Irreversibility and singularities

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Navier-Stokes and reversibility

$$\partial_t u + u\nabla u = -\frac{1}{\rho}\nabla p + \nu\Delta u + F$$

Creeping Flow

$$\nu\Delta u = -F$$

F->-F

u->-u

E=0

Reversible, Equilibrium

Euler flow

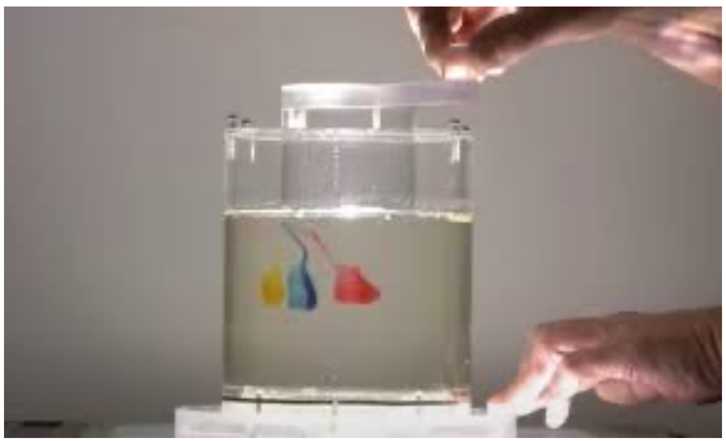
E=cte

$$\partial_t + u\nabla u = -\frac{1}{\rho}\nabla p$$

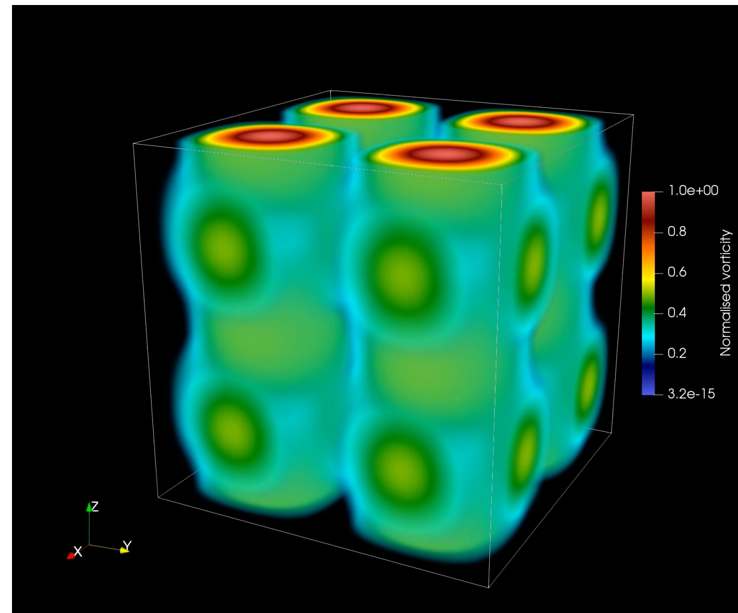
t->--t

u->-u

Reversible, Equilibrium



https://boingboing.net/2020/10/28/liquids-get-mixed-then-unmixed-due-to-stokes-flow.html



