Bridging Classical and Quantum Turbulence, Cargèse, Corsica (France), July 3-15 (2023)

# Visualization study of the law of wall in superfluid helium-4

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GORDON AND BETTY MOORE FOUNDATION

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# GUO CRYOGENICS LAB

## Group members




Collaborators:

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- Dr. S. Bao (Zhejiang University)
- Dr. Y. Zhang (FSU→IIT)
- Prof. Lou Cattafesta (FSU→IIT)

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Graduate students:

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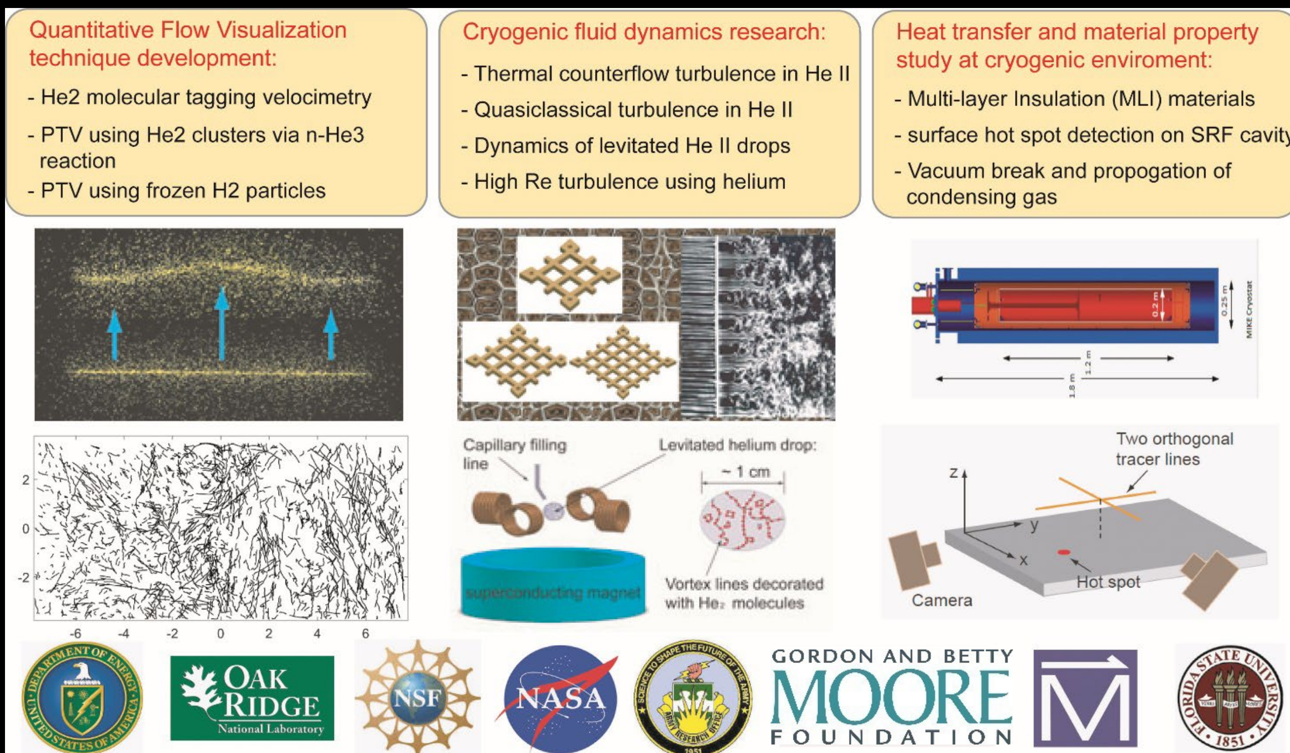
Q. Rababah, G. Mayans

# What do we do?

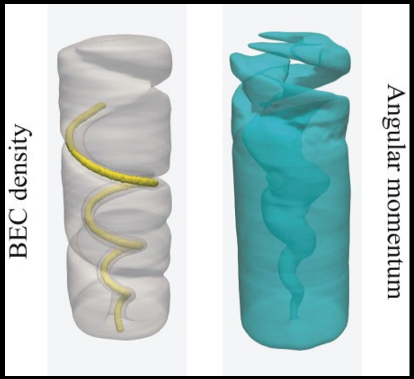



- Quantum fluid dynamics
- Particle accelerator cryogenics
- LHe dark matter
- $LH_2$ aviation
- Qubit: $e^-$ on LHe and solid neon

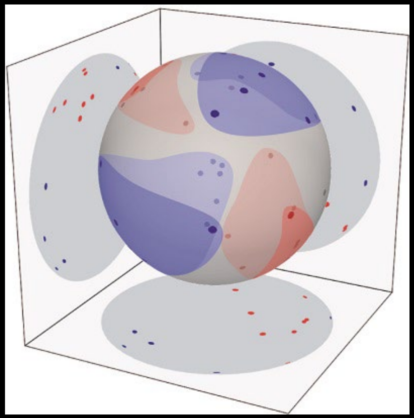
http://www.eng.famu.fsu.edu/~wguo/index.html

- Besides experimental work, we also conduct numerical simulations:

- GPE

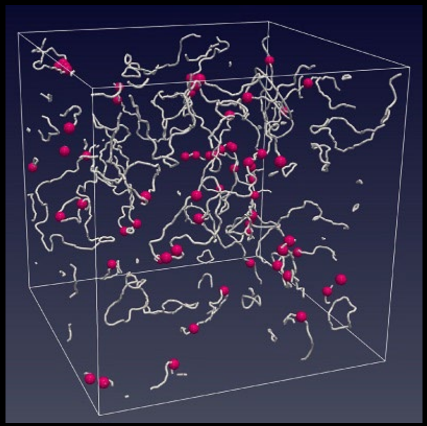



Merging dynamics of rotating superfluid (PRL, 124, 105302 (2020))

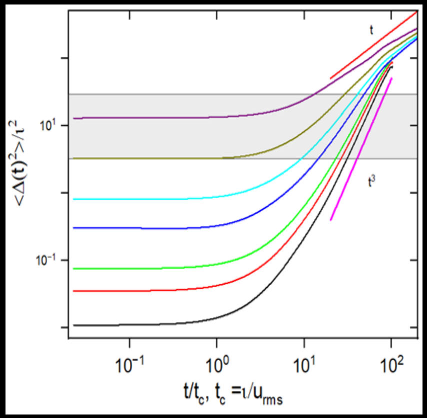


Onsager vortices in 2D superfluid turbulence (PRL, 127, 095301 (2021))

- Vortex filament

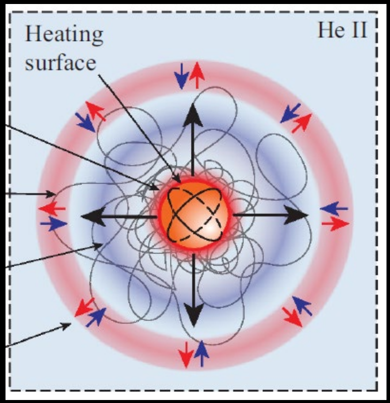


Superdiffusion of vortices (PRL, 124, 105302 (2020))

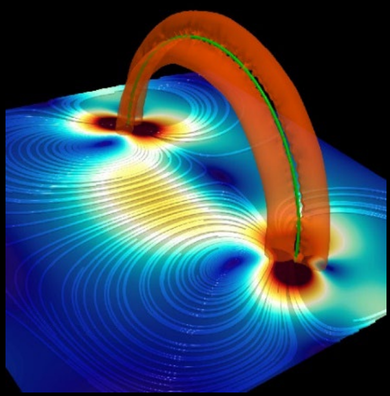



Turbulent dispersion of vortices (in submission)

- HVBK




Counterflow and peak heat flux (PRB, 103, 134510 (2021); PRB, 106, 054501 (2022))



Vortex ring dynamics (Nat. Commun. 14, 2941 (2023))

# Outline

GUO CRYOGENICS LAB

1. Introduction
   - LHe for high Reynolds # turbulence research
   - He II fundamentals
   - Counterflow and quasiclassical flow in He II

2. Flow visualization in He II
   - Particle tracking velocimetry (PTV)
   - Molecular tagging velocimetry (MTV)

3. Law of wall in He II
   - Concept of the law of wall
   - Experimental procedure
   - Near-wall velocity profile

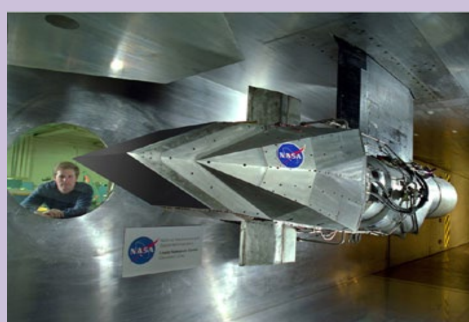
# 1. Introduction

# • LHe for high Reynolds # turbulence research

- Many turbulent flows in nature have extremely high Reynolds (Re) #:


- To generate such flows in laboratory for systematic research, one may either increase U and L or decrease ν

$$Re = \frac{U \cdot L}{\nu}$$



**Increasing *U* or *L***

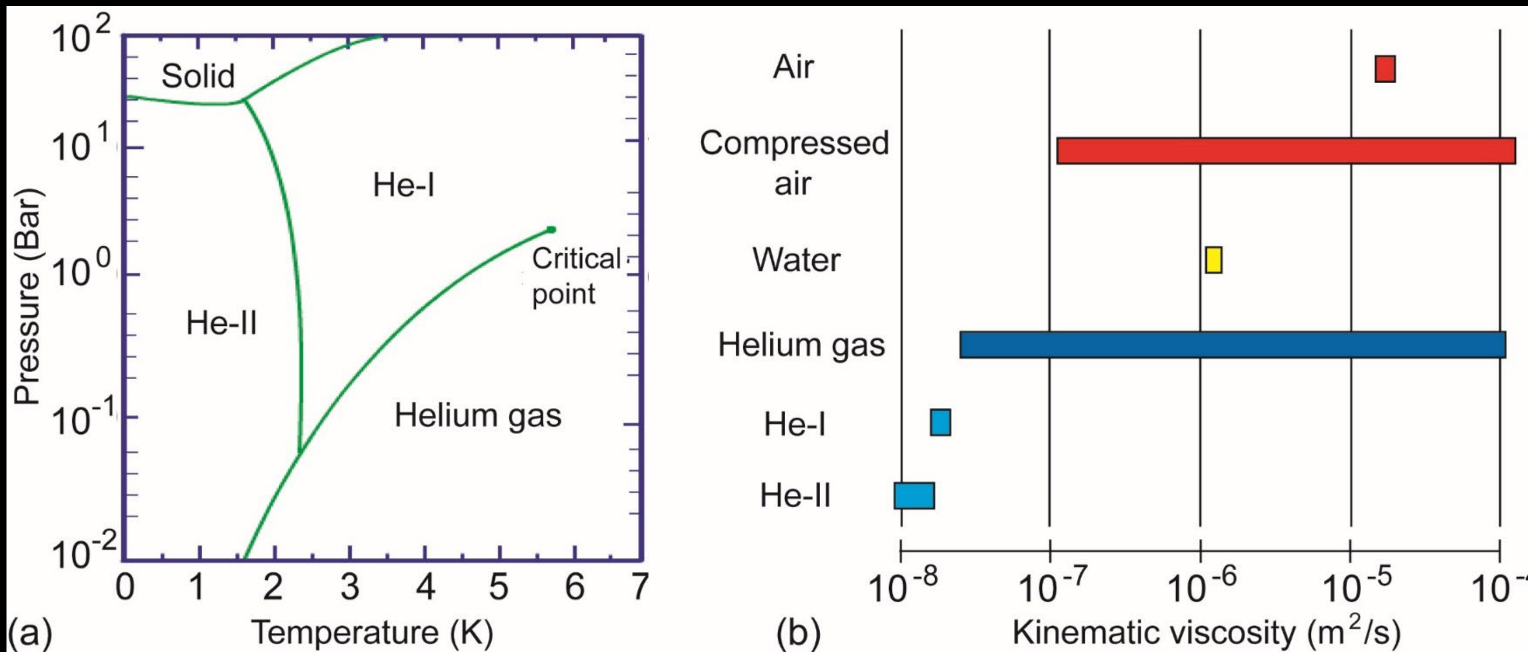
- Compressibility effects
- Cost/energy intensive

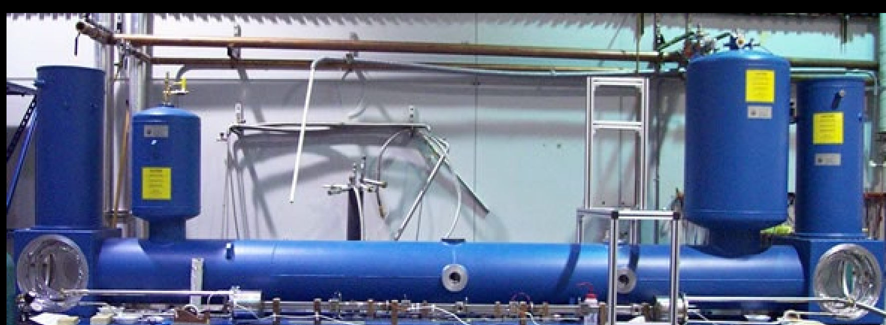
**Decreasing *ν*** [1]

- High Pre. facilities
- Safety

- Large-size facilities are expensive to build and operate
- @ large U, compressibility effect becomes important
- Princeton Superpipe utilized compressed air at 200 bars to achieve small ν. Flow visualization is impractical.

