A self-consistent model of superfluid ^{4}He turbulence

FOUCAULT

L. Galantucci

Bridging Classical and Quantum Turbulence

12 July 2023

Istituto per le Applicazioni del Calcolo "Mauro Picone"

IAC - CNR

Collaborators

Observatoire de la Cote d'Azur

Giorgio Krstulovic

Newcastle University

Carlo Barenghi

Andrew Baggaley

LG, A. Baggaley, C. Barenghi, G. Krstulovic, *Eur. Phys. J. Plus* **135**, 547 (2020)

LG, C. Barenghi, G. Krstulovic, *Phys Rev Fluids* **8**, 014702 (2023)

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Overview

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Helium II - TWO FLUID MODEL

Tisza (1938), Landau(1941)

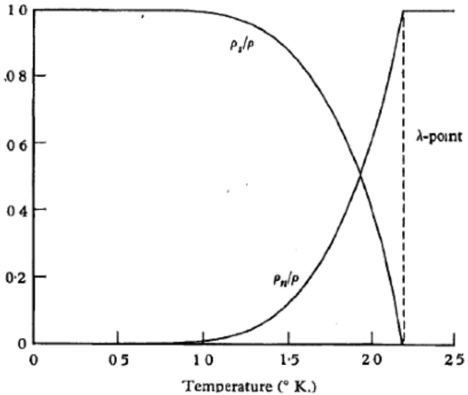
SUPERFLUID

- $\sim$ condensate
  - related to BEC
- $\rho_s, \mathbf{v}_s$
- no entropy
- inviscid $\nu_s = 0$
- $\sim$ Euler fluid

NORMAL FLUID

- thermal excitations
  - phonons
  - rotons $(1.5K < T < 2.1K)$
- $\rho_n, \mathbf{v}_n$
- entropy $s \neq 0$
- viscosity $\nu_n \sim 10^{-8} m^2/s$
- $\sim$ Navier-Stokes fluid

Helium II



T (K)	ρ_s/ρ_n
1.3	21.3
1.4	12.7
1.5	7.98
1.6	5.16
1.7	3.37
1.8	2.19
1.9	1.38
1.96	1.02
2.0	0.80
2.1	0.35

- $\rho = \rho_s + \rho_n = 145\ kg\ m^{-3}$

Helium II - Quantised Vortices

- topological defects of the superfluid
- one–dimensional structures $a_0 \sim 1\text{Å}$
  - $\ell \sim 10^{-4}\,\text{m} \div 10^{-5}\,\text{m}$
  - $D \sim 10^{-3}\,\text{m} \div 10^{-2}\,\text{m}$
- $\boldsymbol{\omega}_s = \nabla \times \mathbf{v}_s$ confined to vortex lines $\mathbf{s}(\zeta, t)$

$$\boldsymbol{\omega}_s(\mathbf{x}, t) = \kappa \oint_{\mathscr{L}} \mathbf{s}'(\zeta, t)\delta^{(3)}(\mathbf{x} - \mathbf{s}(\zeta, t))\, d\zeta$$

- circulation quantized, $\kappa = h/m = 10^{-7}\,\text{m}^2/\text{s}$
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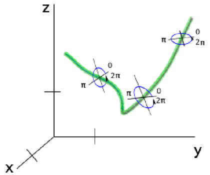

vortex-lines scattering centres for thermal-excitations

$\Downarrow$

mutual friction force $\mathbf{F}_{ns}$

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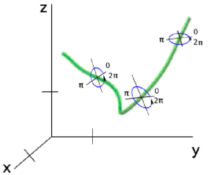

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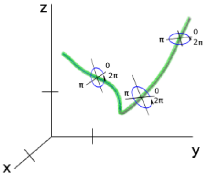

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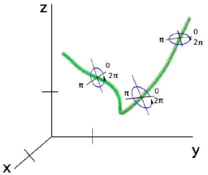

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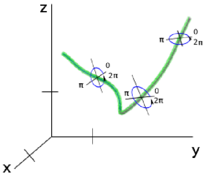

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Helium II - Quantised Vortices

The rotation of liquid helium II

II. The theory of mutual friction in uniformly rotating helium II

By H. E. Hall and W. F. Vinen

The Royal Society Mond Laboratory, University of Cambridge

Hall, H.E. and Vinen, W.F.,

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Proc. R. Soc. London A **215**, 215 (1956)

Helium II - Mutual Friction Force $\mathbf{F}_{ns}$

COFLOWS $\overline{\mathbf{v}}_n \approx \overline{\mathbf{v}}_s$

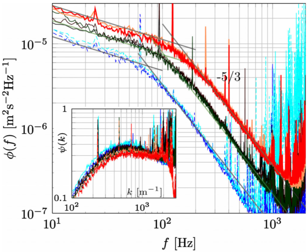
- He II mechanically driven at large scales
  - oscillating grids, objects
  - bellows → channel flow

Large scales $\Delta \gg \ell$

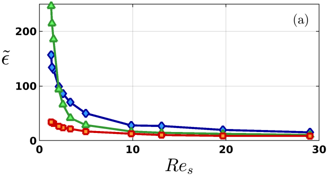
- $\mathbf{F}_{ns}$ couples the two fluids
  $\dot{\mathbf{s}} - \mathbf{v}_n \to 0 \ \Rightarrow \epsilon_{ns} \to 0$
- $E(k) \sim k^{-5/3}$
  [Maurer *et al.* , *EPL* (1998)]
- turbulent decay $L(t) \sim t^{-3/2}$
  [Stalp *et al.* , *Phys Rev Lett* (1999)]

Small scales $\delta < \ell$

- $\mathbf{F}_{ns} \neq 0 \ \Rightarrow \epsilon_{ns} > 0$
  dissipation anomaly in superfluids
  [LG *et al.* , *Phys Rev Fluids* (2023)]



[Salort *et al.* , *Phys Fluids* (2010)]



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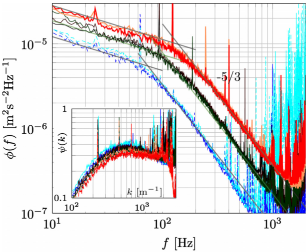

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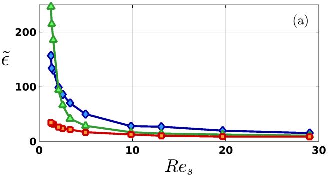

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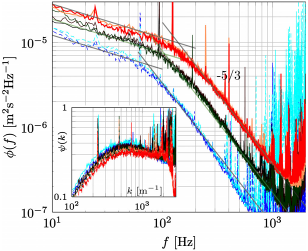

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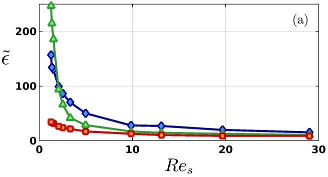

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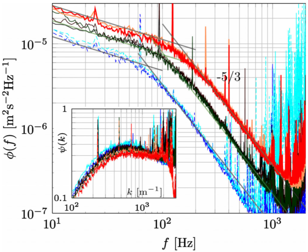

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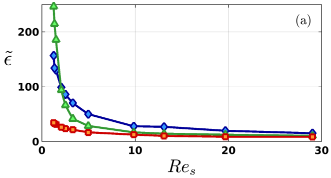

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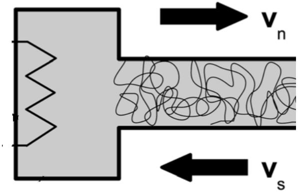

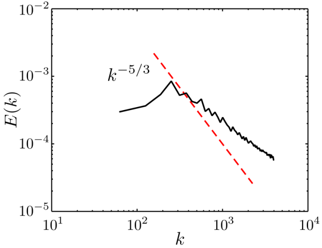

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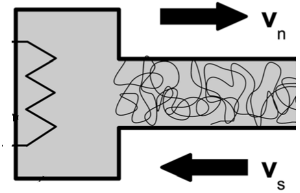

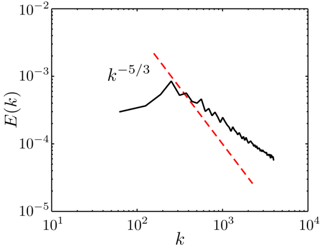

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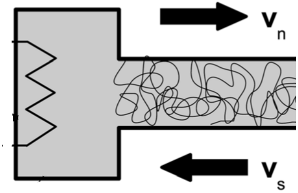