Courtesy J.I. Polenco

Navier-Stokes is irreversible and non-equiibrium due to

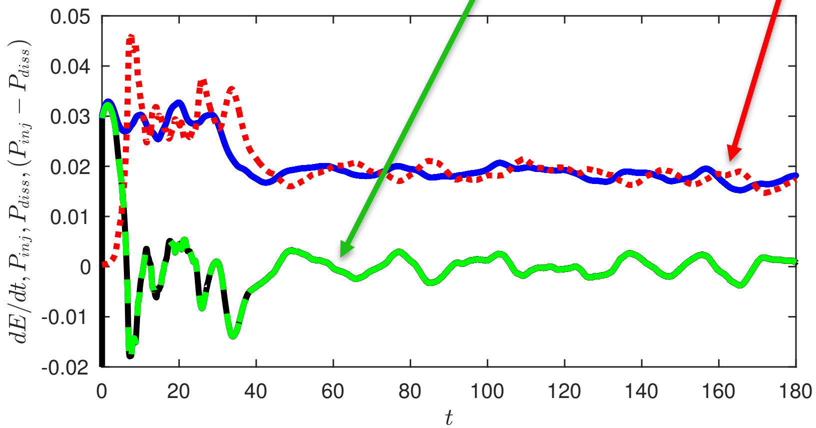
Viscosity and forcing

Entropy production =Dissipation

Non-equilibrium stationary states

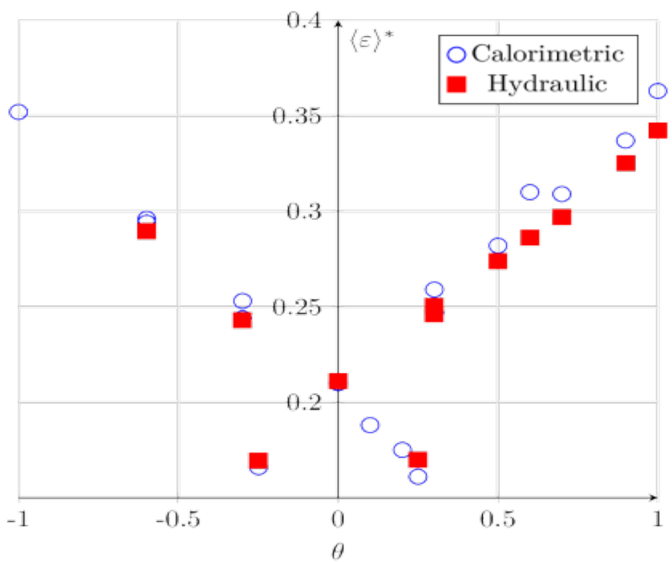
Traditional way: $$\partial_t \int \frac{u^2}{2}\, dx^3 = \int F.u\, dx^3 - \nu \int (\nabla u)^2\, dx^3$$

?



We **impose F at given viscosity**
We measure energy input needed to maintain E statistically constant

$$\epsilon_I(\nu)$$



Efficency of NESS of Navier-Stokes

$$\partial_t \int \frac{u^2}{2}\, dx^3 = \int F.u\, dx^3 - \nu \int (\nabla u)^2\, dx^3$$

$$E \qquad\qquad \epsilon_I \qquad\qquad \epsilon_d(\nu)$$

Traditional protocol:

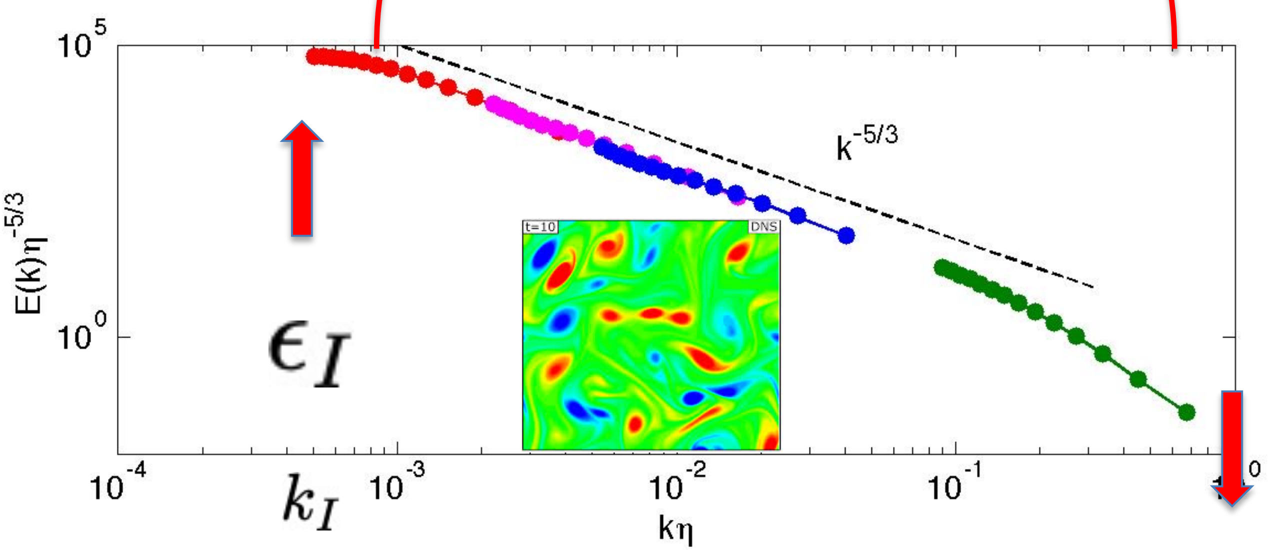
We **impose F at given viscosity**

We measure energy input needed to maintain E statistically constant

Efficiency

$$\mathcal{E} = \frac{\bar{E} k_I}{F}$$

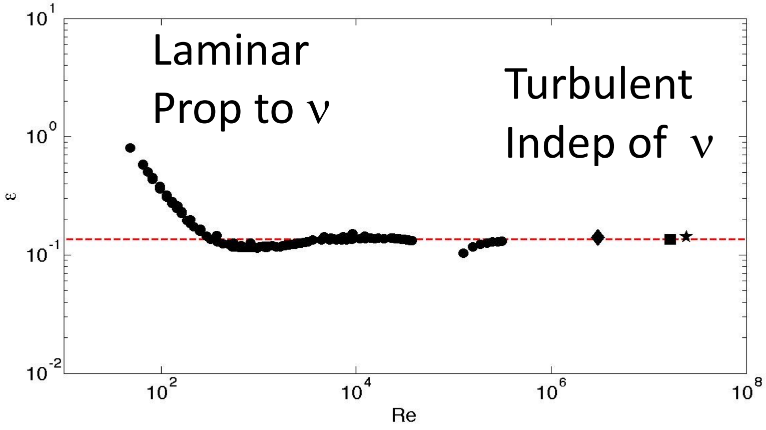
Stored energy E



$\epsilon_d(\nu)$

Dissipation 'anomaly" and Onsager's conjecture

Obs: dissipation does not depend on viscosity (« spontaneous symmetry breaking »)



"...in three dimensions a mechanism for complete dissipation of all kinetic energy, even without the aid of viscosity, is available."

L. Onsager, 1949

See Eyink&Sreenivasan (2006)

$$\frac{1}{2}\partial_t \mathbf{u}^2 + \mathrm{div}\left(\mathbf{u}\left(\frac{1}{2}\mathbf{u}^2 + p\right) - \nu\nabla\mathbf{u}\right) = D(u) - \nu(\nabla\mathbf{u})^2$$

Duchon&Robert. Nonlinearity (2000),

Regular Test function of width ℓ

Inertial dissipation= limit of local energy transfers

$$D(u) = \lim_{\ell\to 0}\frac{1}{4}\int_{r\leq\ell} d^3r\ \nabla\phi_\ell(r)\cdot\delta u_r|\delta u_r|^2$$

$\delta u(\ell) \sim \ell^h$ In the limit of $\ell \approx 0$

$$D(u)[x] \propto \lim_{\ell\to 0}\ell^{3h-1}$$

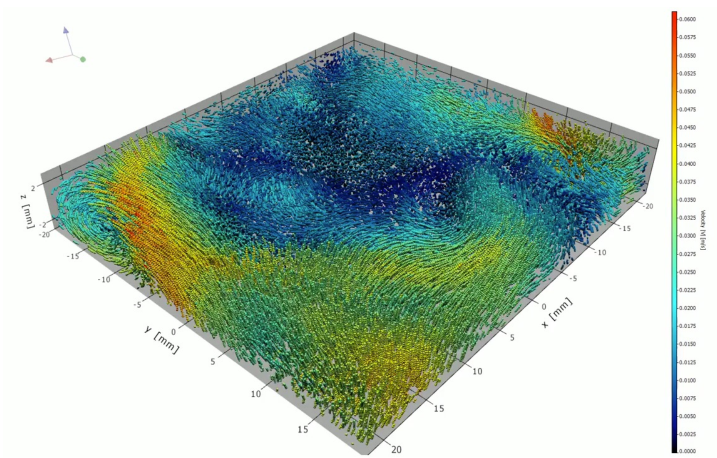
δu=u(x+r)-u(x) Velocity increment

If *h > 1/3* → Euler equation conserves energy, Dissipation in Navier-Stokes by viscosity.
(Eyink 1994, Constantin et al, 1994)

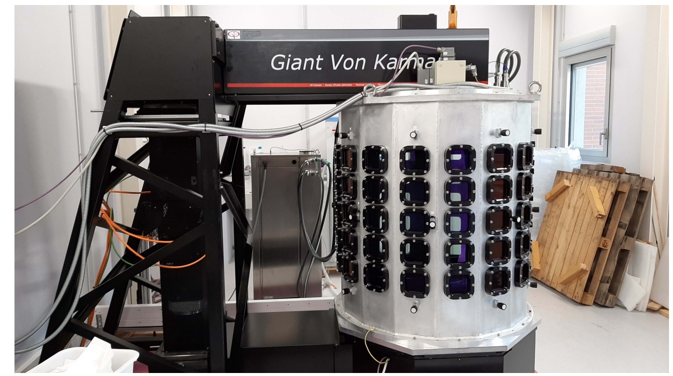
If *h ≤ 1/3* → Dissipation through irregularities (singularities) Without viscosity !
(Isett, 2018)