- LHe has a great potential due to its extremely small viscosity:



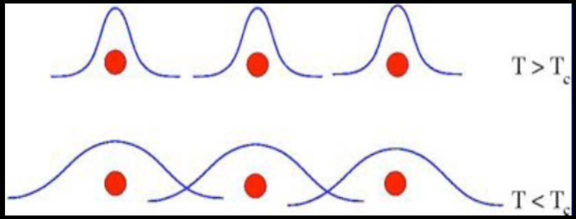



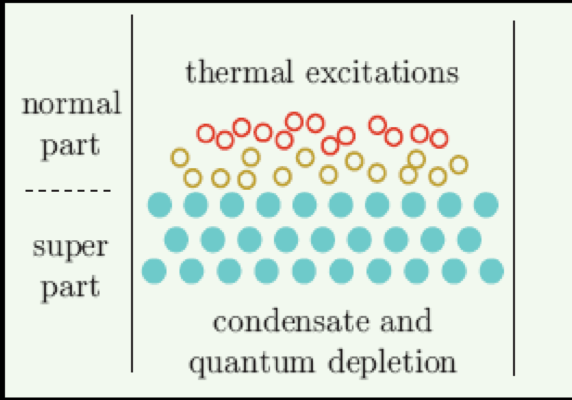
See Rev. Sci. Instrum., 91, 053901 (2020).

Pipe flows (ID=2 cm) with Re~10^7 has already been demonstrated in our cryogenics lab using He-II.

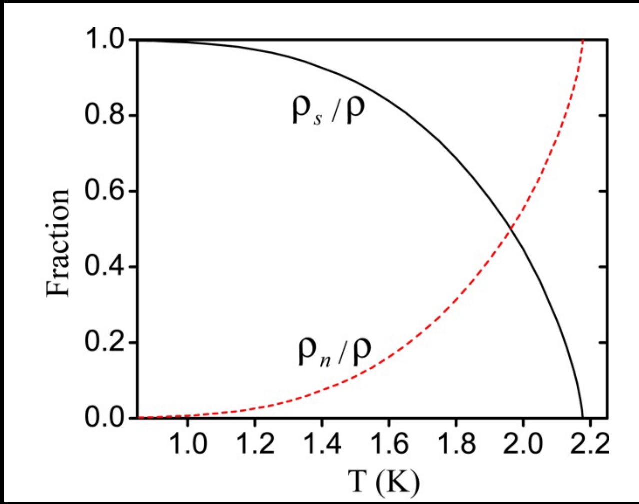
# • He II and quantized vortices

- He4 becomes superfluid below ~2.2 K
- There exist two components:
  → Superfluid component (condensate)
  → Normal-fluid component (excitations)




$$\rho = \rho_s + \rho_n$$




| | Density | Velocity | Viscosity | Entropy |
|---|---|---|---|---|
| Superfluid | $\rho_s(T)$ | $\mathbf{v}_s(\mathbf{r})$ | 0 | 0 |
| Normal fluid | $\rho_n(T)$ | $\mathbf{v}_n(\mathbf{r})$ | $\eta_n(T)$ | $s_n(T)$ |

## ➢ Circulation in the superfluid is quantized:

- Superfluid wavefunction: $\psi(\vec{r},t) = \sqrt{n_0(\vec{r},t)}\,\exp\left[i\phi(\vec{r},t)\right]$

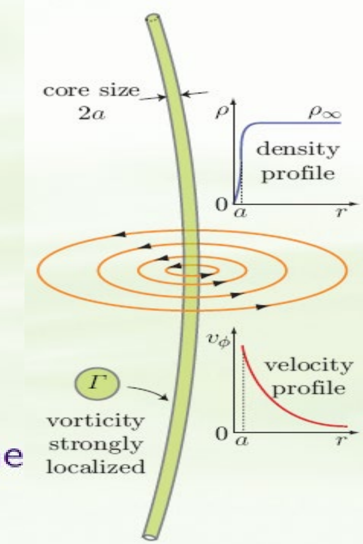
The superfluid velocity

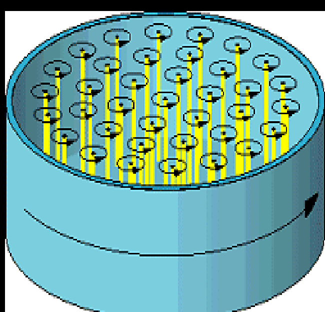
$$\boldsymbol{v}_s = \frac{\hbar}{m}\nabla\Theta(\boldsymbol{r},t).$$

The circulation must be quantized:

$$\Gamma = \oint \mathrm{d}\boldsymbol{r}\cdot\boldsymbol{v}_s = \frac{\hbar}{m}\oint \mathrm{d}\boldsymbol{r}\cdot\nabla\Theta = n\frac{h}{m}.$$

The integer $n$ is the **winding number** of the phase $\Theta(\boldsymbol{r},t)$ around a singular region.



- Hollow vortex tube in the superfluid:

  Velocity near the vortex core: $v = \frac{h}{2\pi m}\cdot\frac{1}{r}$

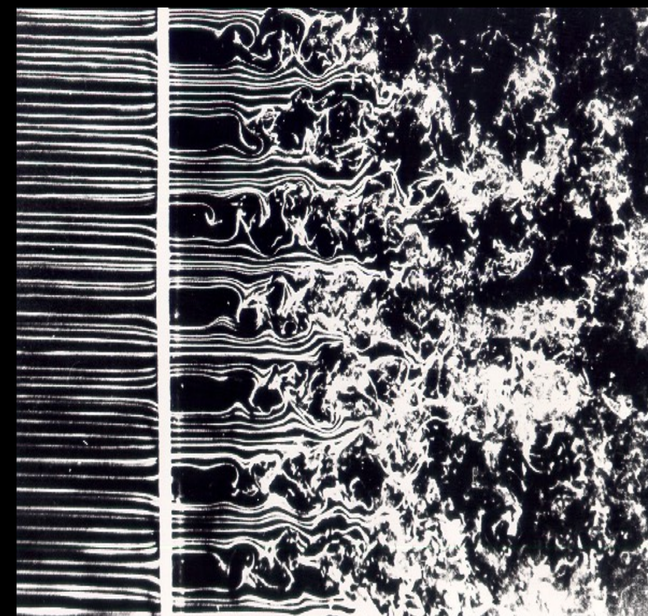
  ➔ Density drops to zero in the core to avoid diverging velocity

## Turbulence in He II:

Superfluid turbulence can be induced by a tangle of quantized vortex lines

Each quantized vortex line has the same conserved circulation. Pure QT may be simpler to model than classical turbulence.

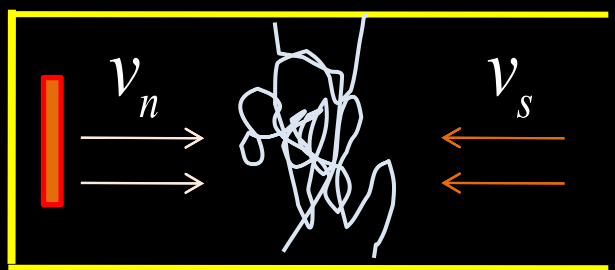
Normal-fluid turbulence is more classical but can be affected by the vortices.



Quasi-particles can scatter off quantized vortices → origin of the mutual friction between the two fluids:

# • Thermal counterflow and quasiclassical flow in He II

Thermal counterflow: heat transfer in He II is by counterflow, where the superfluid moves towards the source of heat while the normal fluid flows in the opposite direction carrying thermal energy.




$$v_n = \frac{w/A}{\rho S T} \qquad v_s = \frac{\rho_n}{\rho_s} v_n$$

- Counterflow can render the highest effective thermal conductivity.
- Both fluids can become turbulent, which affects the heat transfer capability.

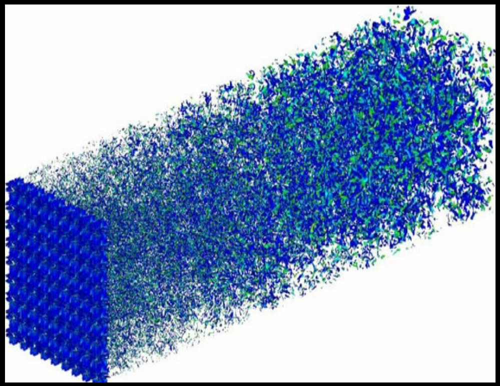
Maglab:
Superconducting Magnets – 45T hybrid; 900Hz NMR, etc.

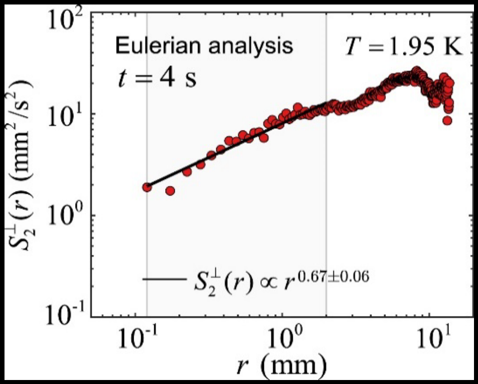
CERN: LHC
(27 Km ring)
LHe-4: 700,000 liters
T=1.8 K

Space telescope
LHe-4: 4.9 kg
T=1.2 K

# • Quasiclassical flows in He II

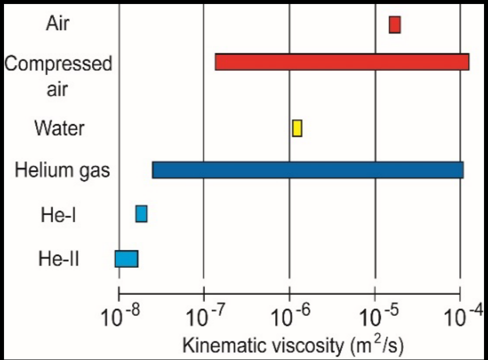
Quasiclassical flows: in mechanically generated flows, the two fluids can become strongly coupled at scales greater than inter-vortex spacing and exhibit classical features.






- Y. Tang, S. Bao, T. Kanai, and W. Guo, Phys. Rev. Fluids, 5, 084602 (2020)
- E. Varga, J. Gao, W. Guo, and L. Skrbek, Phys. Rev. Fluids, 3, 094601 (2018).

- This raises the possibility of using He II for high Reynolds # turbulence research and model testing



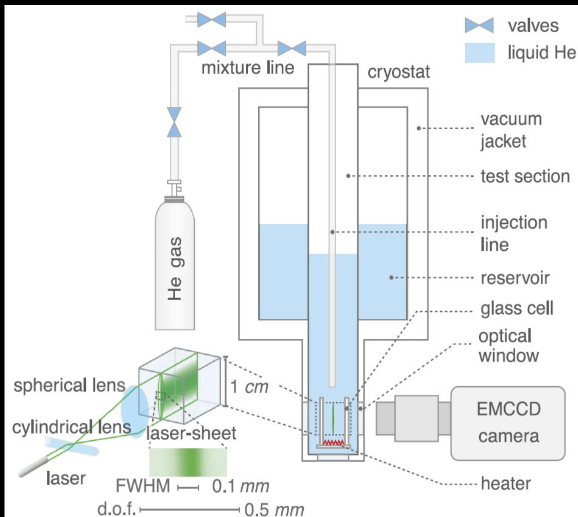

However:

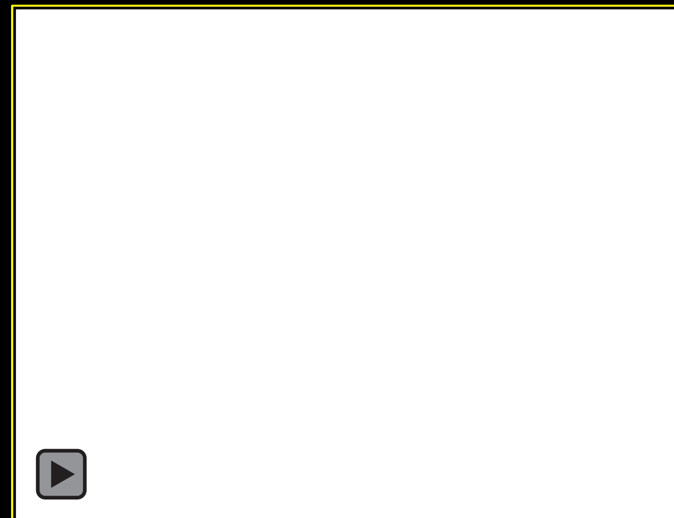
- Doubts exist on whether the vorticity fields in the two fluids are indeed matched (Kivotides, Europhys Lett 112, 36005 (2015))
- Focused studies on QC flows near solid boundaries is extremely limited.

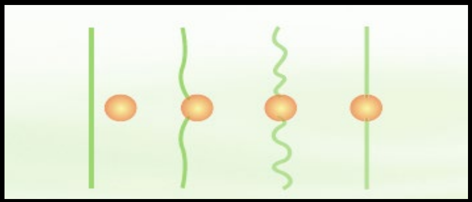
# 2. Flow visualization in He II

## • Particle tracking velocimetry

Tracer particles: solidified H2/D2 ice particles



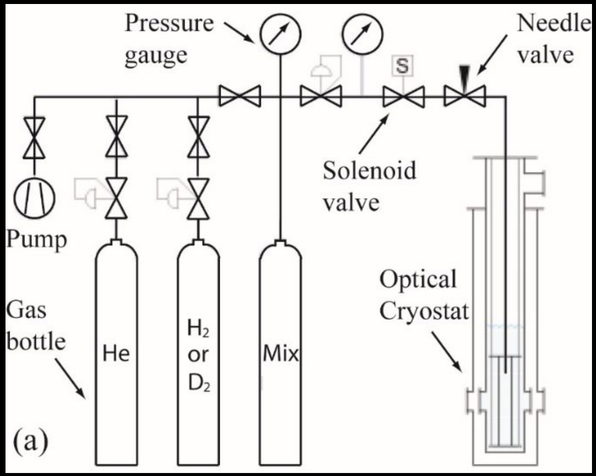



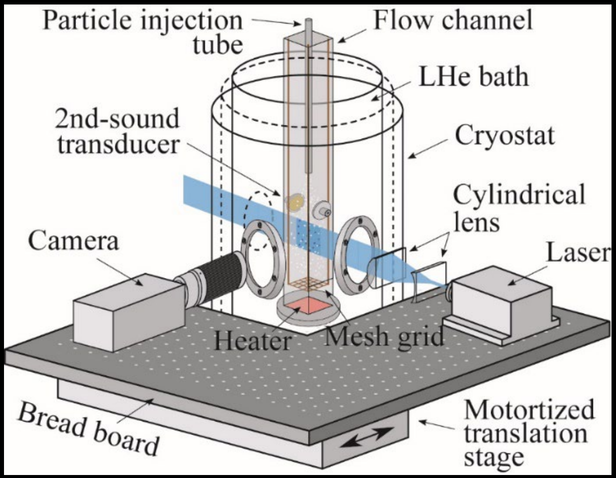


- The micron-sized particles are entrained by the viscous normal-fluid component.
- They can also get trapped on quantized vortices in He II due to Bernoulli pressure.