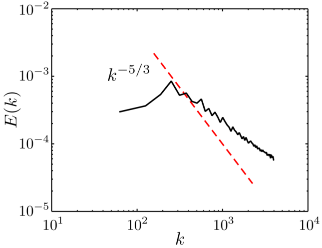

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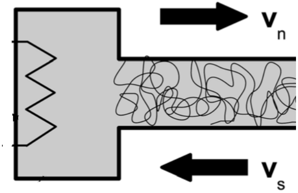

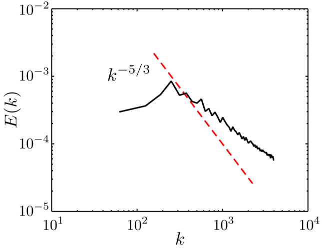

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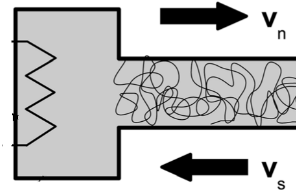

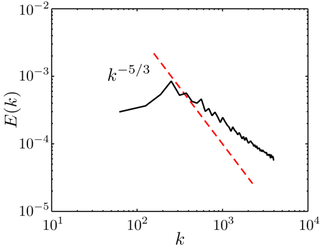

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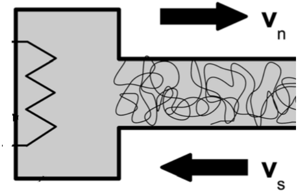

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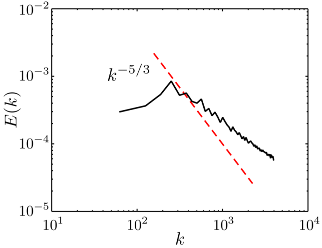


Overview

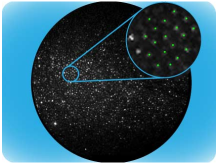
Helium II - Mutual friction force $\mathbf{F}_{ns}$

- pioneering work Hall & Vinen

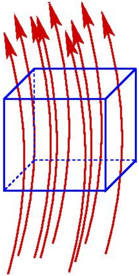
  [Hall & Vinen, *Proc. Roy. Soc. Lond. A* **238**, 204 (1956)]

  [Hall & Vinen, *Proc. Roy. Soc. Lond. A* **238**, 215 (1956)]

- distinct approaches have modeled $\mathbf{F}_{ns}$
  - probed lengthscales $\Delta \gg \ell$, $\delta < \ell$
  - configuration superfluid vortex tangle
  - numerical simulations performed



[Bewley *et al.* , *Nature* **441**, 588 (2006)]



1. coarse-grained framework $\Delta \gg \ell$

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2. local kinematic model $\delta < \ell$, *imposed* $\hat{\mathbf{V}}_n$

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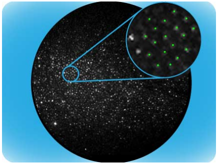

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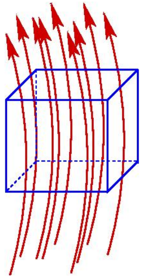

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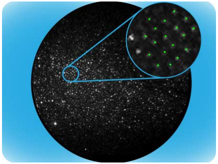

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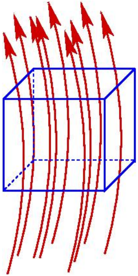
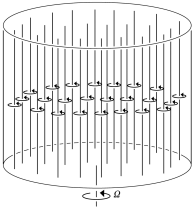
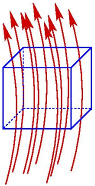


$\mathbf{F}_{ns}$: coarse-grained model HVBK



- I. attentuation of 2nd sound
  - dependence on $\theta((\mathbf{v}_s - \mathbf{v}_n), \hat{\mathbf{\Omega}})$
- II. theory of $\mathbf{F}_{ns}$
  - vortex scattering of rotons
  - dragging of normal fluid
  - Magnus force
- Feynman's model of vortex lines confirmed!



$\Delta \gg \ell$

$$\mathbf{F}_{ns} = -B\frac{\rho_s\rho_n}{\rho}\frac{\boldsymbol{\omega}_s \times [\boldsymbol{\omega}_s \times (\overline{\mathbf{v}}_s - \overline{\mathbf{v}}_n)]}{\omega_s} - B'\frac{\rho_s\rho_n}{\rho}\boldsymbol{\omega}_s \times (\overline{\mathbf{v}}_s - \overline{\mathbf{v}}_n)$$

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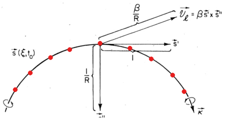

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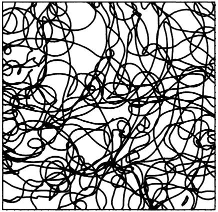

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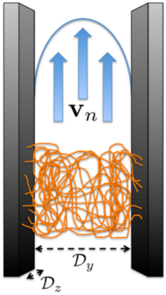
$\mathbf{F}_{ns}$: Local *kinematic* model: VFM

- $\widehat{\mathbf{V}}_n(\mathbf{x}, t) \implies$ tangle evolution $\{\, \mathbf{s}_i(t) \,\}_{i=1,\ldots,N_p}$
  - uniform
  - parabolic
  - Hagen-Poiseuille
    tail-flattened flows
  - vortex tubes
  - ABC flow
  - random waves
  - turbulent NS flows



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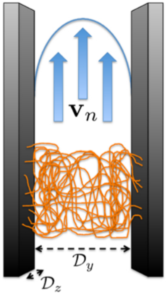

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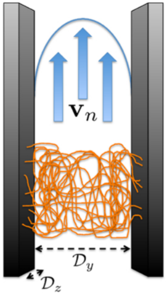

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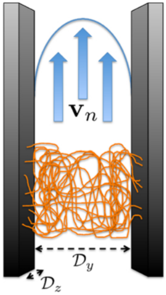

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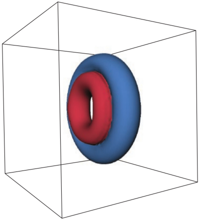

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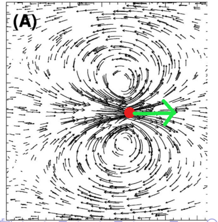

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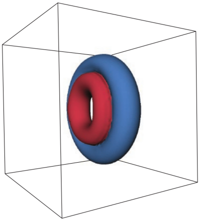

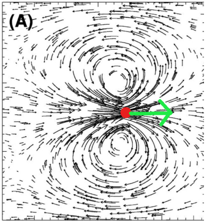

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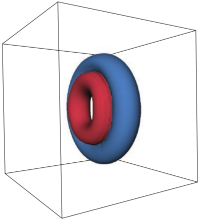

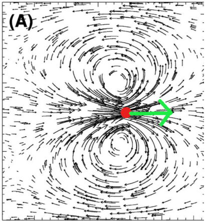

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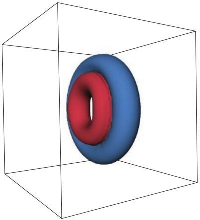

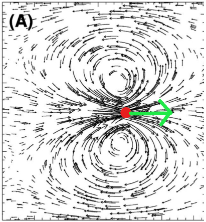

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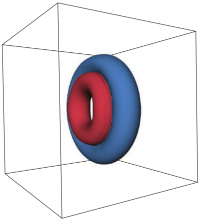

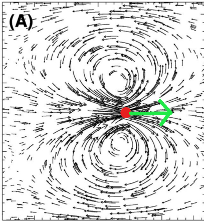

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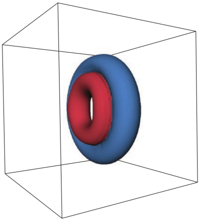

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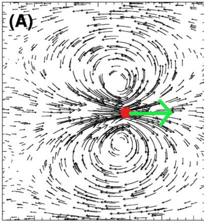

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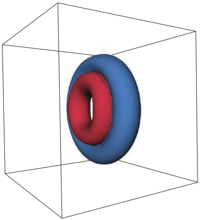