Onsager conjecture links irreversibility and singularity! I

Direct investigation using local dissipation



Local Lagrangian and Eulerian velocity measurements



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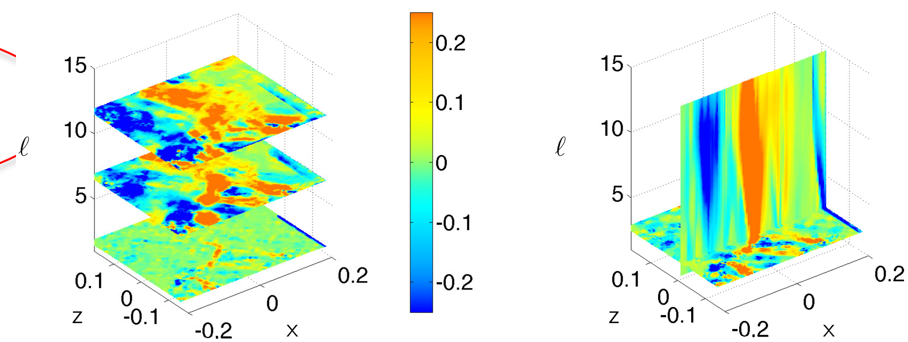
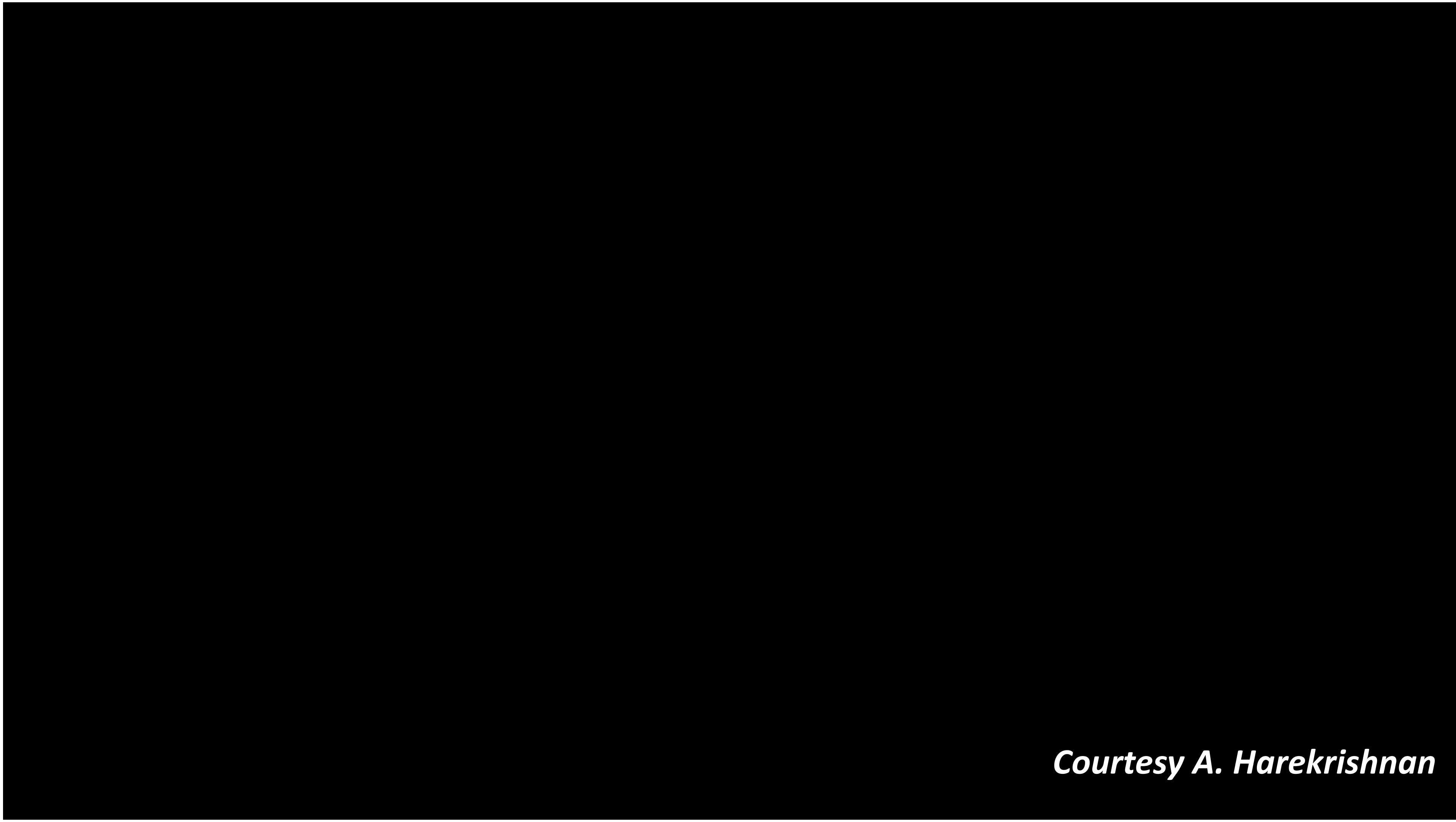
Scalar regularity indicator D_ℓ^I