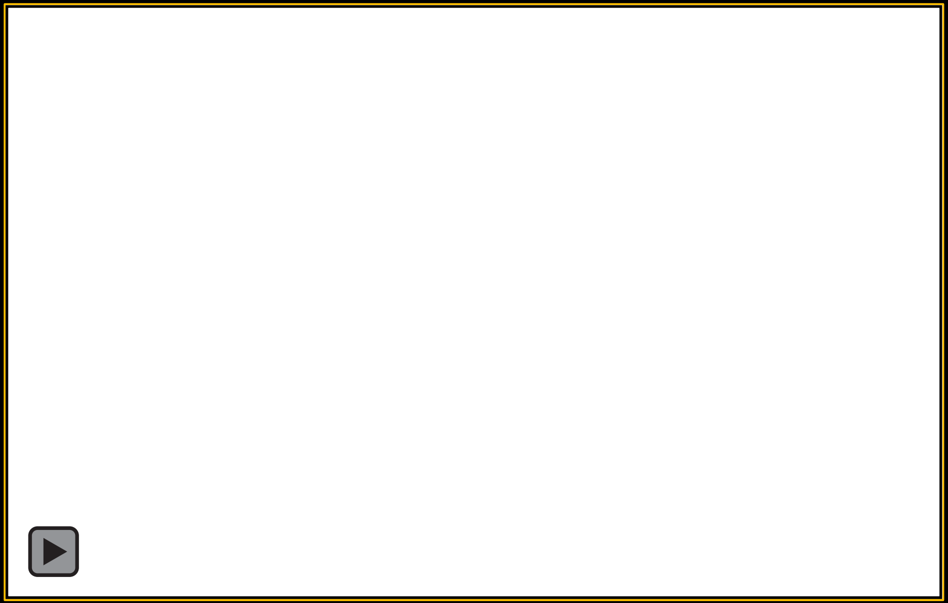
Bewley, Lathrop, and Sreenivasan *Nature* ***441*** *, 588 (2006)*

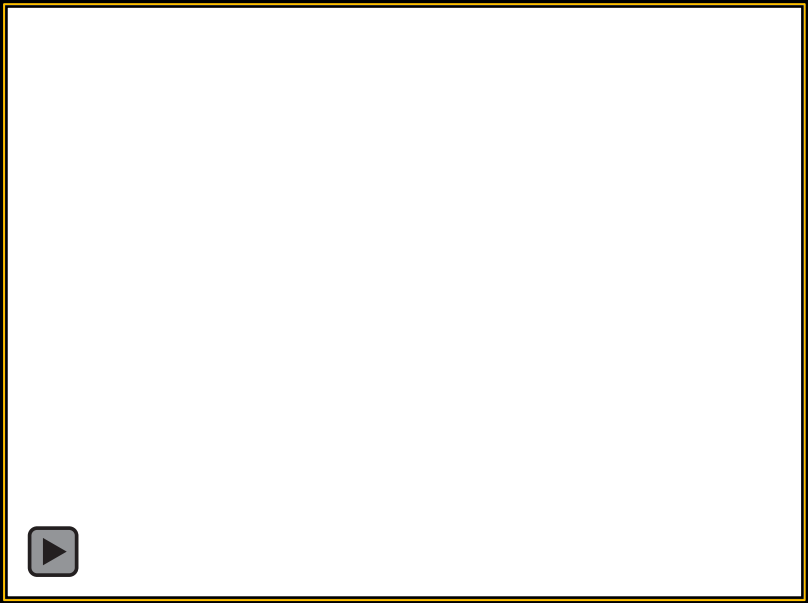
- We have built our own PTV system and have applied it to study counterflow and vortex dynamics



- We have obtained a large amount of video data showing how quantized vortices moving around, reconnecting with each other, generating Kelvin waves :

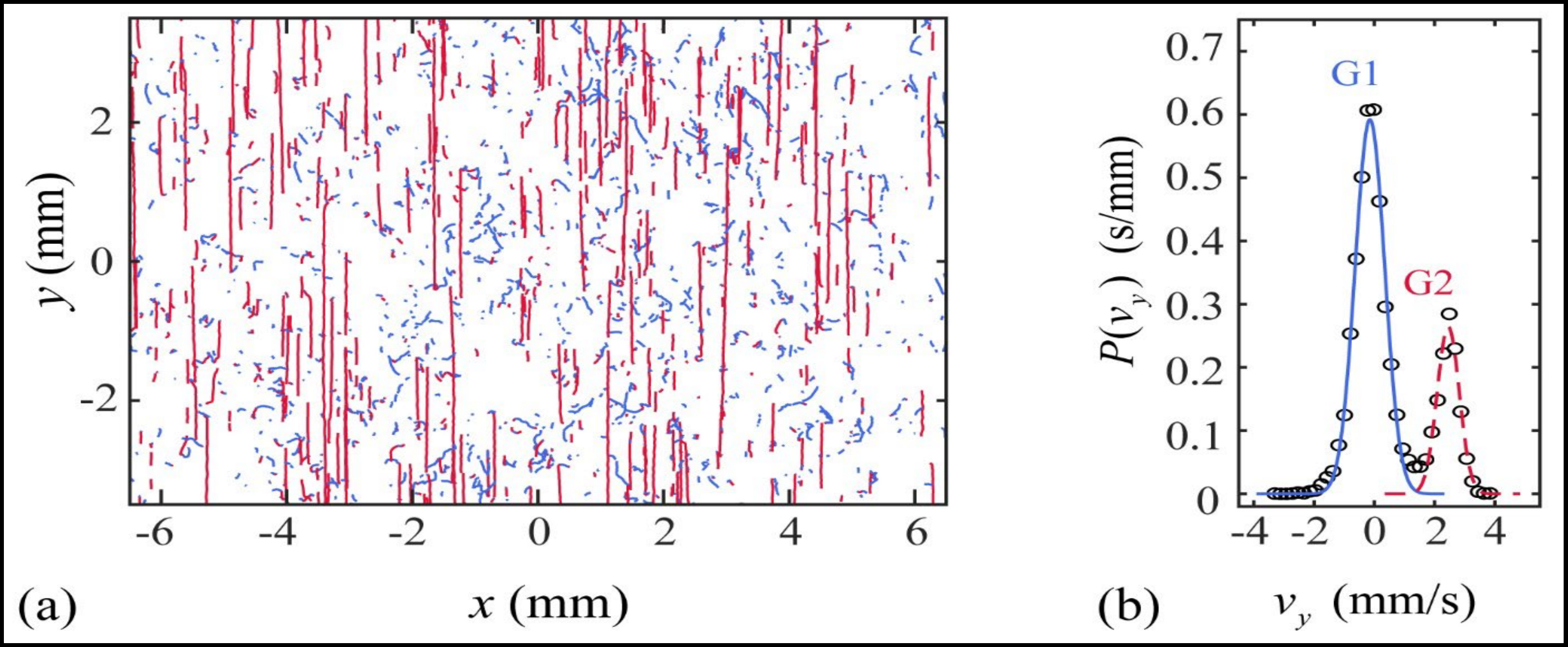




Frame rate can reach 1000 Hz !

## Visualizing thermal counterflow in He II:

- The normal fluid (heat flux) moves up, entraining the untrapped particles
- The vortex tangle moves with the superfluid towards the heater

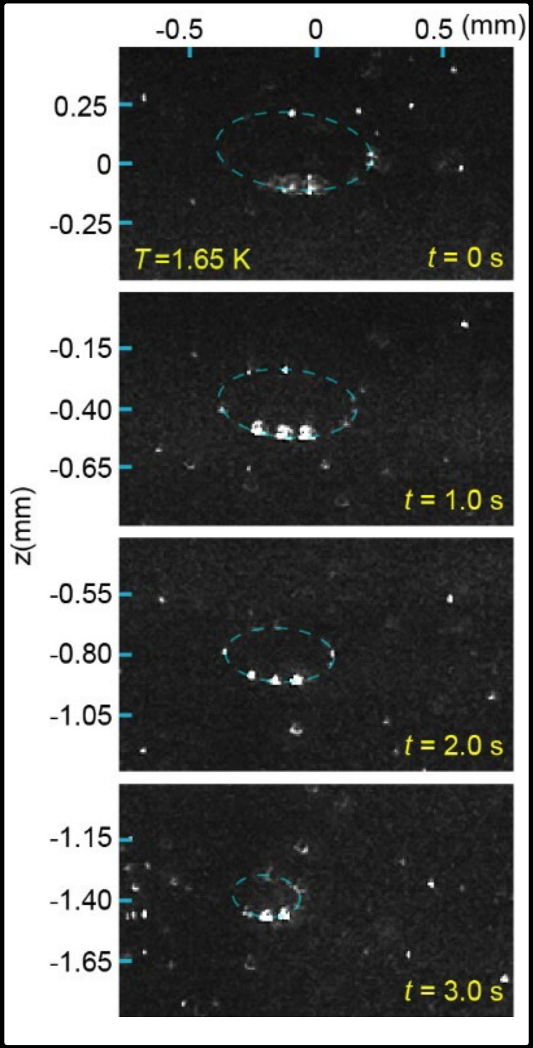


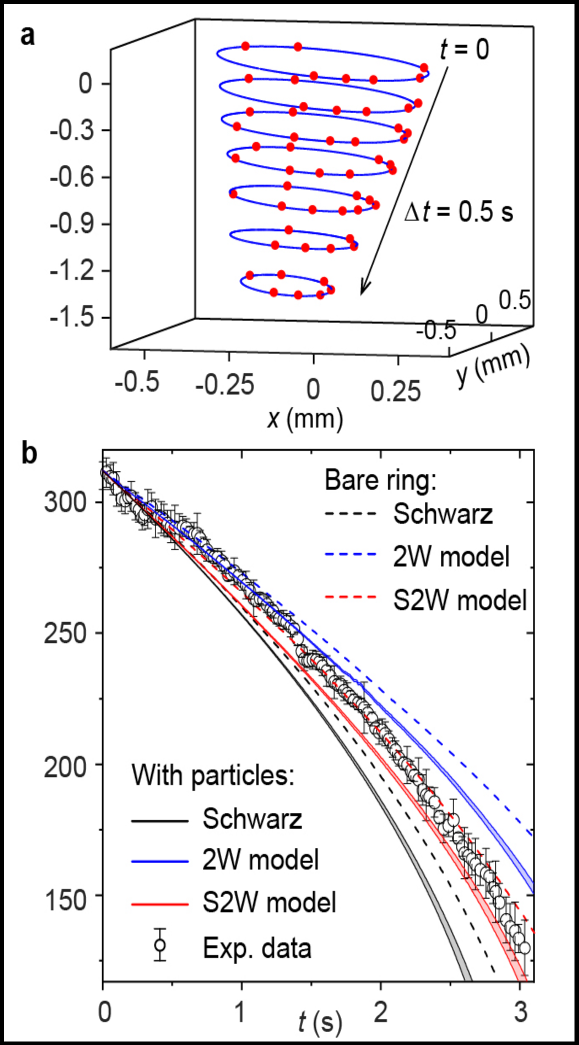

- B. Mastracci and W. Guo, Phys. Rev. Fluids, 4, 023301 (2019) Editor's Suggestion
- S. Yui, H. Kabayashi, M. Tsubota, and W. Guo, PRL, 124, 155301 (2020)
- Y. Tang, S. Bao, and W. Guo, PNAS, 118, e2021957118 (2021)
- S. Yui, et al., PRL, 129, 025301 (2022) Editor's Suggestion

- Recently, we managed to observe vortex rings propagating in He II:

- The rings are likely created by vortex reconnections:

- We can conduct quantitative analysis of high-quality vortex ring event:



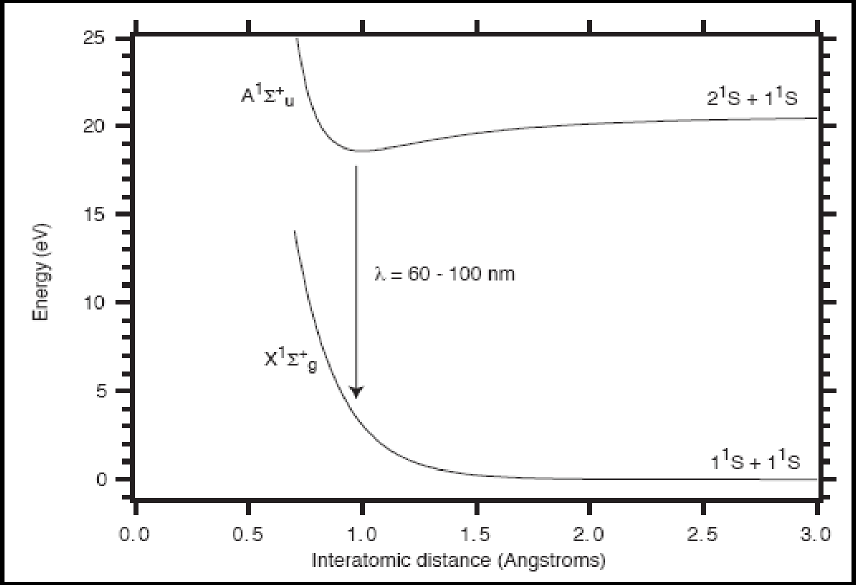




- Y. Tang, et al., Nature Communications, 14, 2941 (2023)

(S2W: Galantucci et al., Eur. Phys. J. Plus, 135, 547 (2020))

# • Molecular tagging velocimetry in He II

➢ Tracer particles: He2 excimer molecules




- Metastable $He_2^*$ molecules can be easily produced as a result of ionization or excitation in LHe4:

$$e^- + He^+ + He \rightarrow He^* + He \rightarrow He^*_2$$

- Metastable $He_2^*$ excimer molecules can easily form in LHe:
  - singlet state $A^1\Sigma_u^+$ lifetime: ~1ns
  - triplet state $a^3\Sigma_u^+$ lifetime: **~13s**

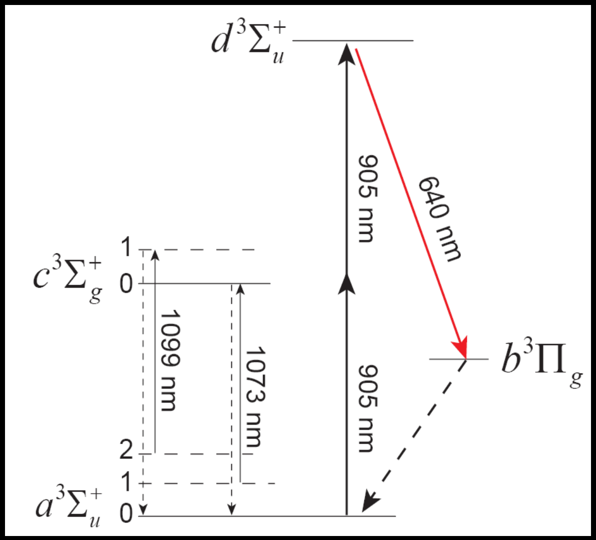
- $He_2^*$ molecules form little bubbles in LHe. ( R~6Å )

➔ Above 1K : molecules trace the normal-fluid component only.

