentum and energy:

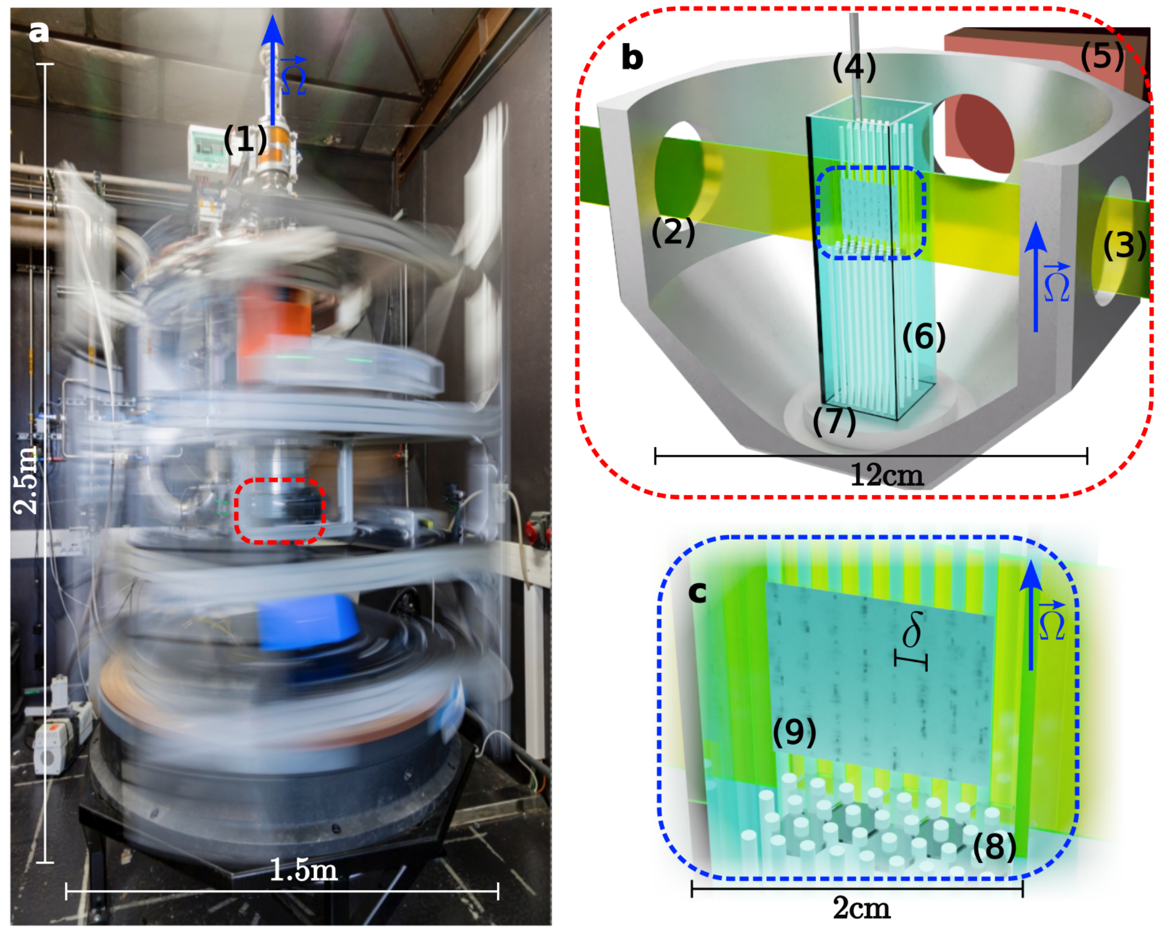
- Directionally of the pulse
- Energy radiated and geometry