We can use this quantity as an indicator pointi Towards less regular regions

B. Dubrulle, JFM perspectives, 2019

Direct investigation using local dissipation



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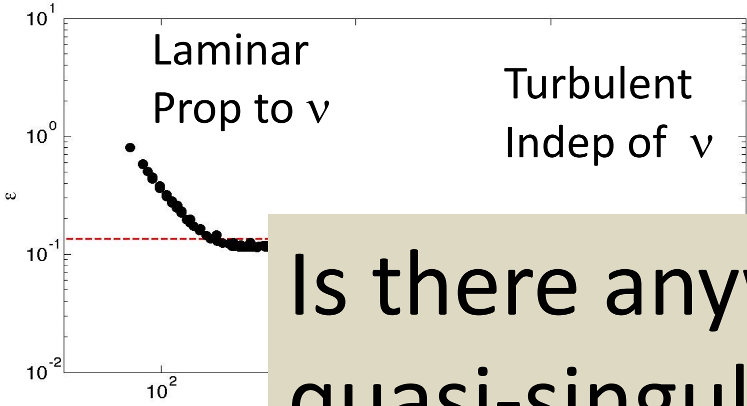

”...in three dimensions a mechanism for complete dissipation of all kinetic energy, even without the aid of viscosity, is available.”

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Sreenivasan (2006)

Is there anyway we can capture quasi-singularities and weak solutions?

$= D(u) - \nu(\nabla \mathbf{u})^2$

Re

inearity (2000),

Inertial dissipation= limit of local energy transfers

$$D(u) = \lim_{\ell \to 0} \frac{1}{4} \int_{r \le \ell} d^3r \; \nabla\phi_\ell(r) \cdot \delta u_r |\delta u_r|^2$$

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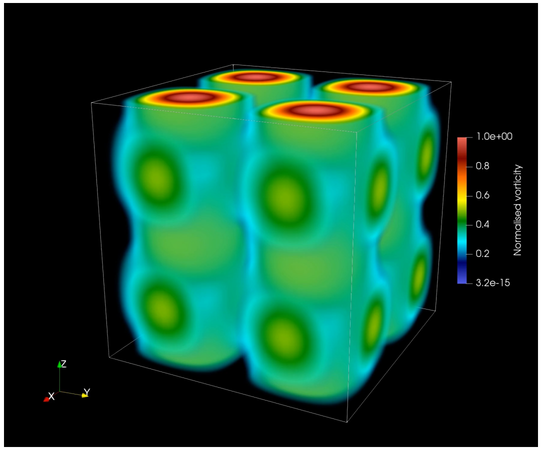
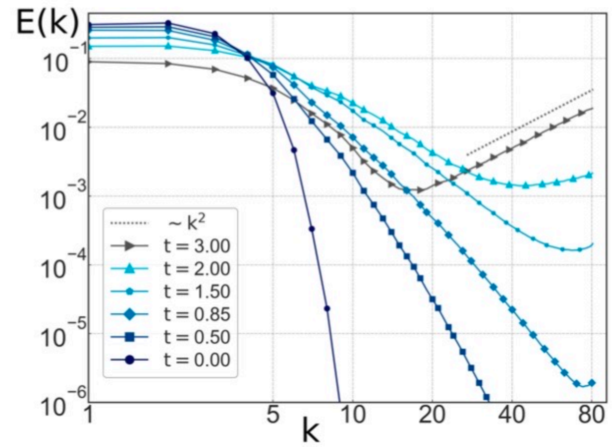
Can we just run an Euler/ NSE equation using DNS?