➔ Below 0.5 K : molecule bubbles can be trapped on vortex lines

(D. Zmeev, et al, Phys. Rev. Lett., 110, 175303 (2013))

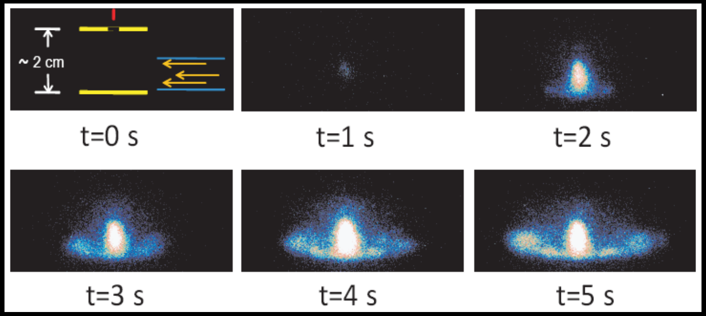
# ➤ Imaging $He_2^*$ molecules: Laser-induced fluorescence



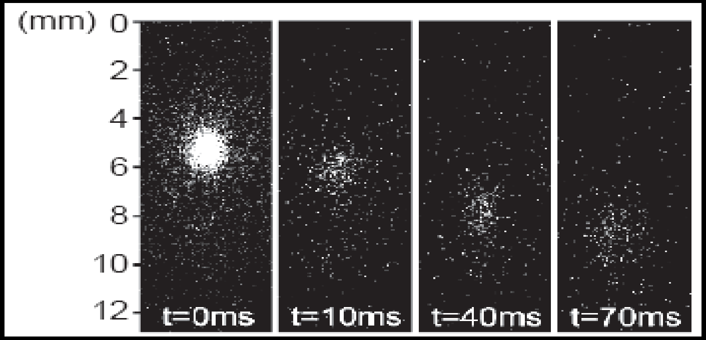

W.G. Rellergert *et al.*, Phys. Rev. Lett, 100 (2008).

For molecules in the triplet ground state a(0):

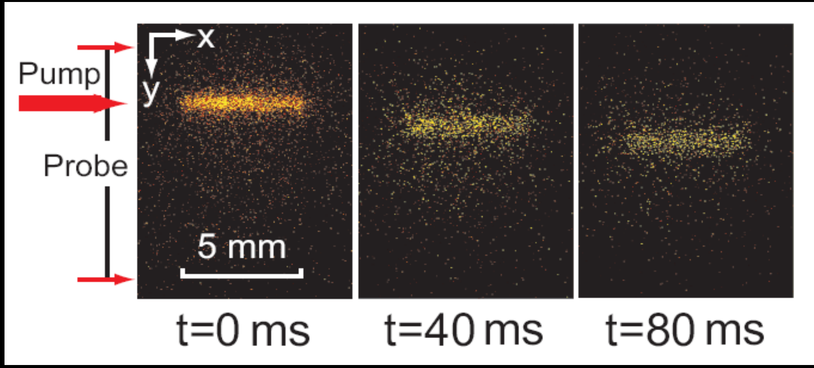
- A 905 nm pulsed laser is used to drive a cycling transition.
- Fluorescent light emitted at 640 nm.




Guo, et al., J. Low Temp. Phys., 158, 346 (2009)



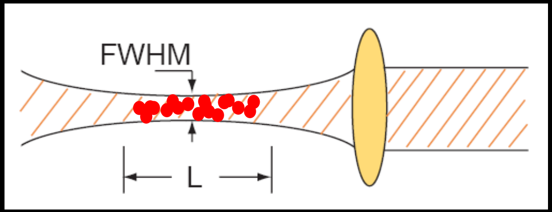

Guo, *et al.*, Phys. Rev. Lett., 102, 235301 (2009)



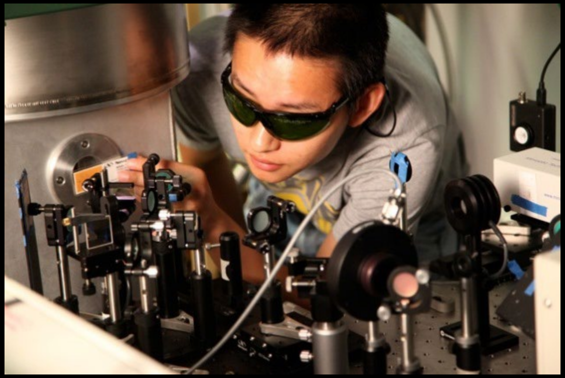

Guo, et al., Phys. Rev. Lett. 105, 045301 (2010).

## ➢ Molecular tagging velocimetry (MTV):

Femtosecond laser field ionization in helium:



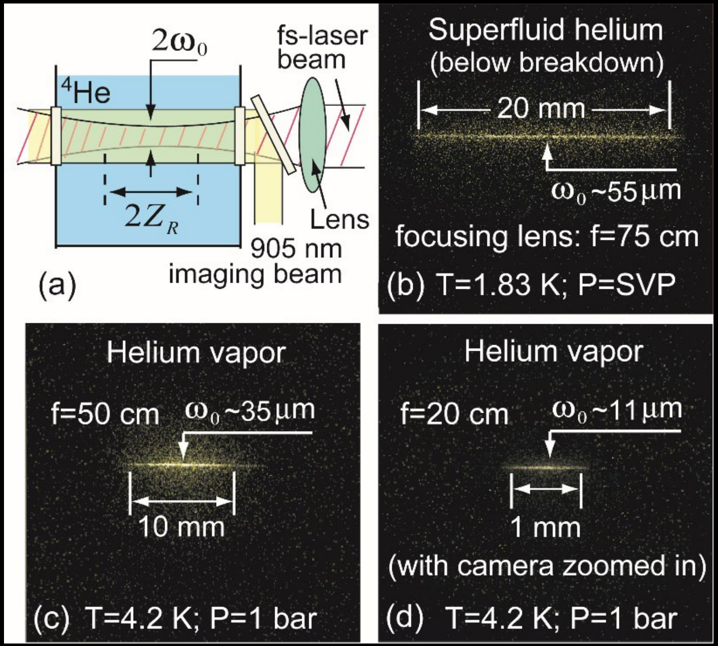

$I \geq 10^{13}\ \mathrm{W/cm^2}$



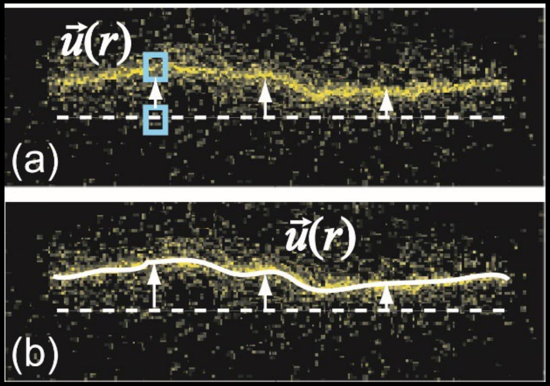
Pulse length: 35 fs

Pulse energy: up to 4 mJ

Rep rate: up to 5 kHz



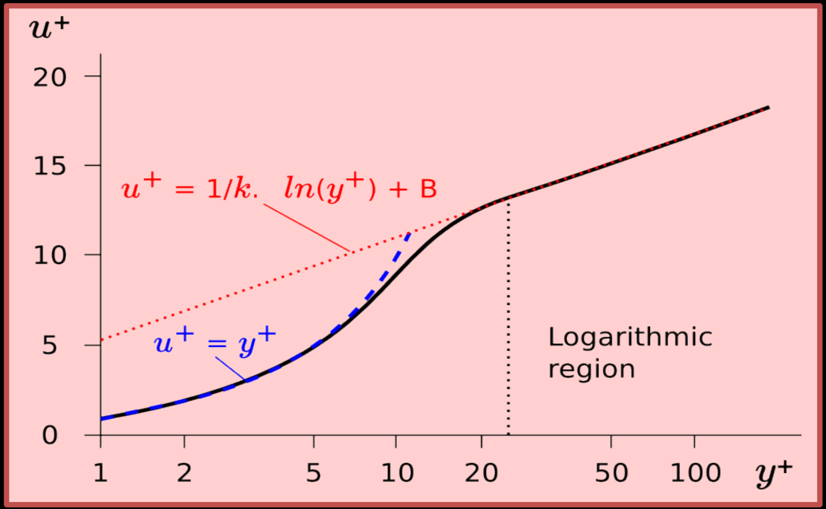

J. Gao, et al., Rev. Sci. Instrum. 86, 093904 (2015)

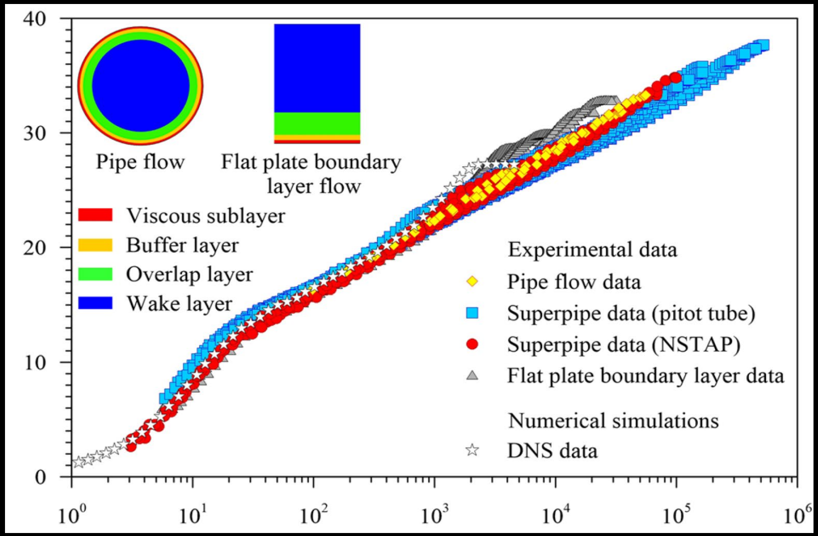



- W. Guo, *et al.*, PNAS, 111, 4653 (2014)
- A. Marakov, et al., PRB 91, 094503 (2015).
- J. Gao, et al, JETP Lett., 103, 732 (2016).
- J. Gao, et al, PRB, 97, 184518 (2018).
- S. Bao, et al, Phys. Rev. Applied, 11, 044003 (2019)

# 3. Law of the wall in He II

# • Concept of the law of wall




Ali and Dey, Phys. Fluids 32, 121401 (2020)

- In classical pipe or boundary-layer flows, the near-wall mean velocity profile takes a universal log-law form:

$$u^+ = \frac{1}{\kappa}\ln y^+ + B$$

where: $u^+ = \bar{u}(y)/u_\tau$ and $y^+ = y/l$

and the viscous velocity and wall unit:

$$u_\tau = (\tau_\omega / \rho)^{1/2} = (f_D \overline{U}^2 / 8)^{1/2}$$

$$l = v/u_\tau$$

- Measurements and simulations revealed universal constants:
  von Karman constant: $\kappa \simeq 0.4$
  Additive constant: $B \simeq 5.0$

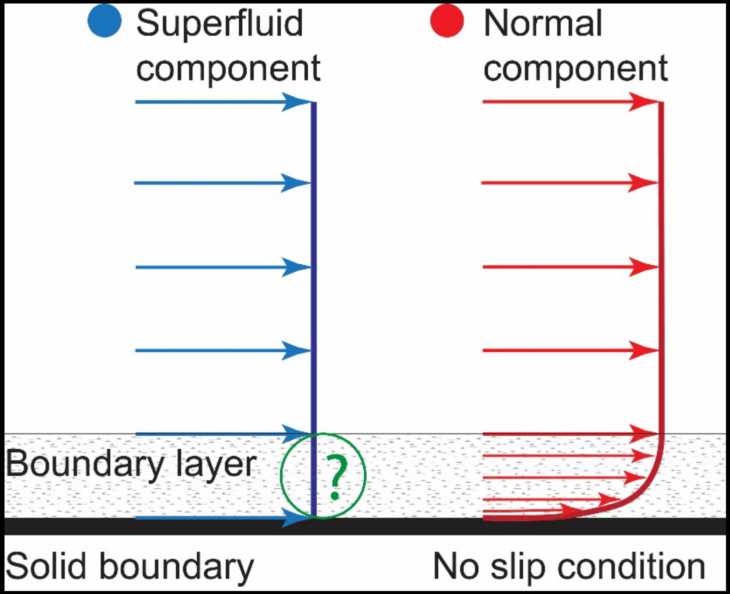
- A clear log-law can be observed at:

$$\sim 10^2 < y^+ < \sim 0.15 Re_\tau$$

- Recent research suggests that κ may increase @ high pressure gradient.