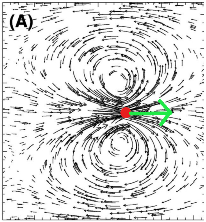

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- friction coefficients are recalculations of $\Delta \gg \ell$

  $(\alpha\ ,\ \alpha')$ [Hall & Vinen, *Proc. Roy. Soc. Lond. A* **238**, 204 (1956)]

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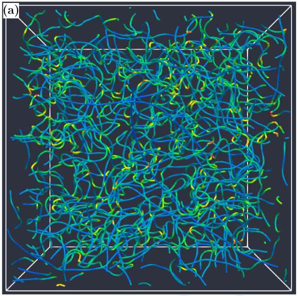
  $(D\ ,\ D_t)$ [Idowu *et al.* , *J Low temp Phys* **120**, 269 (2000)]

- Numerics: low parallelisation
  - $\mathbf{v}_n(\mathbf{x}, t)$ on 128^3 grid
  - $N_p \sim 5 \times 10^4$

small range of scales !

$$D/a_0 \sim 10^8$$



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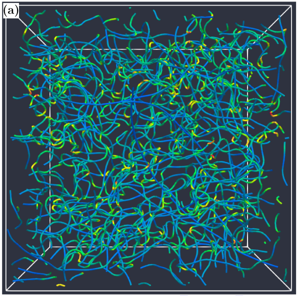

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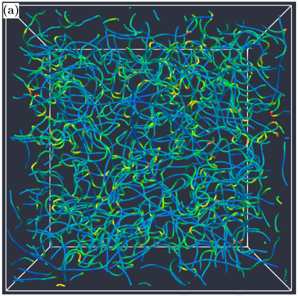

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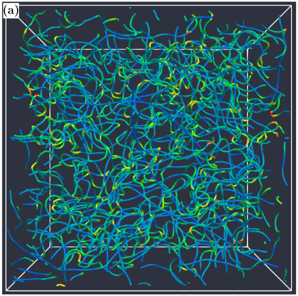

Overview

FOUCAULT

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Fully cOUpled loCAl model of sUperfLuid Turbulence

1. more *realistic* classical model of $\mathbf{F}_{ns}$
2. distribution of $\mathbf{F}_{ns}$ on $\mathbf{v}_n$ grid points
   physically motivated
3. higher parallelisation
   solve wider range of scales

USE tools from Classical Turbulence

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$\mathbf{F}_{ns}$: *fully coupled* local model

- classical, low-Reynolds fluid dynamics

  [Kivotides, *Phys. Rev. Fl.* **3**, 104701 (2018)]

  - $\mathbf{f}_{ns} \propto (\dot{\mathbf{s}} - \mathbf{v}_n) \sim$ Stokes drag
  - vortex locally $\sim$ cylinder
  - $\delta / a_0 \sim 10^4 \div 10^5$
  - $\mathrm{Re} = \dfrac{|\dot{\mathbf{s}} - \mathbf{v}_n| a_0}{\nu_n} \sim 10^{-5} \div 10^{-4}$
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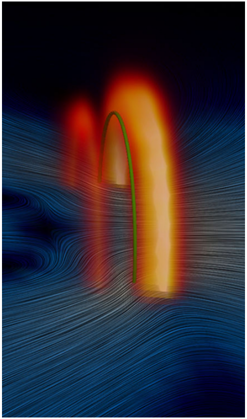

LG, CFB, AWB, GK, *Eur. Phys. J. Plus* **135**, 547 (2020)

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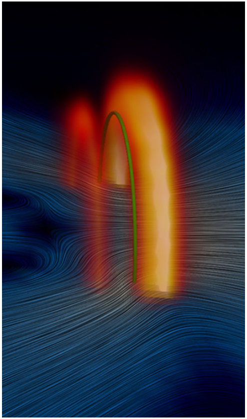

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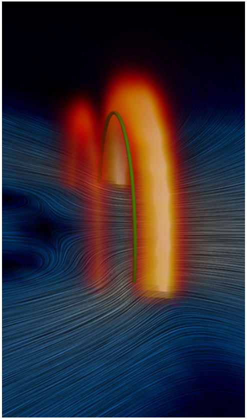

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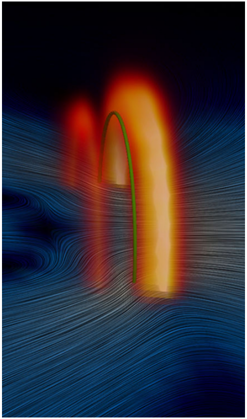

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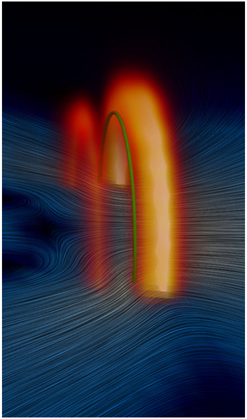

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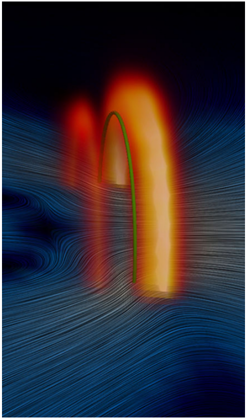

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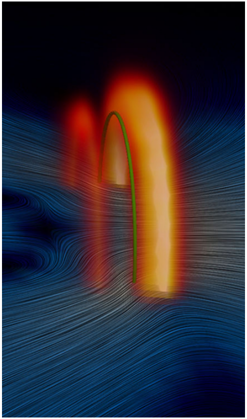

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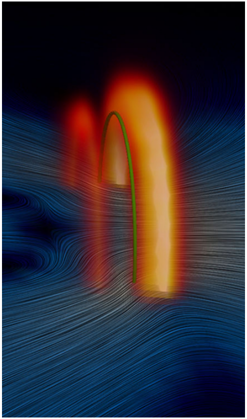

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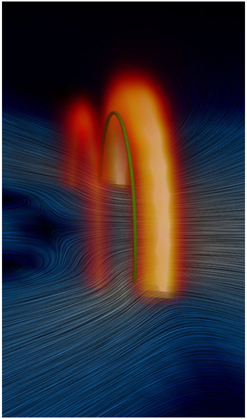

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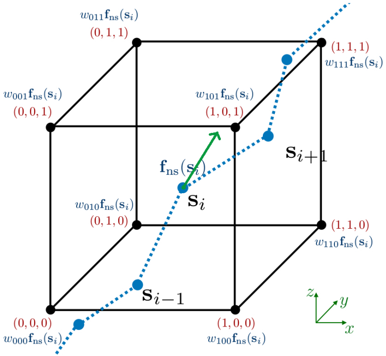

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- $w_{\zeta,\mu,\chi}$
- $\sum_{\zeta,\mu,\chi=0}^{1} w_{\zeta,\mu,\chi} = 1$
- nearest neighbours
  tri-linear extrapolation
- Filtering
  - moving avg N_{filter} points
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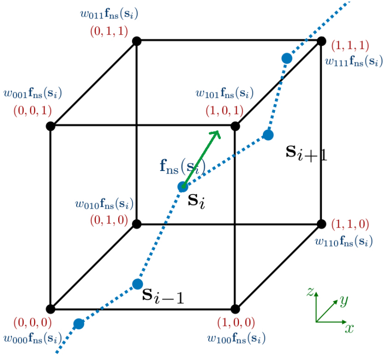
    $\sigma = N_{filter} \Delta x$



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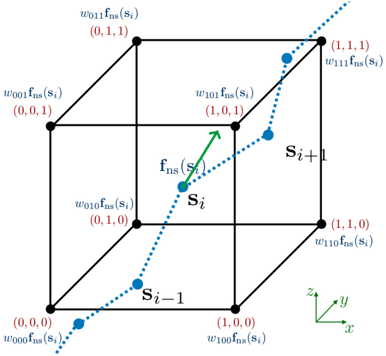