Courtesy J.I. Polenco

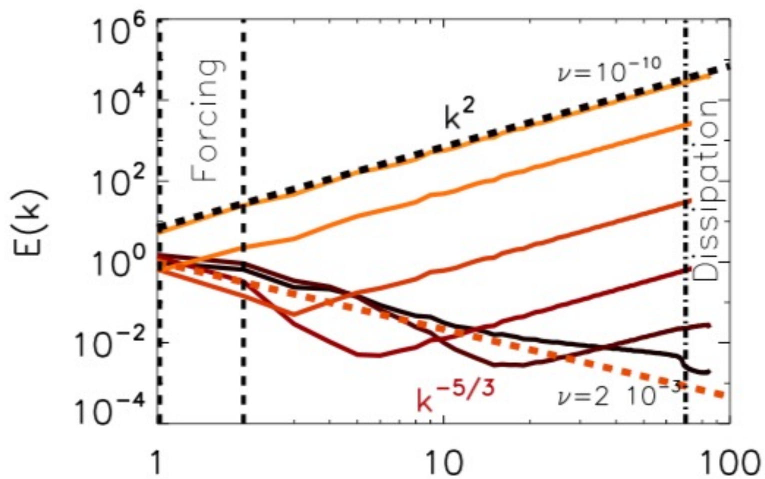
Energy equipartition
k^2 spectrum

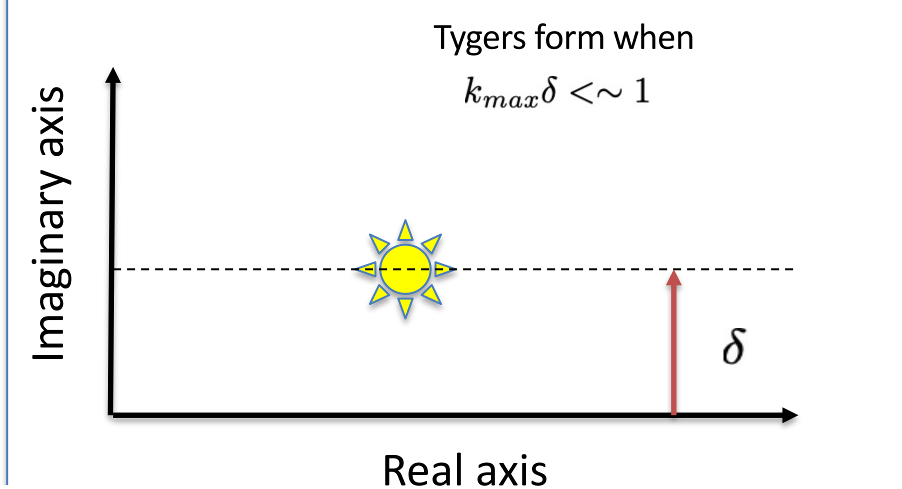


Cichowlas et al, PRL, 2005

Due to thermalization, any truncated Euler equation ends in equilibrium state with k^2 spectrum

Alexakis&Brachet, 2019





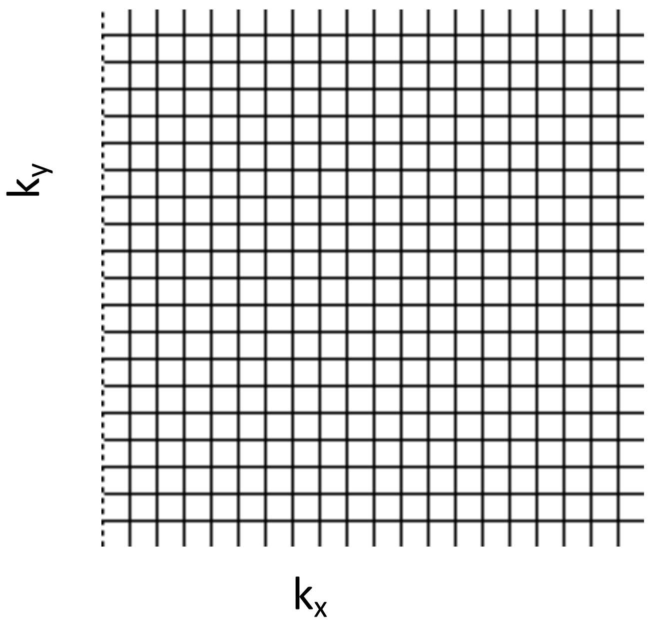
« Thermalization » also happens for Navier-Stokes