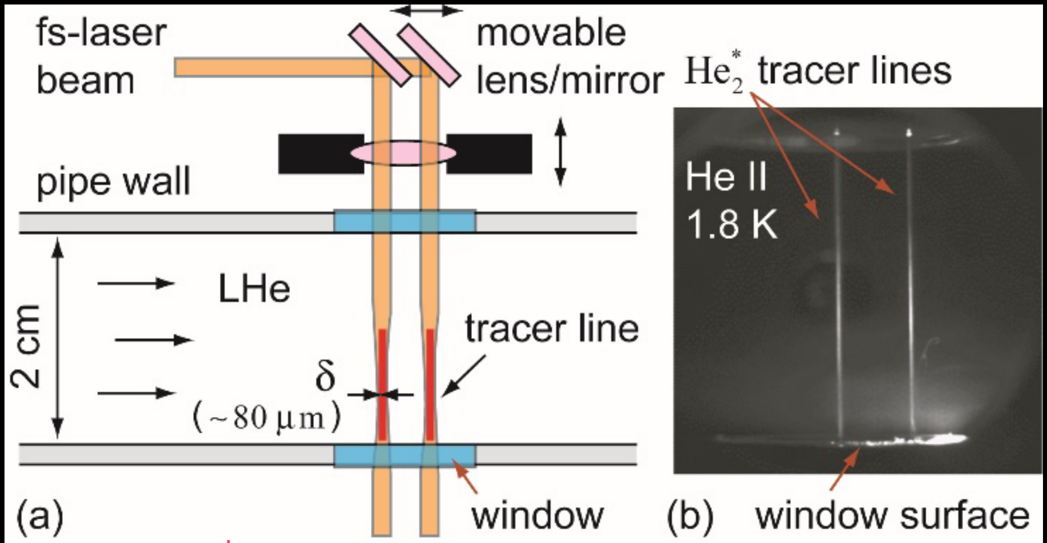
Monkewitz and Nagib, arXiv:2303.08071

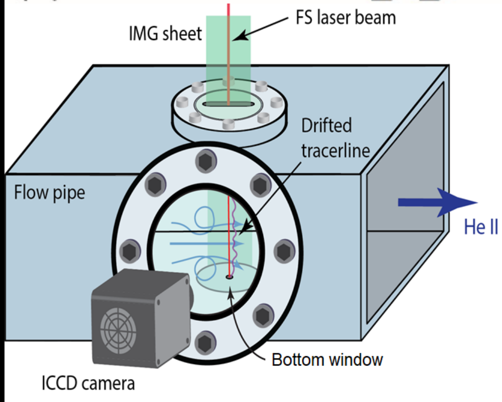
## ➢ What is the near-wall velocity profile in He II pipe flow?




- The normal fluid obeys the no-slip boundary condition, but the superfluid does not.
- For the superfluid to mimic the normal-fluid velocity profile, high density of vortices polarized in a special way are required.
- There is no existing knowledge on whether the two fluids can become fully coupled in the thin boundary layer!
- If the two velocity profiles do not match, mutual friction can alter the near-wall velocity profile.

- Implementation of MTV in our LHe flow visualization facility (LHFVF):

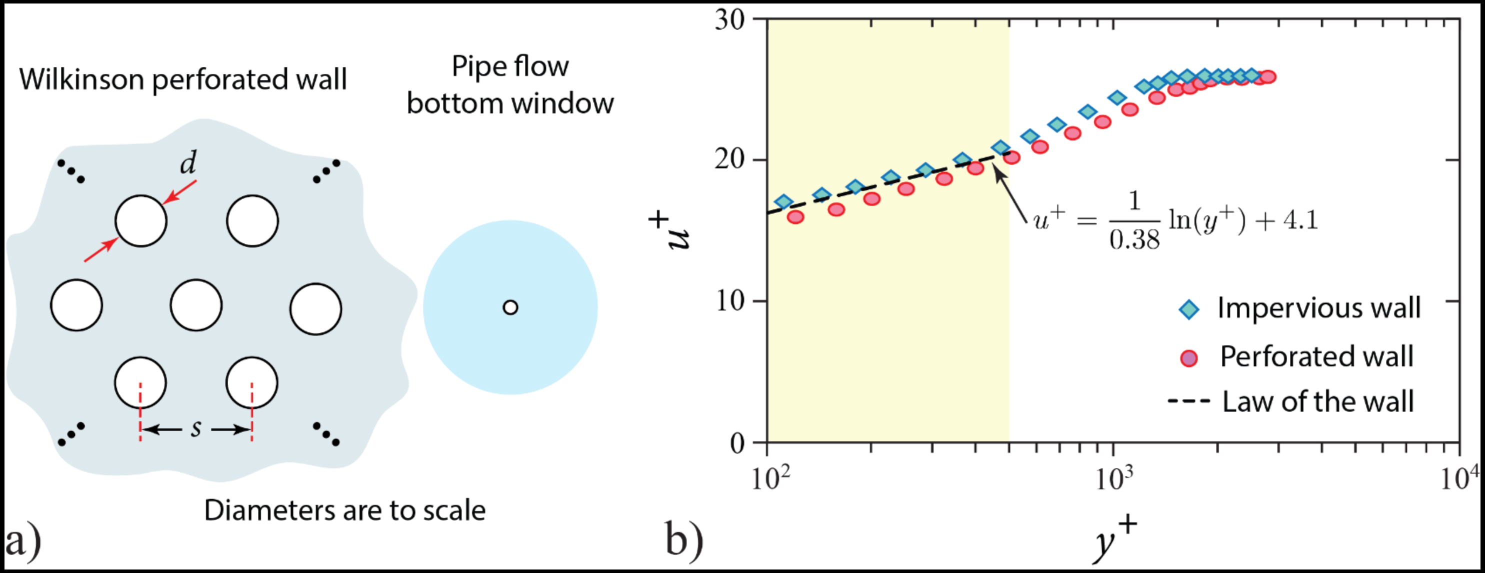



- We can measure not only near-wall velocity profile, but also longitudinal and transverse structure functions

## ➢ Sapphire windows design:

- Log-law velocity profile was observed in classical fluid boundary layer over surfaces with an array of small hole: $d^+ = d/l = 110$

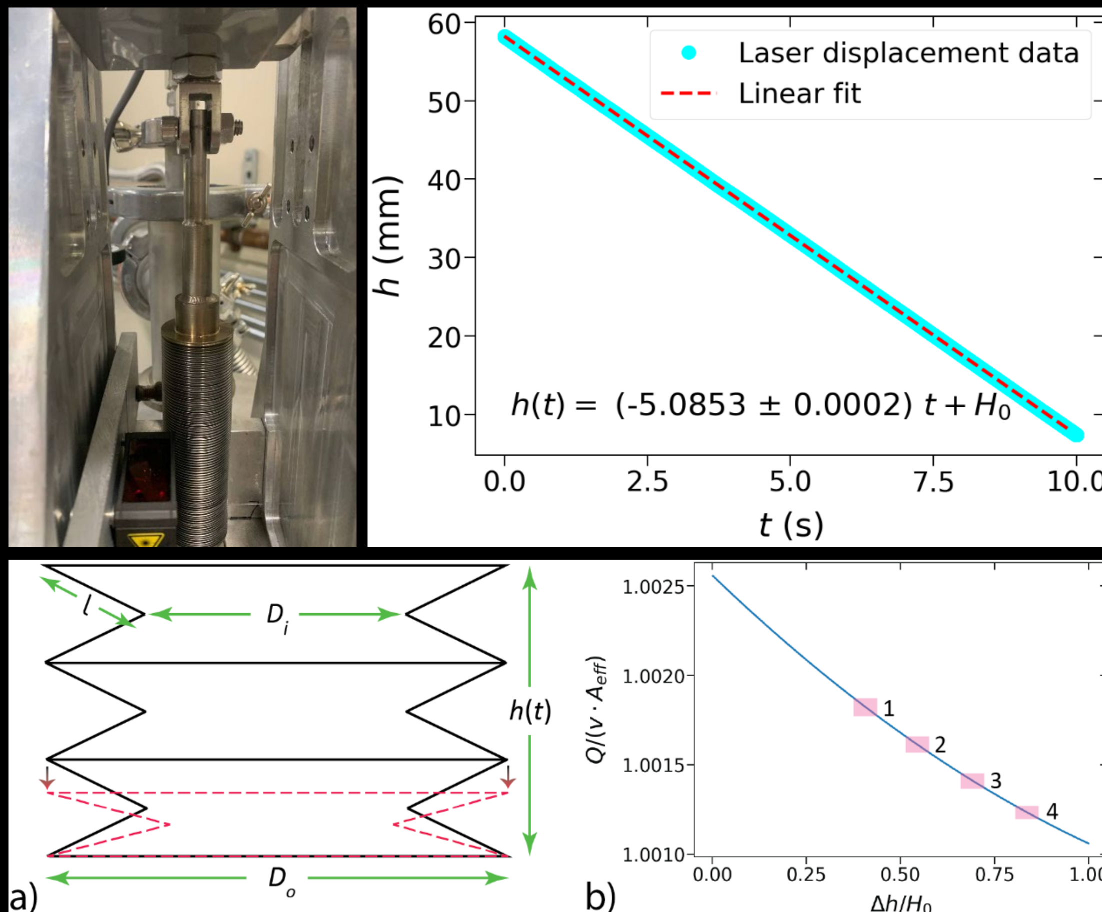



S Wilkinson. "Influence of wall permeability on turbulent boundary-layer properties", 21st Aerospace Sciences Meeting. 1983, p. 294.

- In our typical experimental runs, $d^+ = d/l \approx 100 \sim 200$

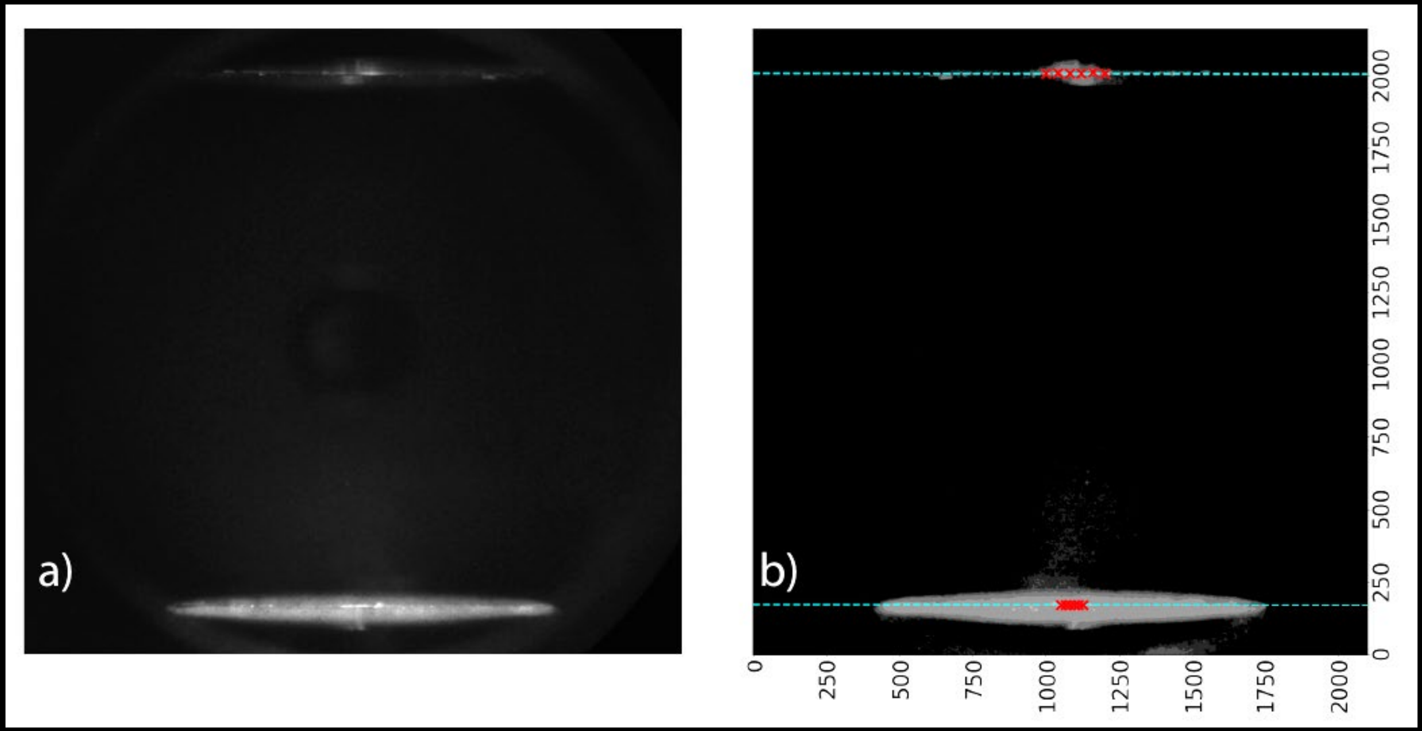
## • Experimental procedure

➢ Controlling the mean velocity:

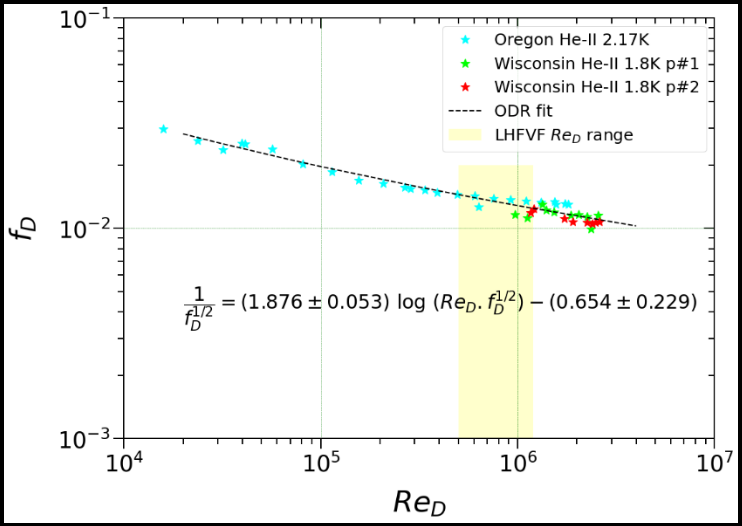



$$Q(t) = \frac{d}{dt}\left[\frac{\pi}{3}\left(\frac{D_0^2 + D_i^2 + D_0 D_i}{4}\right) h(t)\right]$$