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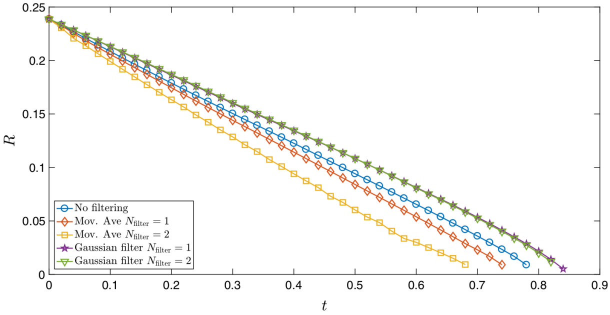
- single vortex ring $R_0 = 4 \times 10^{-3}$cm
- initially quiescent normal fluid
- shrinking vortex dynamics



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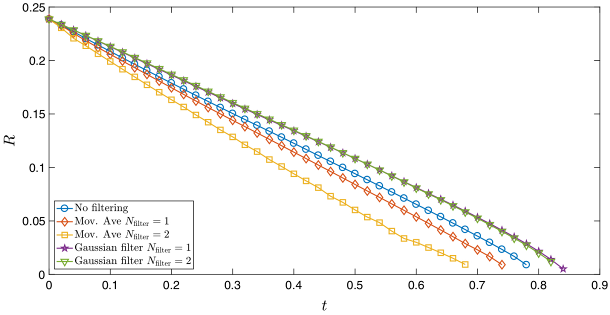


SPURIOUS!

$\mathbf{F}_{ns}$: *fully coupled* local model

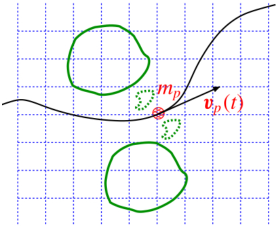
- physically consistent regularisation
- active matter systems
  strongly localised response of
  point-like agents
  - particles (PIV, PTV)
  - bacteria
  - swimmers

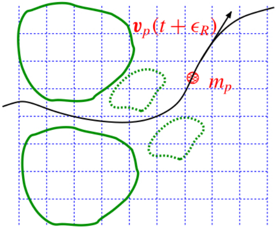
  [Gualtieri *et al.* , *J Fluid Mech* **773**, 520 (2015)]

  [Gualtieri *et al.* , *Phys Rev Fl* **2**, 034304 (2017)]

- Re$\sim 10^{-4} \div 10^{-5}$
- generation localised vorticity $\boldsymbol{\omega}_n$
- diffused by viscosity ν_n

  [LG *et al.*, *Eur. Phys. J. Plus* **135**, 547 (2020)]





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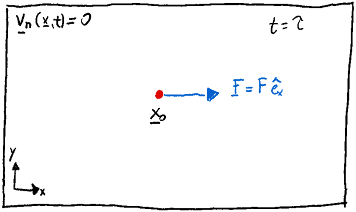

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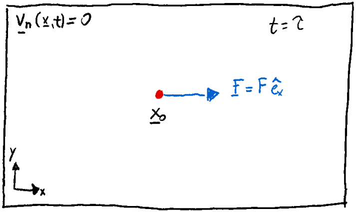

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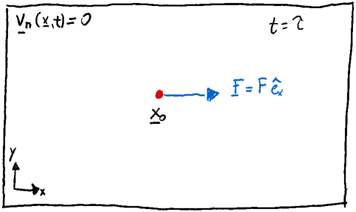

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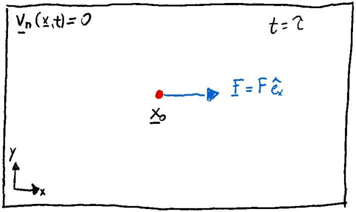

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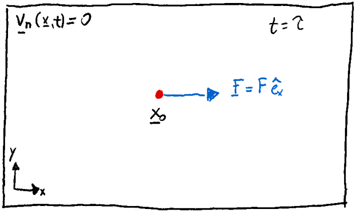

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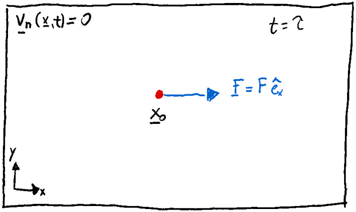
$$g[\mathbf{x}-\mathbf{x}', t-t'] = \frac{1}{[4\pi\nu(t-t')]^{3/2}} e^{-\frac{|\mathbf{x}-\mathbf{x}'|^2}{4\nu(t-t')}}$$

$\mathbf{F}_{ns}$: generation and diffusion of vorticity by a vortex



$$\boldsymbol{\omega}(\mathbf{x},t) = F\,\hat{\mathbf{e}}_x \times \nabla' g[\mathbf{x}-\mathbf{x}', t-\tau]\Big|_{\mathbf{x}'=\mathbf{x}_0},$$

$$\boldsymbol{\omega}(\mathbf{x},t) = = \frac{2\pi F}{[4\pi\nu(t-\tau)]^{5/2}}\, e^{-\frac{|\mathbf{x}-\mathbf{x}_0|^2}{4\nu(t-\tau)}}\,[\hat{\mathbf{e}}_x \times (\mathbf{x}-\mathbf{x}_0)],$$

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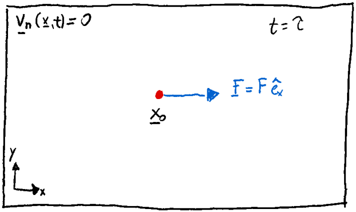

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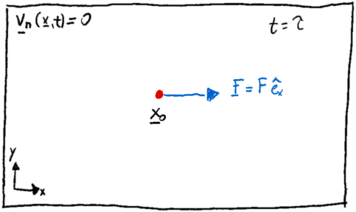

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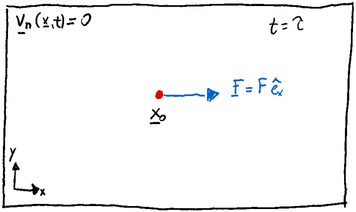

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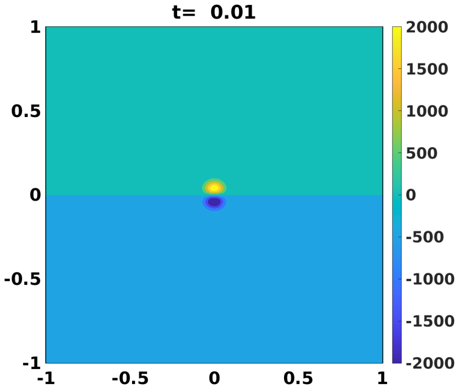

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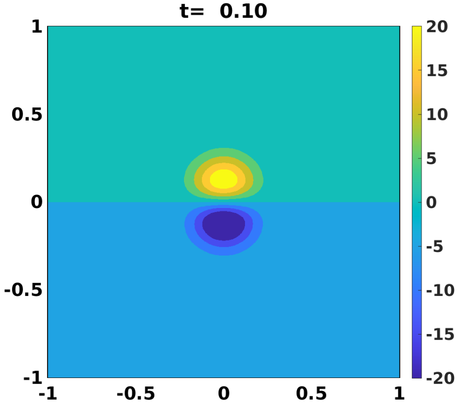

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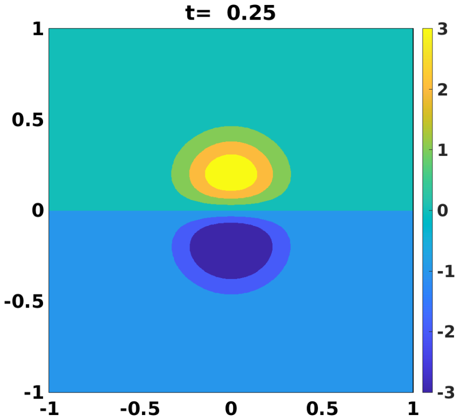

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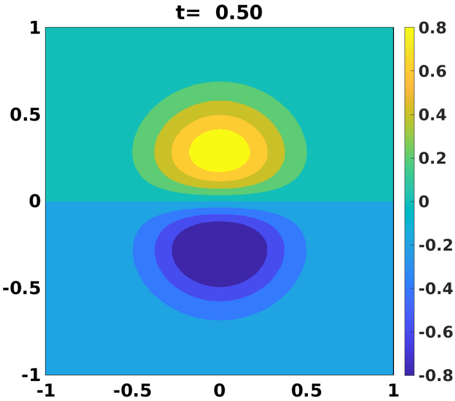