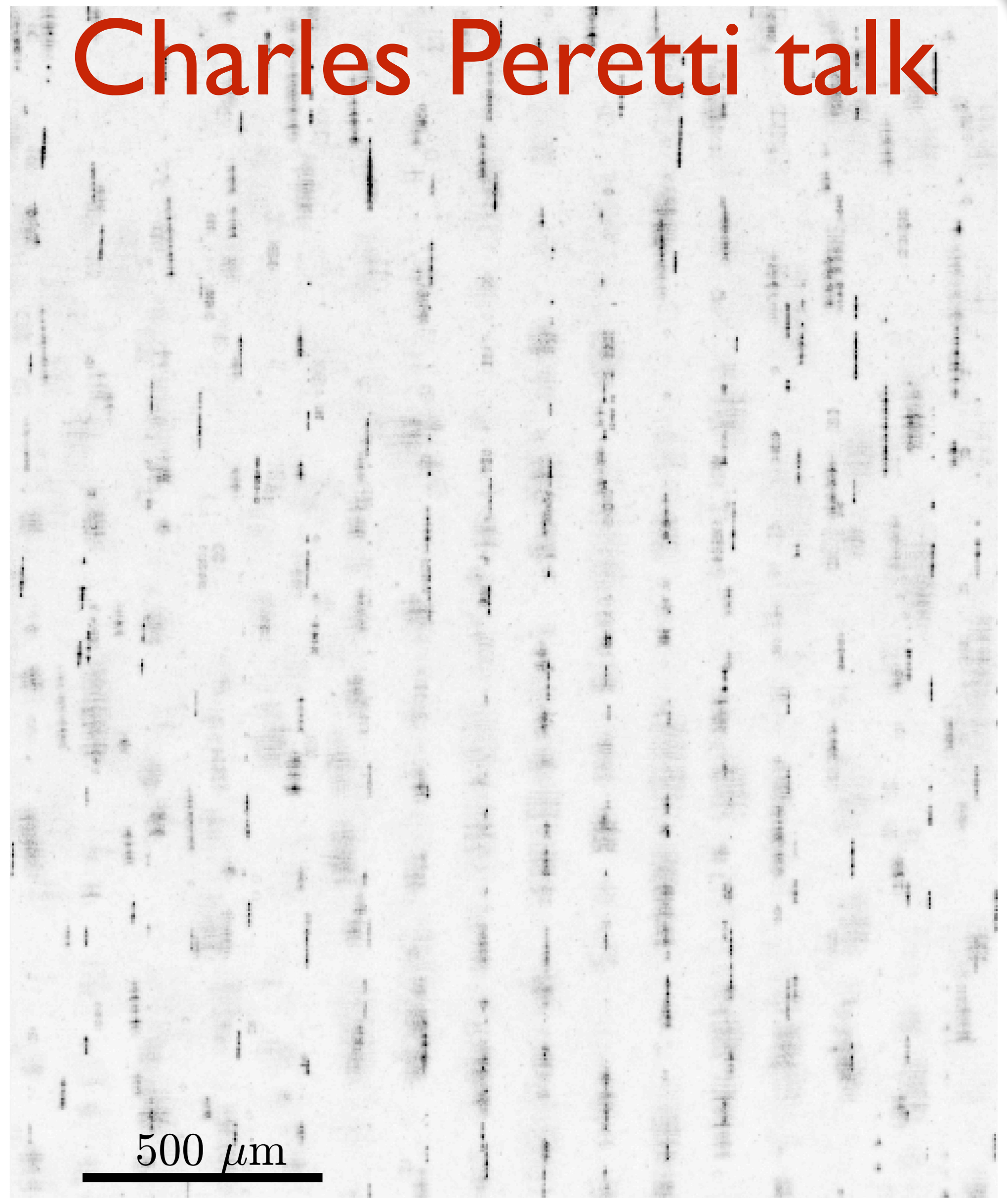
Proment and Krstulovic
PRFluids **5**,704701 (2020)

QuantumVIW project

Cryolem

Mathieu Gibert
Institut Néel (Grenoble)




Charles Peretti talk

FOUCAULT

Fully cOUpled loCAl model of sUperfLuid Turbulence.
(VF and NS)