Real axis
Ray et al, 2011

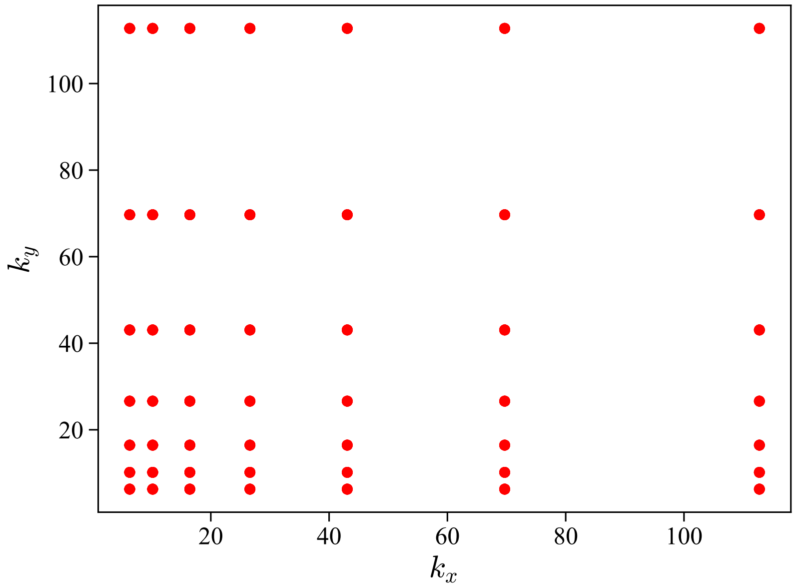
From DNS to log-lattices

$$\partial_t \hat{u}_i = P_{ij}\left(-ik_q \hat{u}_q * \hat{u}_j + \hat{f}_j\right) - \nu_r k^2 \hat{u}_i,$$

Fourier grid



Log grid



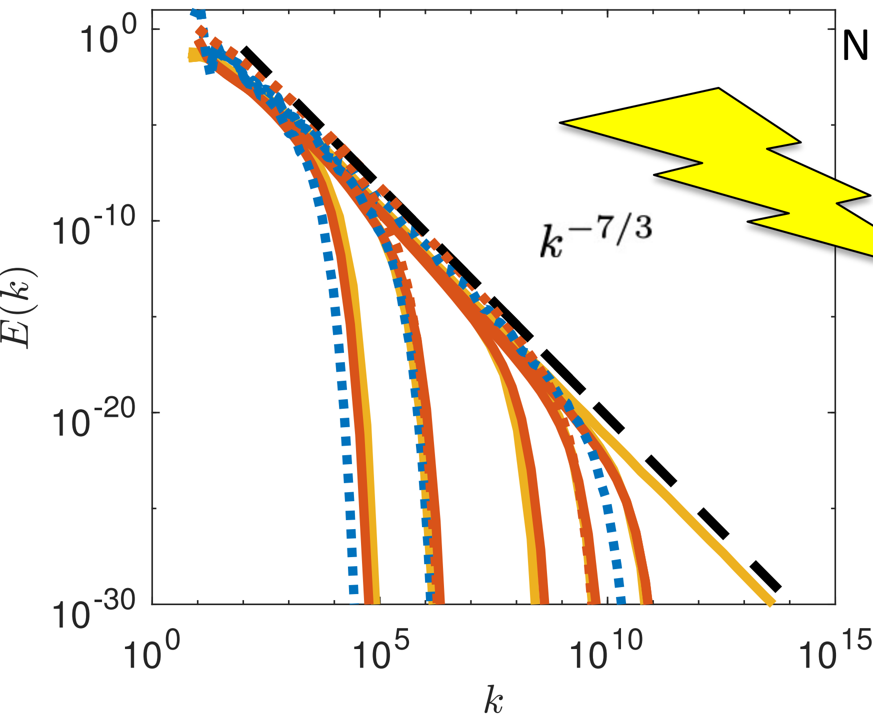
$u * v \qquad m = n + q,\ (\mathrm{m,n,q}) \in \mathbb{Z}^3$

$$\lambda^m = \lambda^n + \lambda^q,\ (\mathrm{m,n,q}) \in \mathbb{Z}^3$$

$$\lambda = 2 \quad (z = 3^D).$$
$$\lambda = \sigma \approx 1.325 \quad (z = 12^D)$$
$$\lambda = \Phi \approx 1.618 \quad (z = 6^D)$$
$$1 = \lambda^b - \lambda^a, 0 < a < b$$

Campolina&Mailybaev, 2018

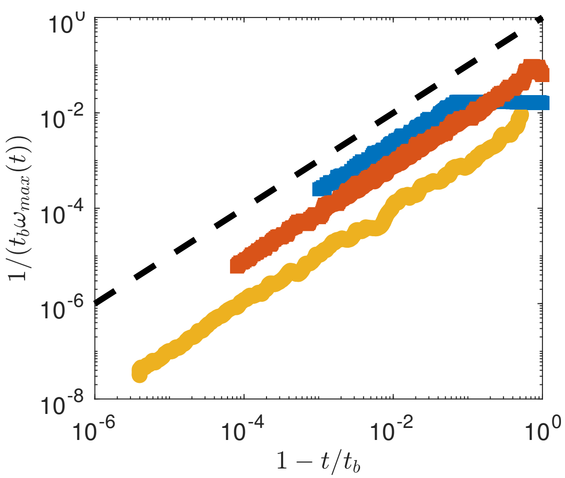
LL-Euler, adaptative grid, Eqyilibrium reversible