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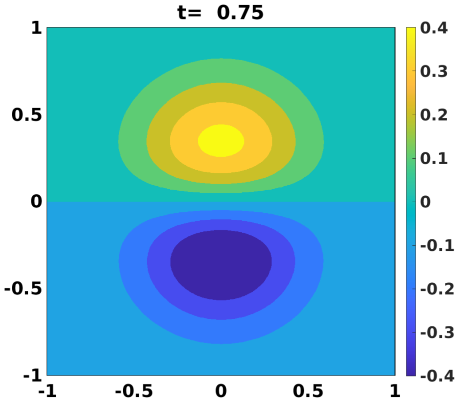

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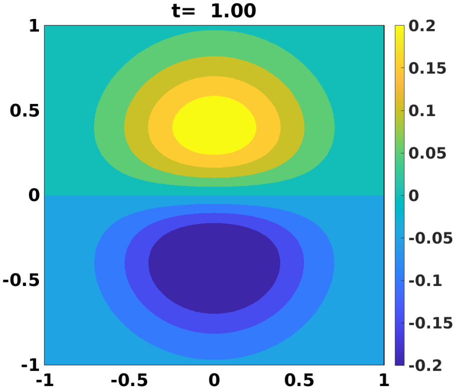

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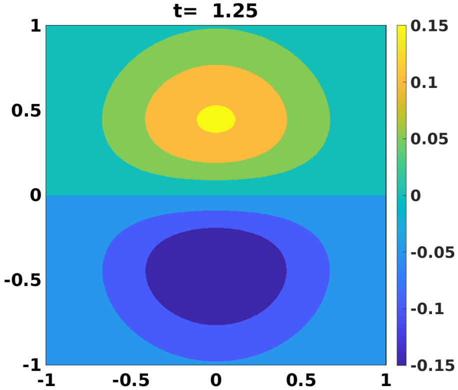

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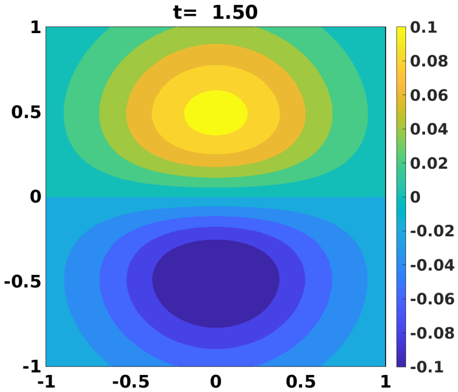

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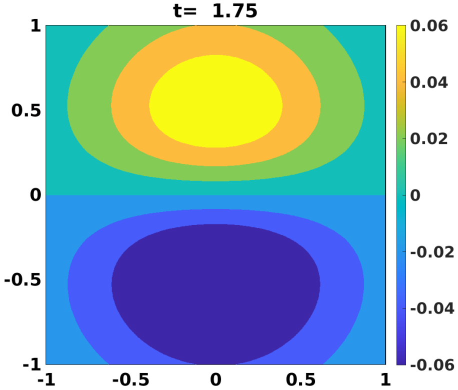

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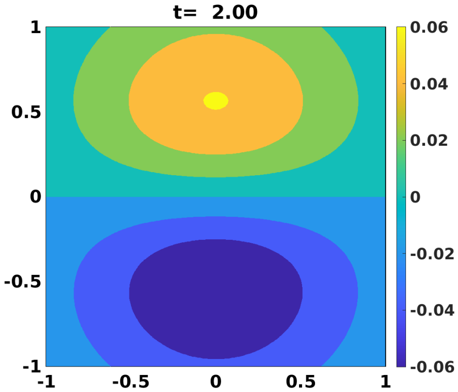

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- introduce time delay ϵ_R:
  diffusion time $\omega_n \rightarrow \Delta x$

  $\sigma_R = \sqrt{2\nu_n \epsilon_R} = \Delta x$

- $\mathbf{f}^i_{ns}(t)\delta(\mathbf{x} - \mathbf{s}_i(t))$

  $\Downarrow$

- $\mathbf{f}^i_{ns}(t-\epsilon_R) g[\mathbf{x} - \mathbf{s}_i(t-\epsilon_R), \epsilon_R]$

- $g[\mathbf{x} - \mathbf{s}_i(t-\epsilon_R), \epsilon_R] =$

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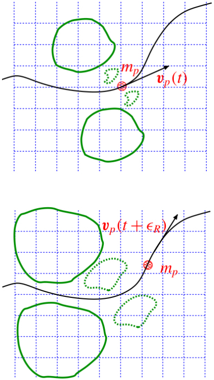

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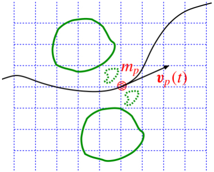

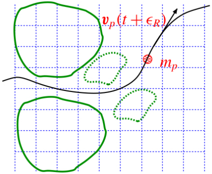

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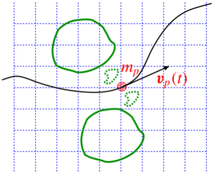

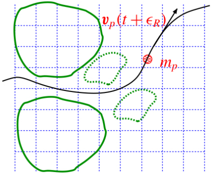

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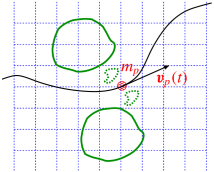

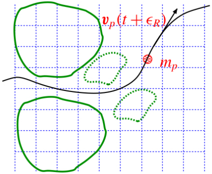

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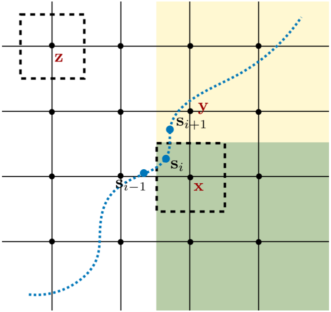
$\mathbf{F}_{ns}$: Regularisation, weights

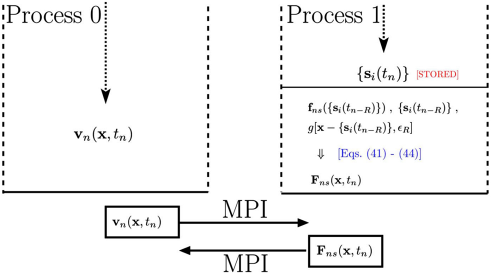


$$w_{\zeta,\mu,\chi} = w_\zeta[s_i^x]\, w_\mu[s_i^y]\, w_\chi[s_i^z] \;,$$

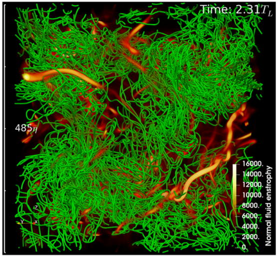
$$w_\zeta[s_i^x] = \zeta + (1-2\zeta)\frac{1}{2}\left(1 + Erf\left[-\frac{\tilde{s}_i^x - \frac{1}{2}}{\sqrt{2}(\sigma_R/\Delta x)}\right]\right) \;,$$

$$\tilde{s}_i^x = \frac{s_i^x - \lfloor s_i^x \rfloor}{\Delta x} \in [0,1]$$

Numerical Architecture



- $\mathbf{v}_n(\mathbf{x}, t)$ on 512^3
- past: 128^3 (40^3)
- $N_p \sim 2 \times 10^5$
- wider range of scales



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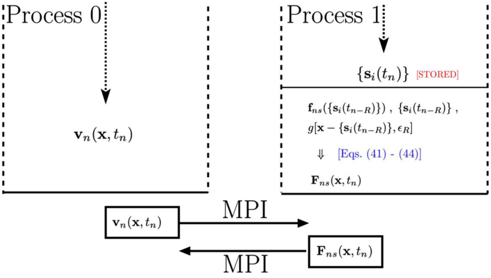