Luca Galantucci talk

Recently funded project

PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

ANR

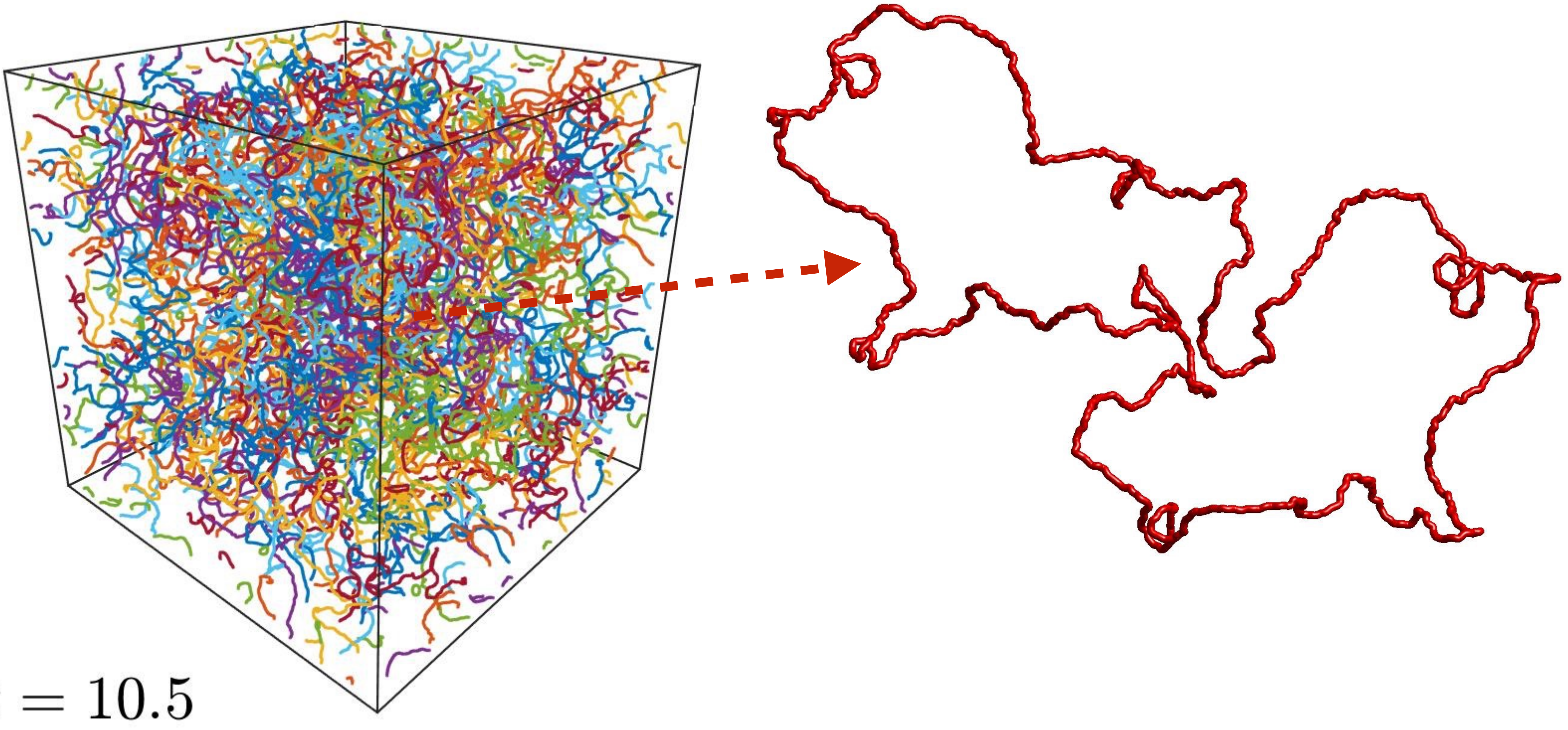
With

M. Gibert
P.P. Cortet
J.P. Polanco

Postdoc position available

Summary

Kelvin waves



Kelvin wave cascade exists and is predicted by the wave turbulence theory

Vortex reconnections