- Scale calibration and normalization:



## Scale calibration and normalization:




- The friction factor in He II pipe flow has been measured. We fit the reported data to obtain $f_D$
- The wall unit can be calculated, and the length can be normalized.

$$l = v/u_\tau \qquad u_\tau = (f_D \bar{U}^2/8)^{1/2}$$

- The expected log-law range is shown below:

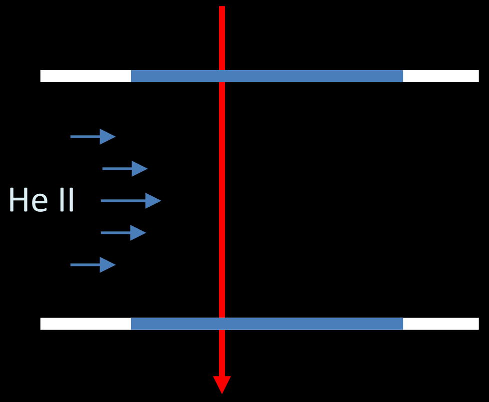
$$\sim 10^2 < y^+ < \sim 0.15 Re_\tau$$

| Flow parameters | | | | | Wall coordinate | | Real coordinate | |
|---|---|---|---|---|---|---|---|---|
| $U$ | $Re_D$ | $f_D$ | $u_\tau$ | $Re_\tau$ | $y^+_{min}$ [10] | $y^+_{max}$ [30] | $y_{min}$ | $y_{max}$ |
| (m/s) | (-) | $(10^{-2})$ | $(10^{-2}$m/s) | (-) | (-) | (-) | ($\mu$m) | (mm) |
| 0.3 | $6.48 \times 10^5$ | 1.383 | 1.247 | 13465 | 400 | 1600 | 297 | 1.2 |
| 0.4 | $8.64 \times 10^5$ | 1.316 | 1.622 | 17512 | 400 | 2100 | 228 | 1.2 |
| 0.5 | $1.08 \times 10^6$ | 1.267 | 1.990 | 21480 | 400 | 2600 | 186 | 1.2 |
| 0.6 | $1.30 \times 10^6$ | 1.229 | 2.352 | 25387 | 400 | 3100 | 158 | 1.2 |

## MTV procedure:

- Fs-laser pulse is sent in at 1000 Hz
- Each fs-laser pulse is followed by an imaging pulse at 905 nm with a delay time of 0.7 ms.
- A tracer line created by one fs-laser pulse is therefore imaged at 0.7 ms, (0.7+1) ms, (0.7+2) ms, …
- Images are obtained by superimposing 600 exposures

t=0 : 1st fs pulse passing through

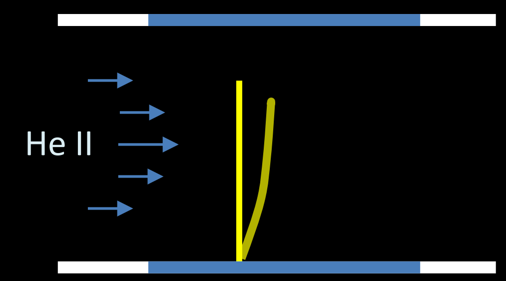



- Deformation of the superimposed line is due to the mean velocity profile.

## MTV procedure:

- Fs-laser pulse is sent in at 1000 Hz
- Each fs-laser pulse is followed by an imaging pulse at 905 nm with a delay time of 0.7 ms.
- A tracer line created by one fs-laser pulse is therefore imaged at 0.7 ms, (0.7+1) ms, (0.7+2) ms, …
- Images are obtained by superimposing 600 exposures

A tracer line is created

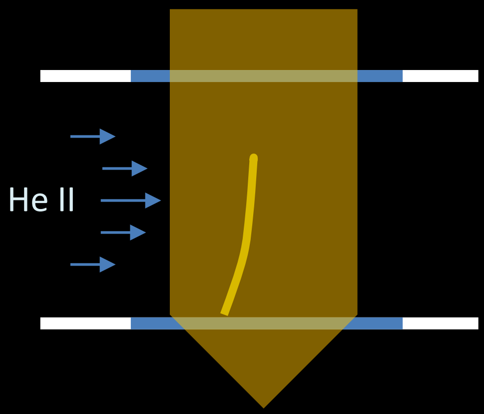



- Deformation of the superimposed line is due to the mean velocity profile.

## ➢ MTV procedure:

- Fs-laser pulse is sent in at 1000 Hz
- Each fs-laser pulse is followed by an imaging pulse at 905 nm with a delay time of 0.7 ms.
- A tracer line created by one fs-laser pulse is therefore imaged at 0.7 ms, (0.7+1) ms, (0.7+2) ms, …
- Images are obtained by superimposing 600 exposures

t=0.7 ms : imaging pulse passing through

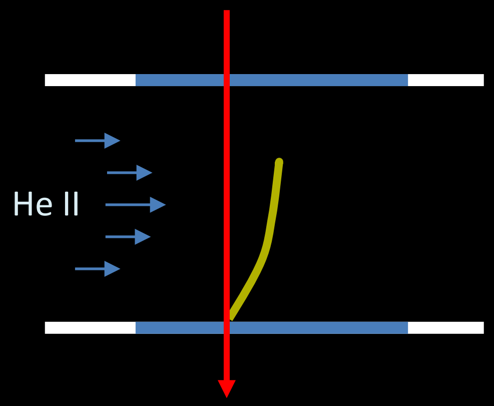



- Deformation of the superimposed line is due to the mean velocity profile.

## MTV procedure:

- Fs-laser pulse is sent in at 1000 Hz
- Each fs-laser pulse is followed by an imaging pulse at 905 nm with a delay time of 0.7 ms.
- A tracer line created by one fs-laser pulse is therefore imaged at 0.7 ms, (0.7+1) ms, (0.7+2) ms, …
- Images are obtained by superimposing 600 exposures

t=1 ms : fs pulse passing through

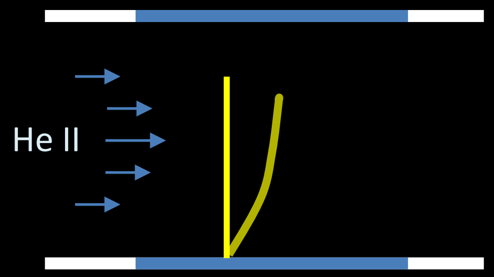



- Deformation of the superimposed line is due to the mean velocity profile.

## ➢ MTV procedure:

- Fs-laser pulse is sent in at 1000 Hz
- Each fs-laser pulse is followed by an imaging pulse at 905 nm with a delay time of 0.7 ms.
- A tracer line created by one fs-laser pulse is therefore imaged at 0.7 ms, (0.7+1) ms, (0.7+2) ms, …
- Images are obtained by superimposing 600 exposures

A new excimer tracer line is created

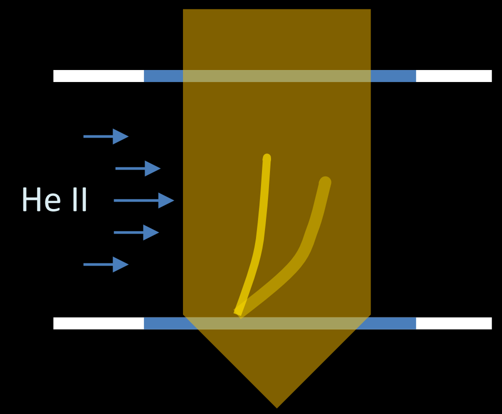



- Deformation of the superimposed line is due to the mean velocity profile.

## ➢ MTV procedure:

- Fs-laser pulse is sent in at 1000 Hz
- Each fs-laser pulse is followed by an imaging pulse at 905 nm with a delay time of 0.7 ms.
- A tracer line created by one fs-laser pulse is therefore imaged at 0.7 ms, (0.7+1) ms, (0.7+2) ms, …
- Images are obtained by superimposing 600 exposures

t=1.7 ms : imaging pulse passing through

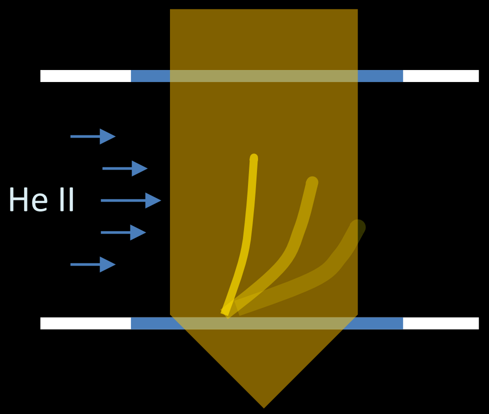



- Deformation of the superimposed line is due to the mean velocity profile.

## MTV procedure:

- Fs-laser pulse is sent in at 1000 Hz
- Each fs-laser pulse is followed by an imaging pulse at 905 nm with a delay time of 0.7 ms.
- A tracer line created by one fs-laser pulse is therefore imaged at 0.7 ms, (0.7+1) ms, (0.7+2) ms, …
- Images are obtained by superimposing 600 exposures

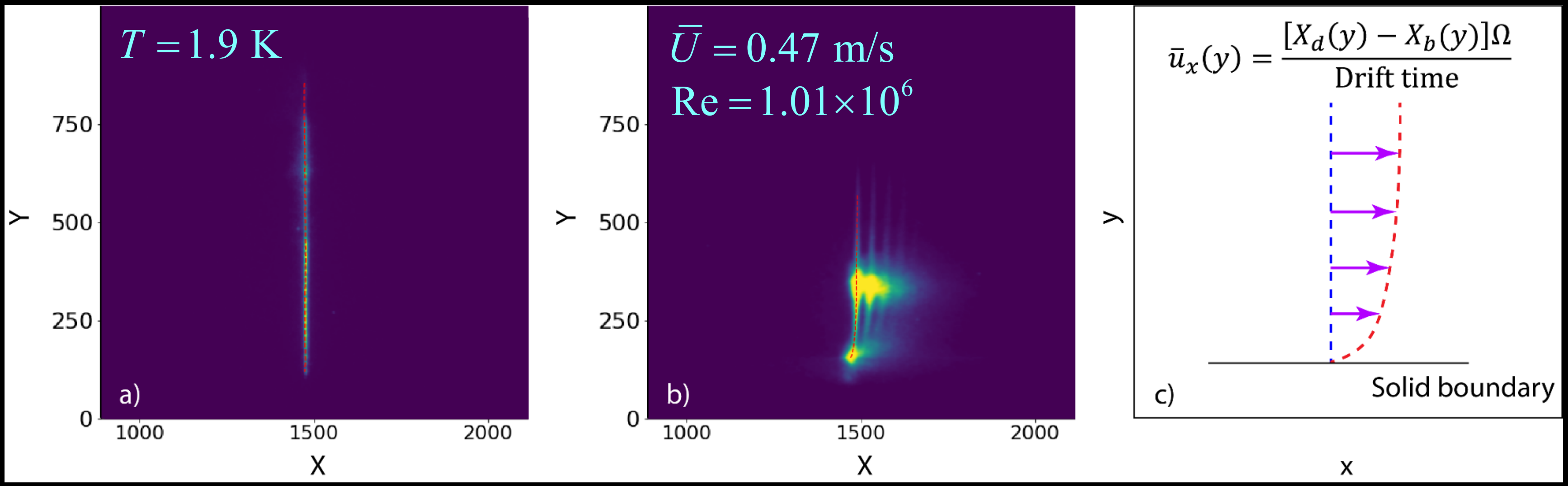
t=2.7 ms : imaging pulse passing through




- Deformation of the superimposed line is due to the mean velocity profile.