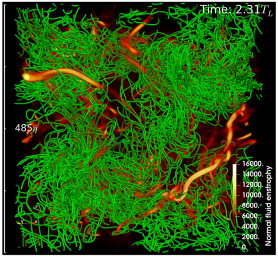

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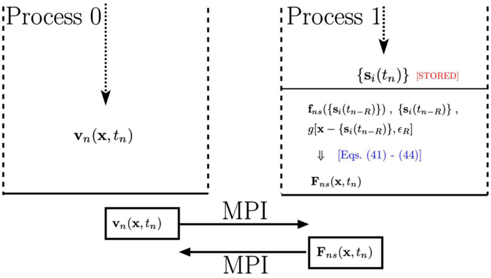

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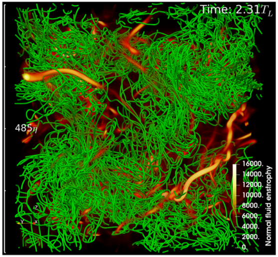
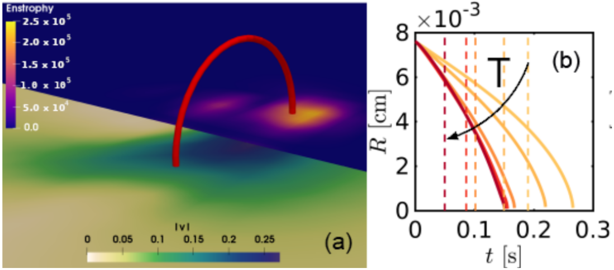


Overview

Shrinking vortex ring in quiescent normal fluid



[LG, Krstulovic, Barenghi, *Phys Rev Fluids* **8**, 014702 (2023)]

- shorter lifetime as T increases
- larger lifetime compared to 1-way coupling [Schwarz, *Phys. Rev. B* **18**, 245 (1978)]
- dipole size $\sim 5\mu m$ $\sim$ size of particles

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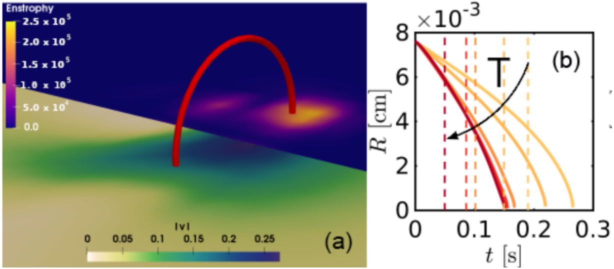

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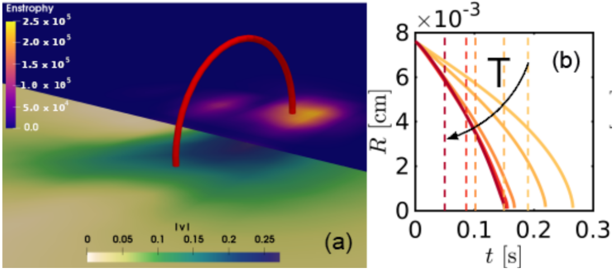

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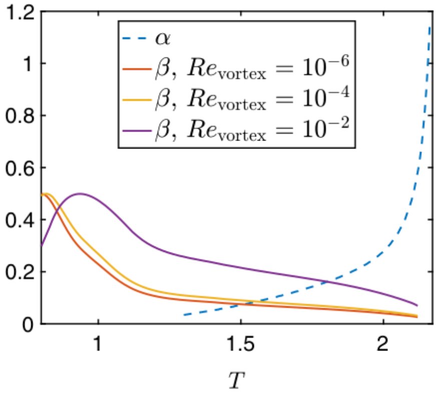

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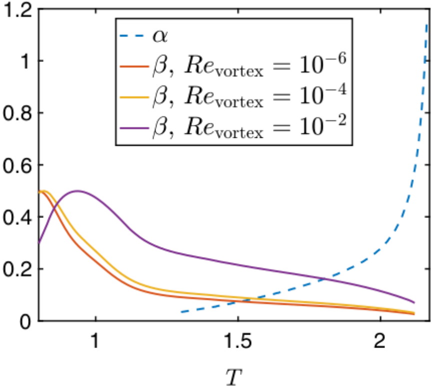

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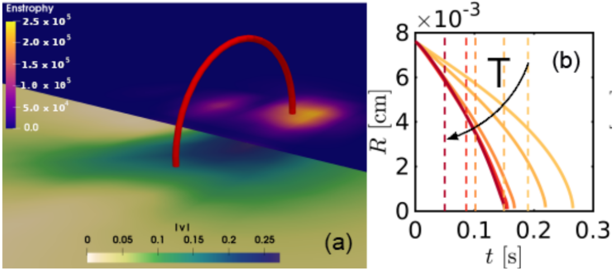

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- important for statistics of particles
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  [Tang *et al.* *Nat Comm* **14**, 2941 (2023)]

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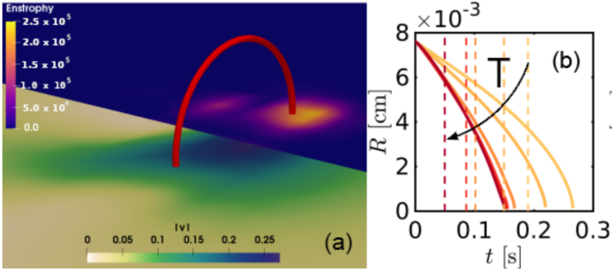

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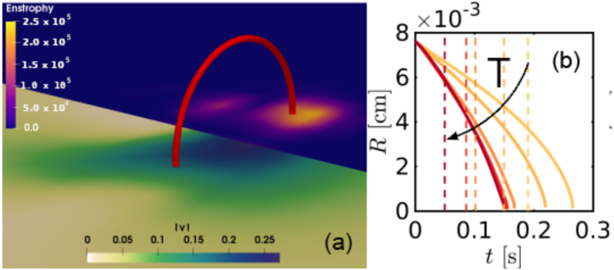
Shrinking vortex ring in quiescent normal fluid



[LG, Krstulovic, Barenghi, *Phys Rev Fluids* **8**, 014702 (2023)]

- velocity wake
- important for statistics of particles
  - PDF(v_p), PDF(a_p)
  - Lagrangian velocity correlations
- consistent with experiments
  [Tang *et al. Nat Comm* **14**, 2941 (2023)]

Experiments: Toroidal vortex bundle

- Borner experiments
  - [Borner *et al.* , *Phys B* **108**, 1123 (1981)]
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- travelled long distance $> 20R$
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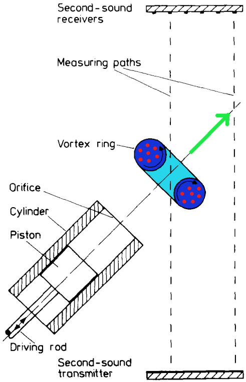

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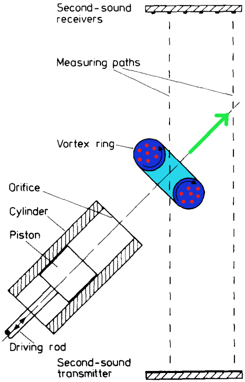

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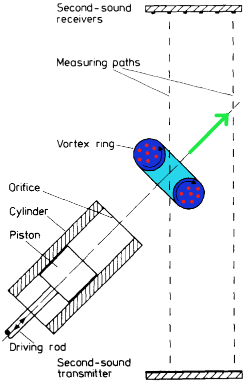

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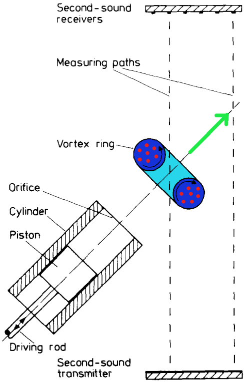

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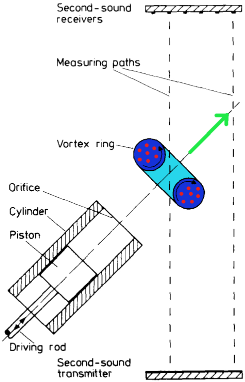

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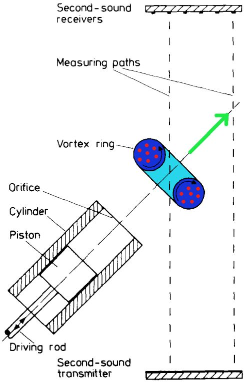