• Near-wall velocity profile

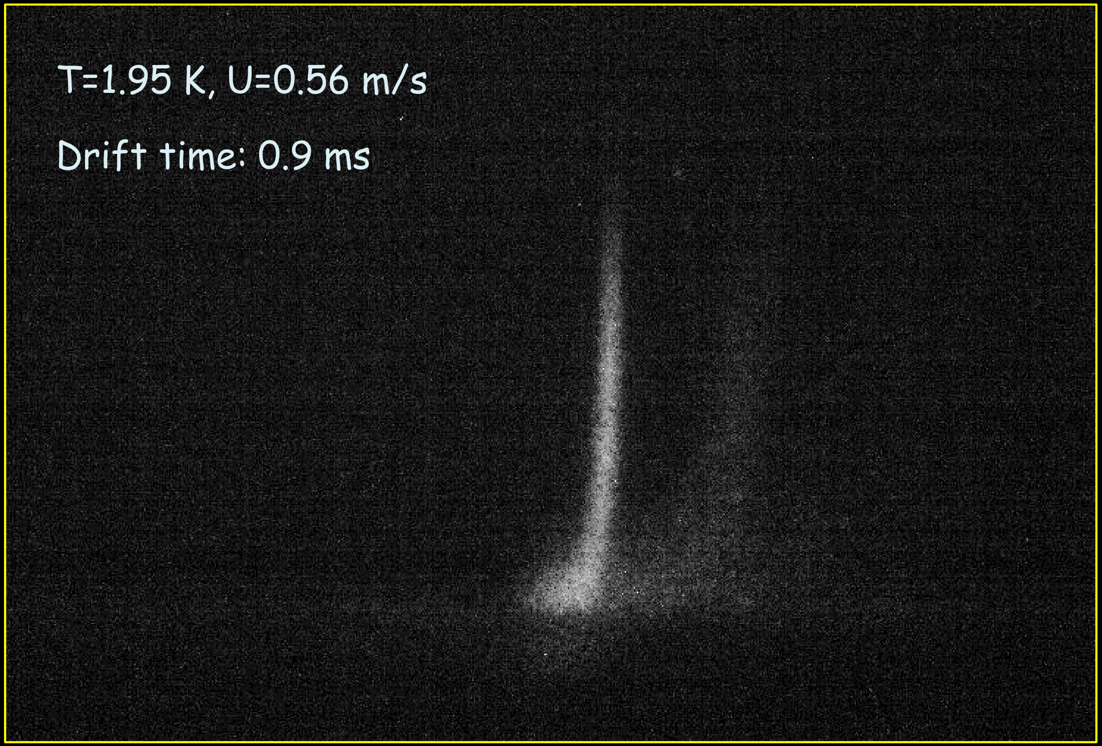
➢ Measuring the mean streamwise velocity profile:



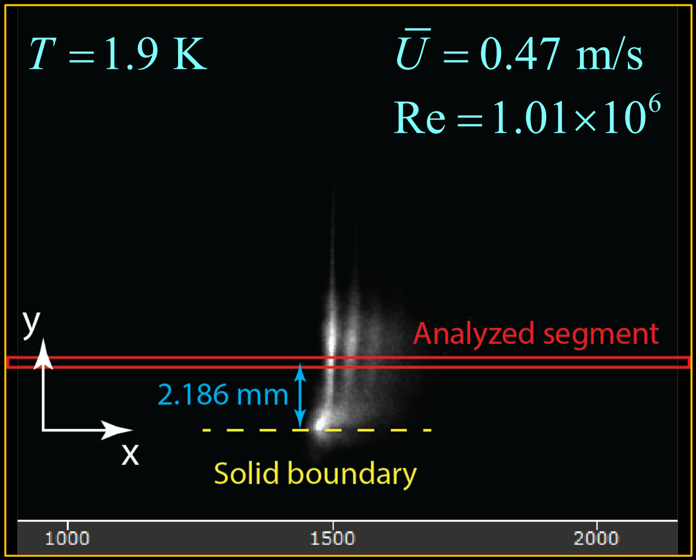

Baseline image

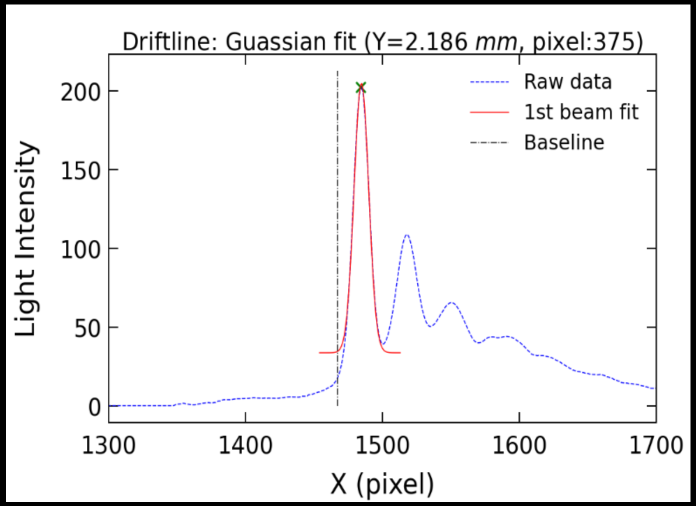
Drifted line image

- We determine the streamwise velocity based on the x-displacement of each line segment.



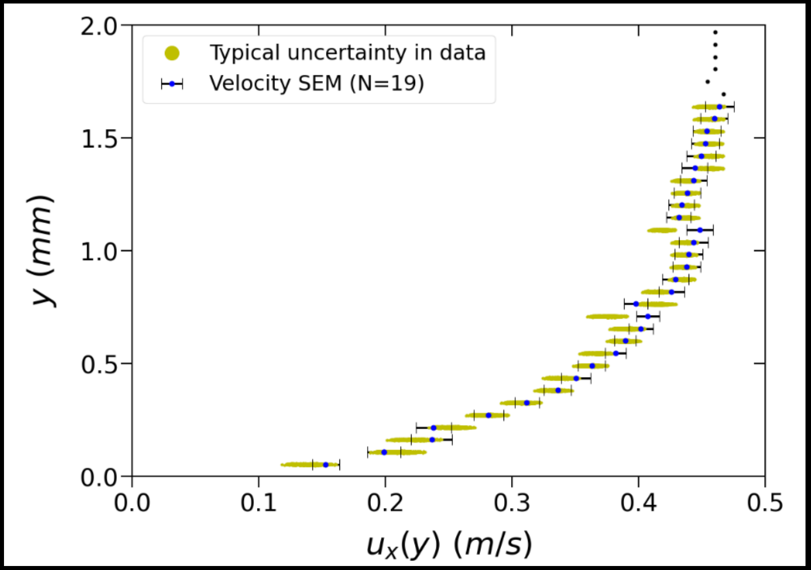

- New images obtained in our recent experiment @ 1.95 K at different drift time.



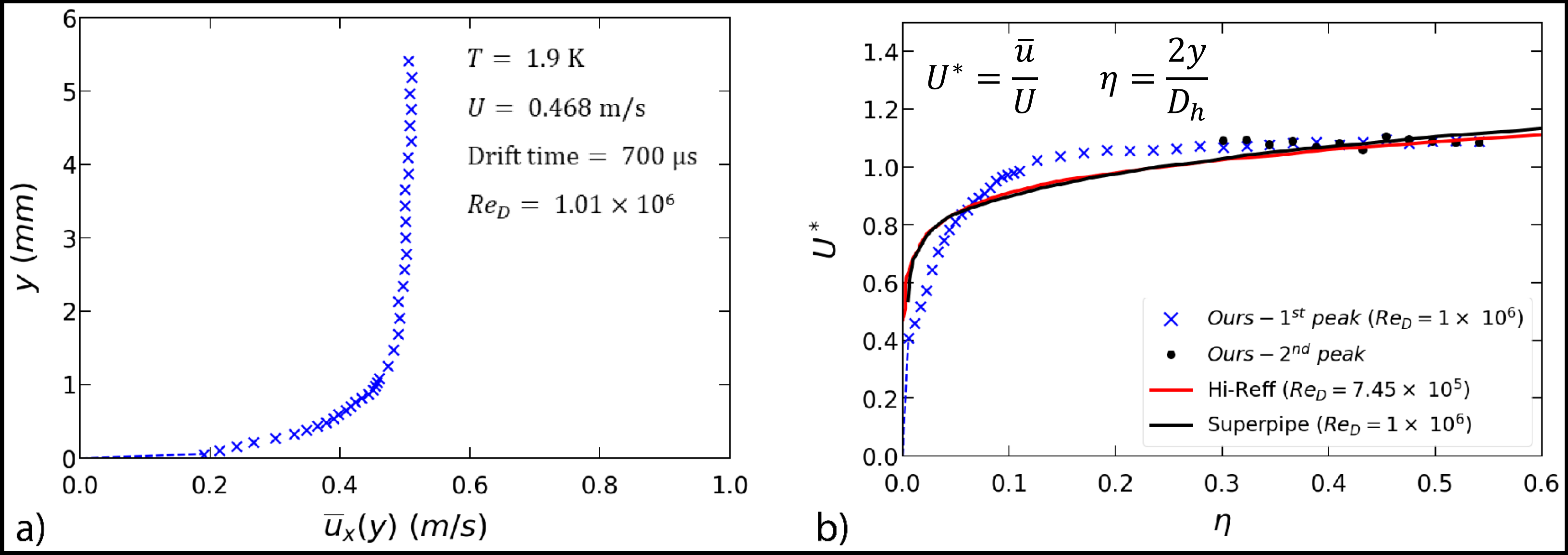




- Cut each image into horizontal stripes with width Δy ~ 100 um. Plot light intensity integrated over Δy as a function of x.
- The center of a tracer-line segment is determined through a Gaussian fit to the light intensity
- The velocity of a segment at y is determine based on its displacement:

$$u_x(y) = \frac{x(y, \text{driftline}) - x(y, \text{baseline})}{\text{Drift time}} \times \text{pixel size}$$

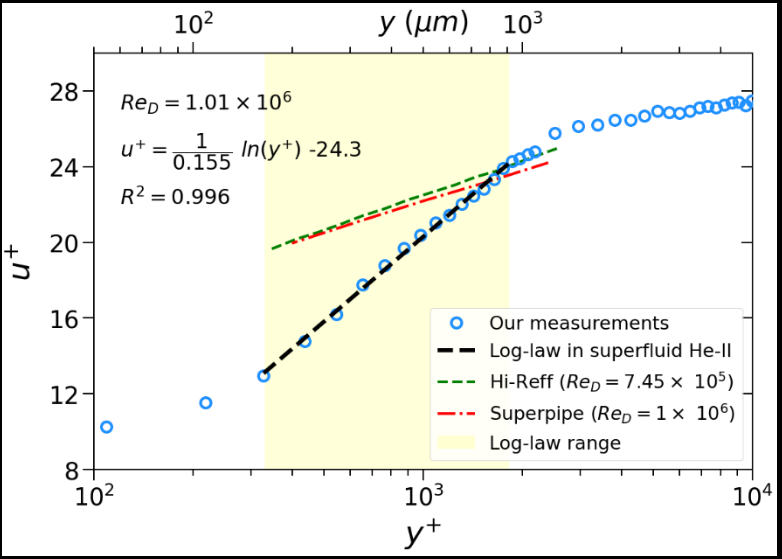


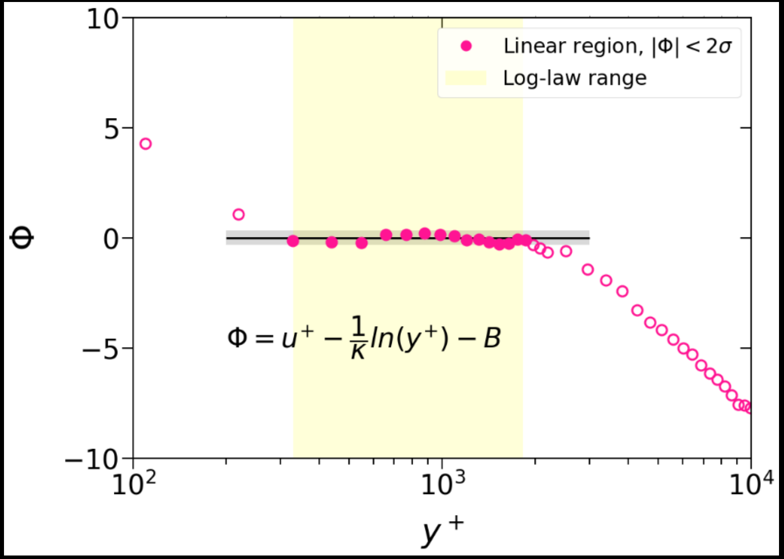
## ➤ Measuring the mean streamwise velocity profile:




- The mean streamwise velocity $u_x(y)$ profile is obtained.
- Normalized velocity profile exhibits clear differences from those obtained in Superpipe (high Re air pipe flow) and Hi-Reff (high Re air flow over plate)
  - Hi-Reff: Furuichi, et al., Phys. Fluids, 27, 095108 (2015).
  - Superpipe: Hultmark, et al., PRL 108, 094501 (2012).

## ➤ Log-law region: (normalize $u_x(y)$ by the viscous velocity u+ and y by the wall unit $l$ )

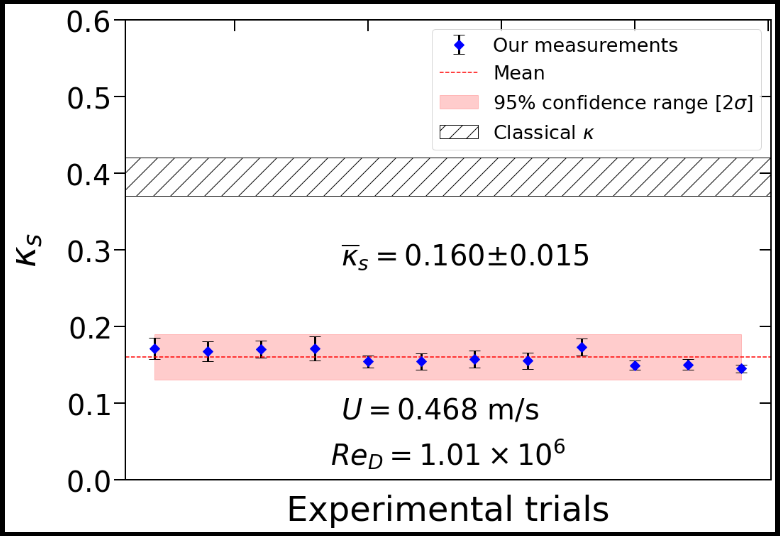


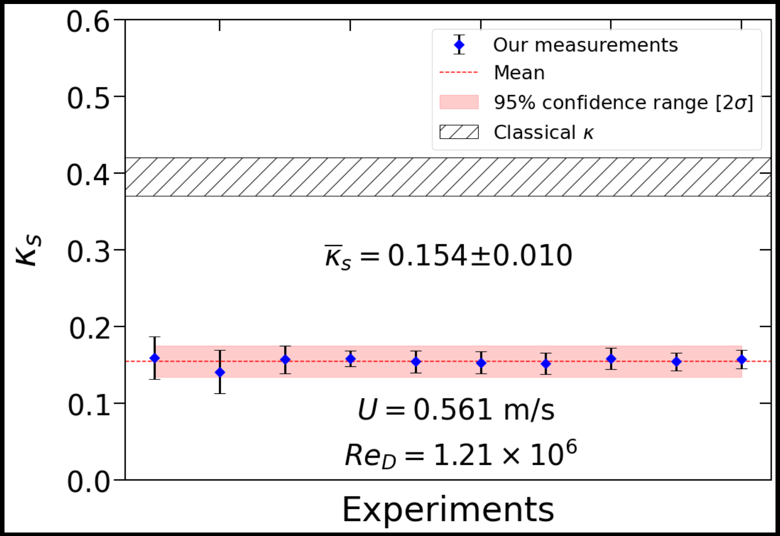




- A log-law region is observed in the expected range of y+:

$$\sim 10^2 < y^+ < \sim 0.15 Re_\tau$$

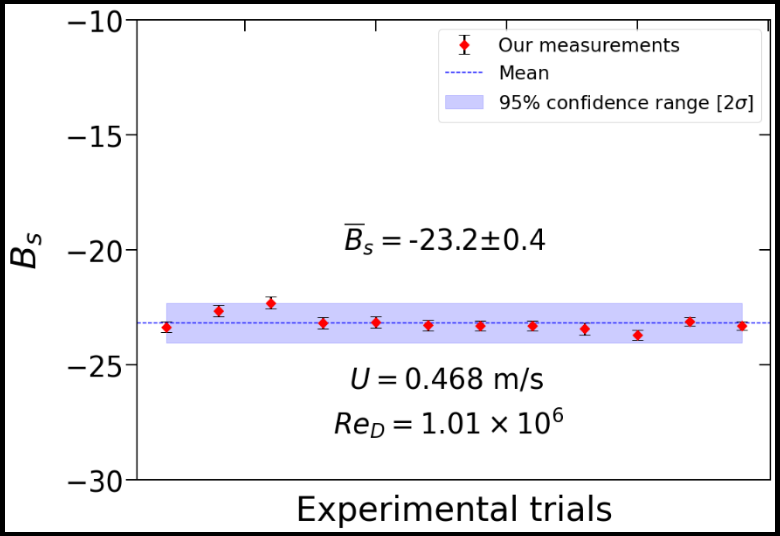
- The Karman constant and the additive constant can be determined through a self-consistent fitting procedure introduced by Hi-Reff.
    - Hi-Reff: Furuichi, et al., Phys. Fluids, 27, 095108 (2015).
    - Superpipe: Hultmark, et al., PRL 108, 094501 (2012).