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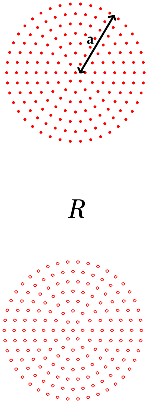
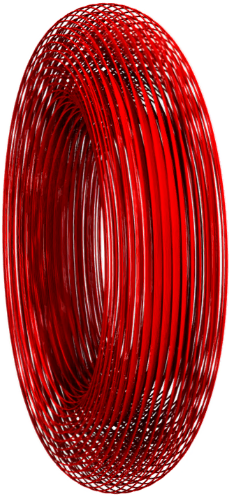
Toroidal vortex bundle: initial conditions





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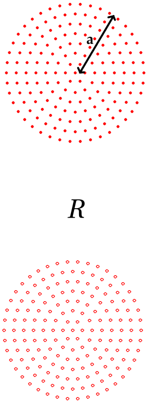

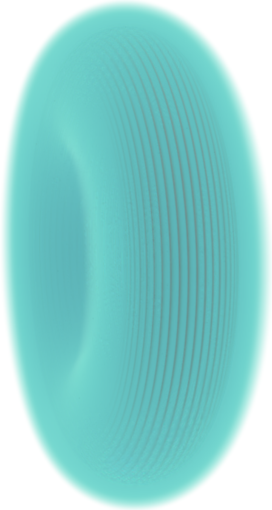

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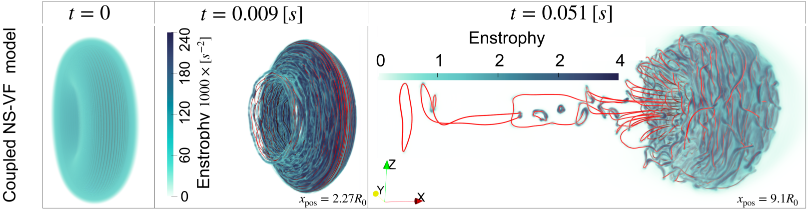
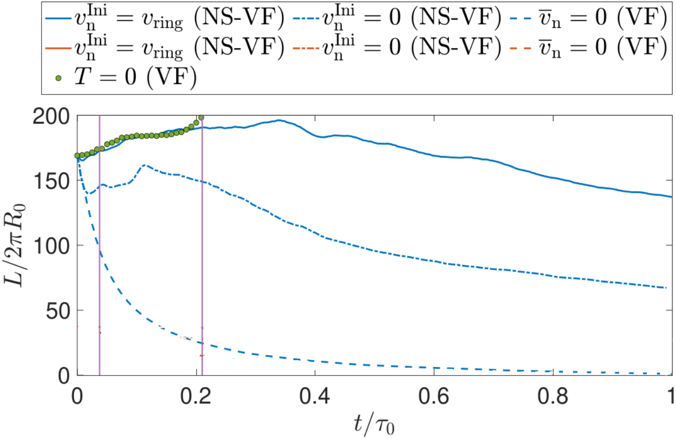


Toroidal vortex bundle: Length



[LG, Krstulovic, Barenghi, *Phys Rev Fluids* **8**, 014702 (2023)]

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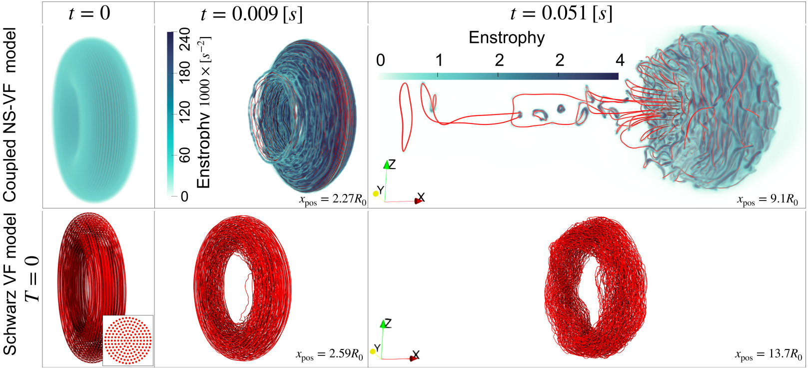

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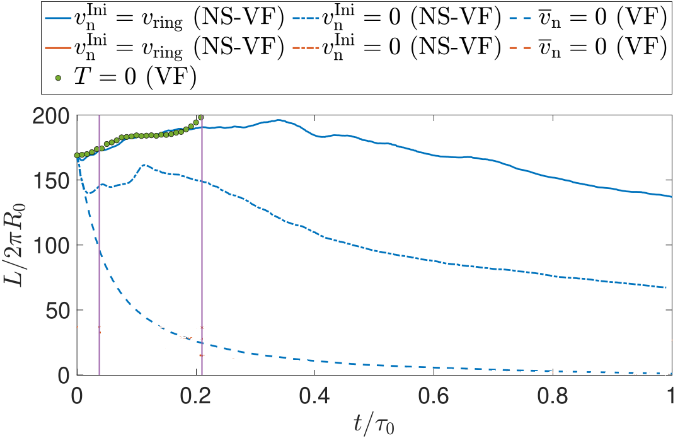

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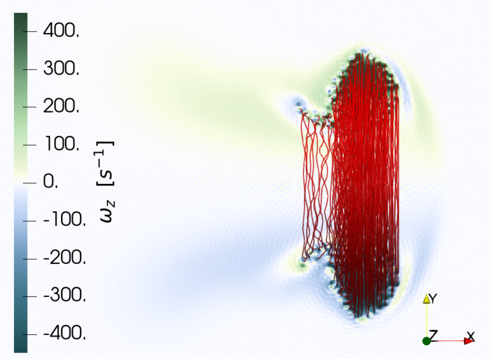
Toroidal vortex bundle: Hydrodynamic cooperation



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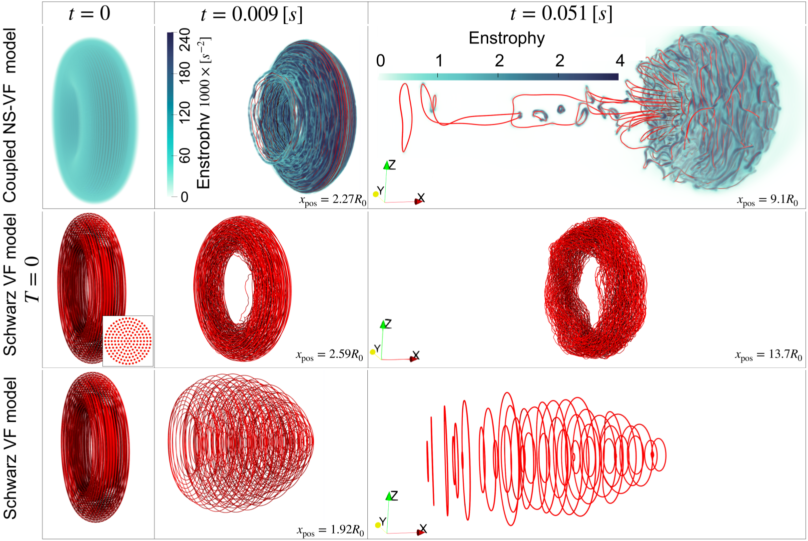

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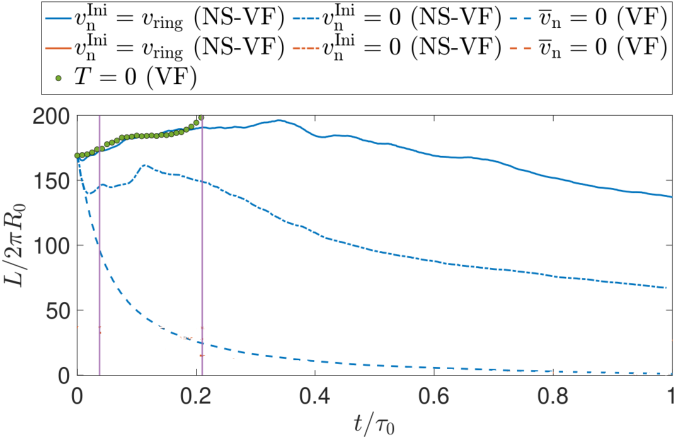