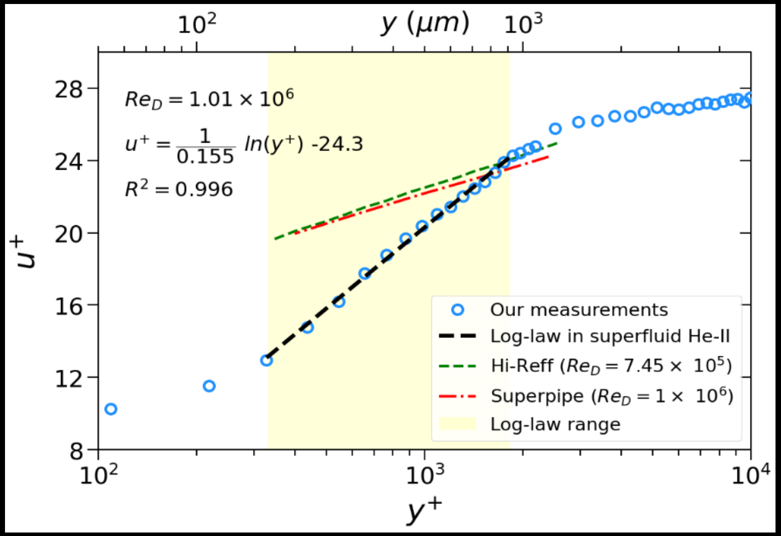



- Repeated runs under the same temperature (1.9 K) and mean velocities.
- Error bars count in all system errors and measurement errors.
- Results are consistent ➔ $\kappa < 0.2$ , which is less than 50% of the classical value 0.41

## ➤ Log-law constants in high-Re He II pipe flow:



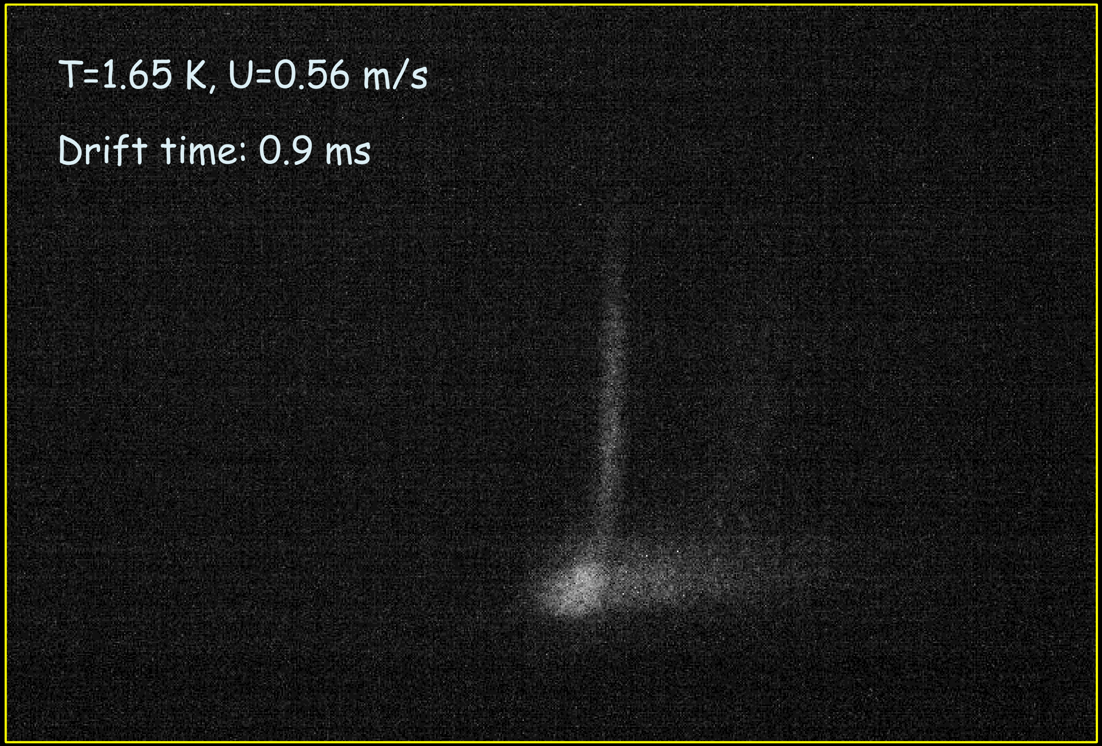




- The additive constant is expected to be very different when the Karman constant changes: $u^+ = \kappa^{-1} \ln y^+ + B$

**Conclusion:**

- A log-law near-wall mean velocity profile in high Re He II pipe flow is observed.
- But the von Karman constant is less than 50% of the classical value. (why??)

- On-going/future work:




- Temperature effect: measuring the log-law constants @ different T.

  $\kappa \approx 1.5$ @ 1.65 K

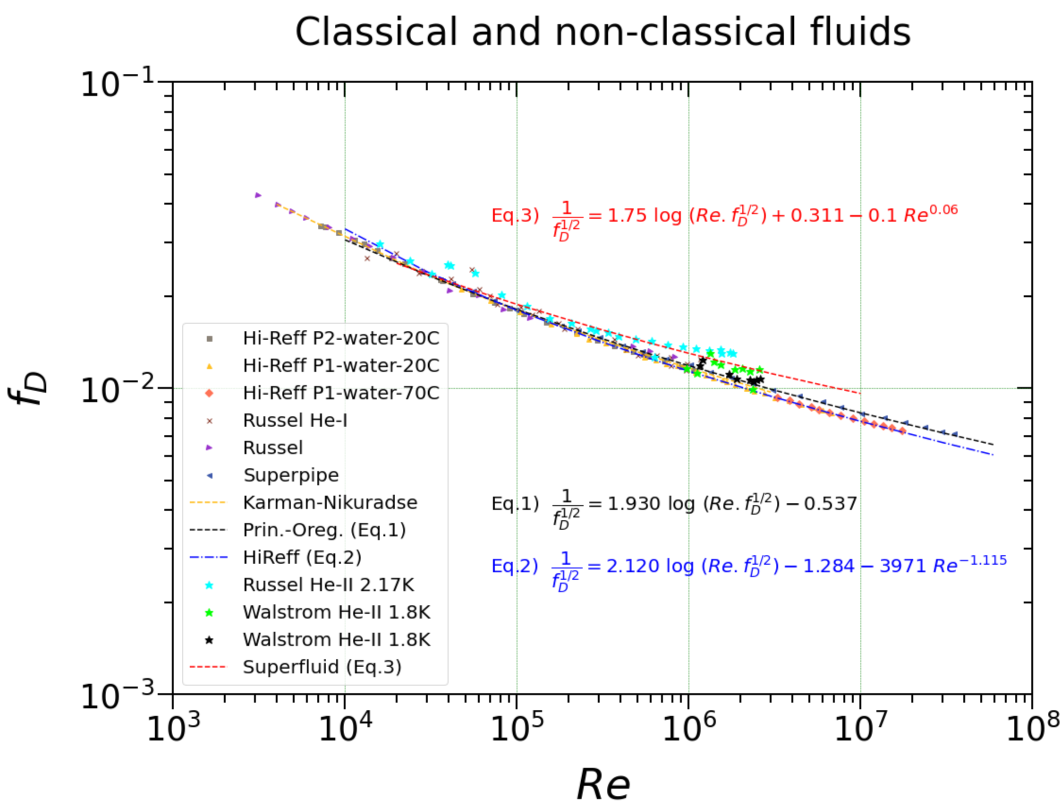
- Near-wall velocity profile in counterflow.

# Questions?

# Friction Factor for Classical and non-classical Fluids

State-of-the-art data in both classical and non-classical fluids are compiled including Princeton superpipe, Japan Hi-Reff, and Oregon Turbulent pipe.

Karman-Nikuradse equation, Princeton-Oregon correlation, Hi-Reff correlation and the fit to the superfluid data are depicted.

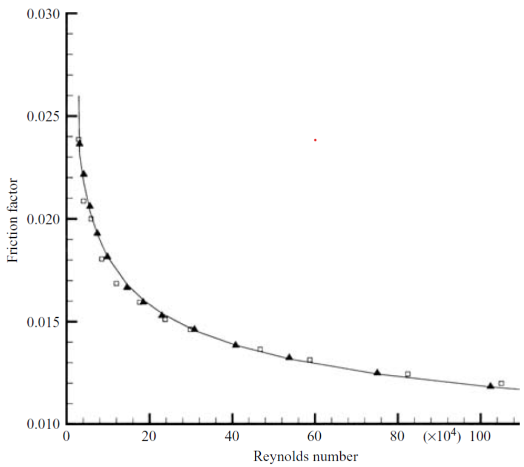



State-of-the-art friction factor data

# Friction factor data in superfluid $^4$He

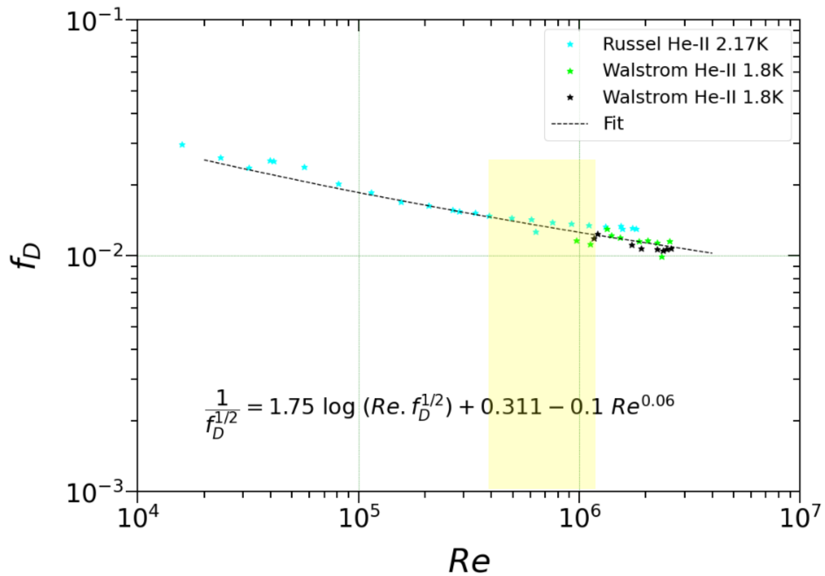
1. Oregon data in classical fluids are precisely validated versus superpipe data to the extent that they provide a unified correlation for friction factor [1]

2. Oregon data in superfluid helium were obtained utilizing the same experimental apparatus and analysis techniques. The relative roughness is also in the ... 2]




Unified Princeton-Oregon correlation

3. However, there are still limited data sets in He-II. So here we perform the analysis using the uncertainty associated with the fit and include it in the uncertainty analysis




He-II friction factor data and fitted correlation.
*Re* range of current experiments is shown in yellow highlight


[1] McKeon, B.J., Swanson, C.J., Zagarola, M.V., Donnelly, R.J. and SMITS, A.J., 2004. Friction factors for smooth pipe flow. Journal of Fluid Mechanics, 511, pp.41-44.
[2] Swanson, C.J., Donnelly, R.J. and Ihas, G.G., 2000. Turbulent pipe flow of He I and He II. Physica B: Condensed Matter, 284, pp.77-78.

## Friction Factor $(f_D)$

- Friction factor is only a function of the Reynolds number *Re* if pipe is smooth, entrance-free and isothermal [1]

Our flow pipe:

- ✓ Entrance length: $100D_h > 25D_h$ [9]
- ✓ Flow pipe is isothermal

Is our flow pipe smooth ?

### 1. Flow Pipe Surface Roughness

- Nikuradse[10] showed: $k_s^+ = k_s u_\tau / \nu$ where $k_s$ is equivalent sandgrain roughness height
- Pipe is smooth for $k_s^+ \leq 5$
- Zagarola and Smits[11] showed $k_s \approx 3k_{rms}$ where $k_{rms}$ is the root-mean-square roughness height
- **Our flow pipe:** $k_s^+ = 2$ (micro−inch for polished stainless steel)/ $0.5\ \mu m = 0.3 \ll 5$

[11] Zagarola, M.V. and Smits, A.J., 1998. Mean-flow scaling of turbulent pipe flow. Journal of Fluid Mechanics, 373, pp.33-79.