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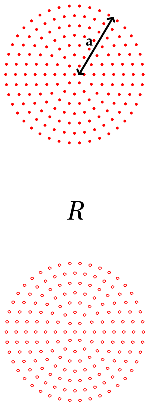
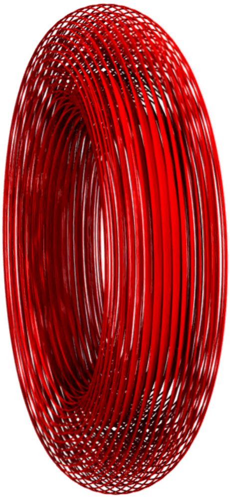
3 Toroidal vortex bundles: different a





1. **a**
2. **2a**
3. **4a**

- $\mathbf{v}_n(\mathbf{x},0)=0$

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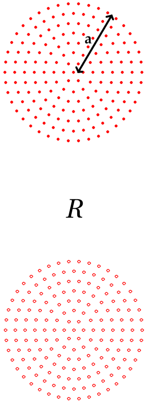

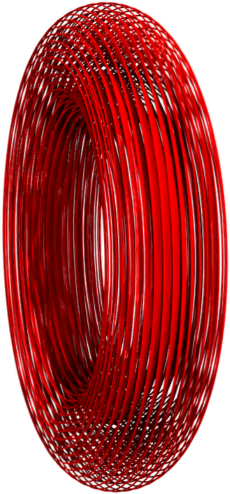

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3 Toroidal vortex bundles: dissipation reduction χ

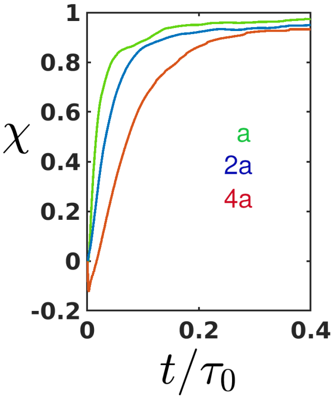
- superfluid kinetic energy dissipation $\epsilon(t)$

$$\epsilon(t) = \oint_{\mathscr{L}} \mathbf{f}_{\rm ns}(\mathbf{s}) \cdot \dot{\mathbf{s}}(\xi, t)\, \mathrm{d}\xi$$

- relative dissipation reduction $\chi(t)$

$$\chi(t) = \frac{\epsilon(0) - \epsilon(t)}{\epsilon(0)}$$

  - $\chi(t) > 0$ dissipation reduction
  - $\chi(t) \to 1 \;\Rightarrow\; \epsilon(t) \to 0$



hydrodynamic cooperation!

single, isolated vortex ring travelling in a quiescent $\mathbf{v}_n$

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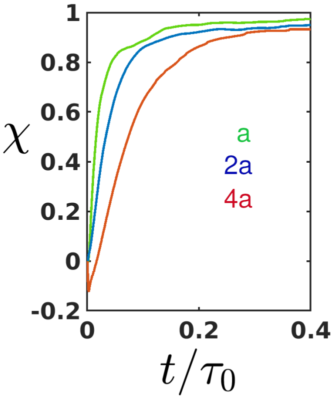

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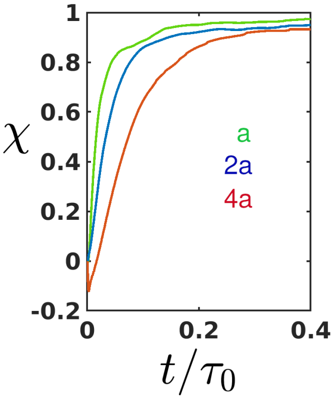

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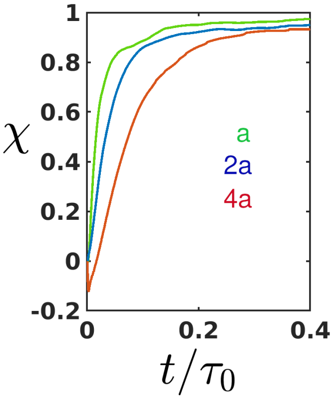

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Superfluids as Active Fluids ?

1. more than one vortex needed for dissipation reduction
2. dissipation reduction more efficient when vortices are closer

⇓

superfluid ^{4}He is an active fluid

hydrodynamic cooperation between superfluid vortices

reduce dissipation

Active fluids

- bacterial suspensions
- fungal spores in air
- cyclists in pelotons

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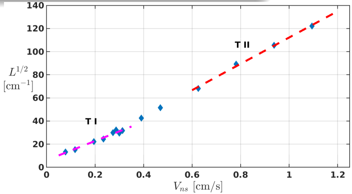
Preliminary Results with FOUCAULT

T I - T II transition ^{4}He counterflow

- $T = 1.5\text{K}$
- $L^{1/2} = \gamma V_{ns}$
- $\gamma_1 = 83.22\text{cm/s}^2$
- $\gamma_2 = 113.8\text{cm/s}^2$



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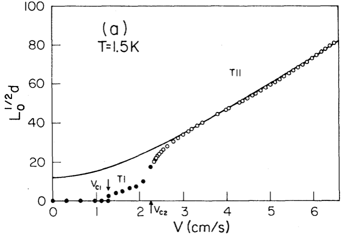
- $\gamma_2 = 145\text{cm/s}^2$



[Martin & Tough, *Phys Rev B* **27**, 2788 (1983)]

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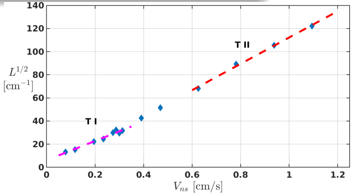

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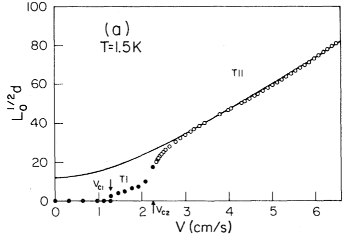

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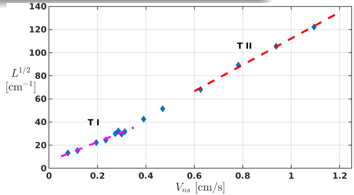

- $\gamma_1 < 85\text{cm/s}^2$
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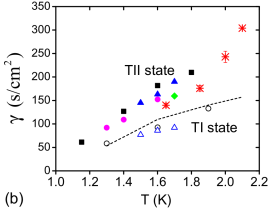


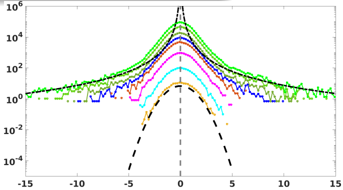
[Gao *et al.* , *Phys Rev B* **96**, 094511 (2017)]

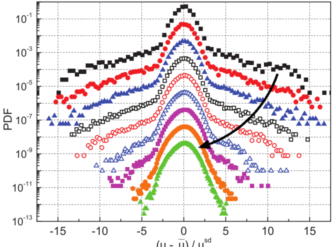
Preliminary Results with FOUCAULT

particles dynamics ^{4}He counterflow

$$\frac{d\mathbf{u}_p}{dt} = \frac{1}{\tau_s}(\mathbf{v}_n - \mathbf{u}_p) + \frac{\rho_n}{\rho}\frac{D\mathbf{v}_n}{Dt} + \frac{\rho_s}{\rho}\frac{D\mathbf{v}_s}{Dt}$$

- $T = 1.8\mathrm{K}$
- $V_{ns} = 0.55\mathrm{cm/s}$
- T I or T II ?
- no trapping





- $\delta < \ell \quad PDF(v) \sim v^{-3}$
- $\Delta > \ell \quad PDF(v)\ \ Gaussian$

Summary

- novel fully coupled algorithm FOUCAULT

  [LG, Baggaley, Barenghi, Krstulovic, *EPJP* **135**, 547 (2020)]

- reproduce experimental results

  [LG, Krstulovic, Barenghi, *Phys Rev Fluids* **8**, 014702 (2023)]

- Superfluids can be described as active fluids

Perspective

- N-body GPU algorithm for vortex dynamics
  - intermittency
  - well resolved statistics
- inertial particle dynamics
  - QT *vs* CT
- wall-bounded flows

  boundary layers