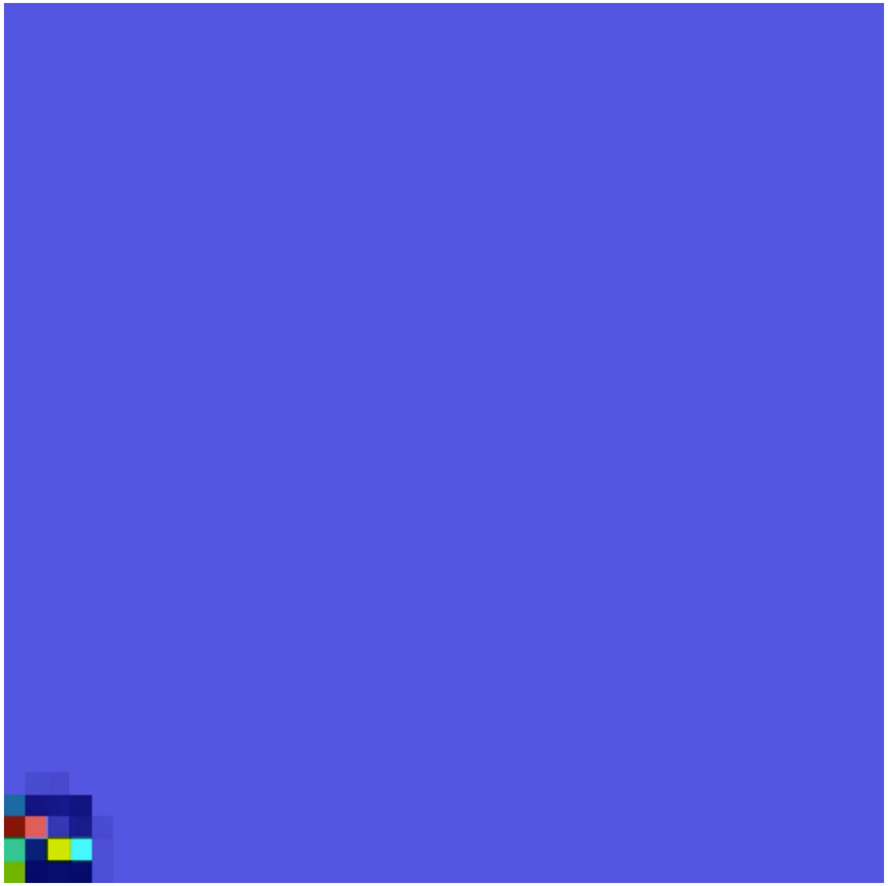


Non-dissipative Singularity !

Self-similar blow up in finite time

(h=2/3)





Campolina&Mailybaev, 2018

A new protocol to capture weak solutions

Traditional way: $\partial_t \int \frac{u^2}{2} dx^3 = \int F.u\, dx^3 - \nu \int (\nabla u)^2\, dx^3 + \int D(u) d^3x$?

We **impose F at given viscosity**
We measure energy input needed to maintain E statistically constant $\qquad \epsilon_I(\nu)$

Alternative way using RNS

We **impose F at given E**
We measure the viscosity needed to sustain constant E $\qquad \nu(E)$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = -\nabla p + \nu_{\mathrm{r}} \nabla^2 \mathbf{u} + \mathbf{f},$$

Gallavotti, PRL, 1996

Control parameter: $\mathcal{R}_r = \frac{1}{\mathcal{E}} = \frac{F}{E k_I}$

If No anomaly $\quad \nu_{\mathrm{r}}[\mathbf{u}] = \frac{\int_{\mathcal{D}} \mathbf{f} \cdot \mathbf{u}\, d\mathbf{x}}{\int_{\mathcal{D}} (\nabla \times \mathbf{u})^2\, d\mathbf{x}}.$

For weak solutions $\quad \nu_r = \frac{< f \cdot u > - D(u)}{< \omega^2 >}$

Navier-Stokes and reversibility

$$\partial_t u + u\nabla u = -\frac{1}{\rho}\nabla p + \nu\Delta u + F$$

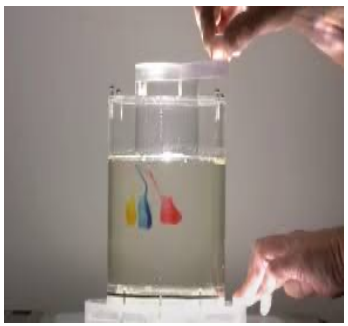
Creeping Flow

Reversible Equilibrium

F->-F

u->-u

E=0



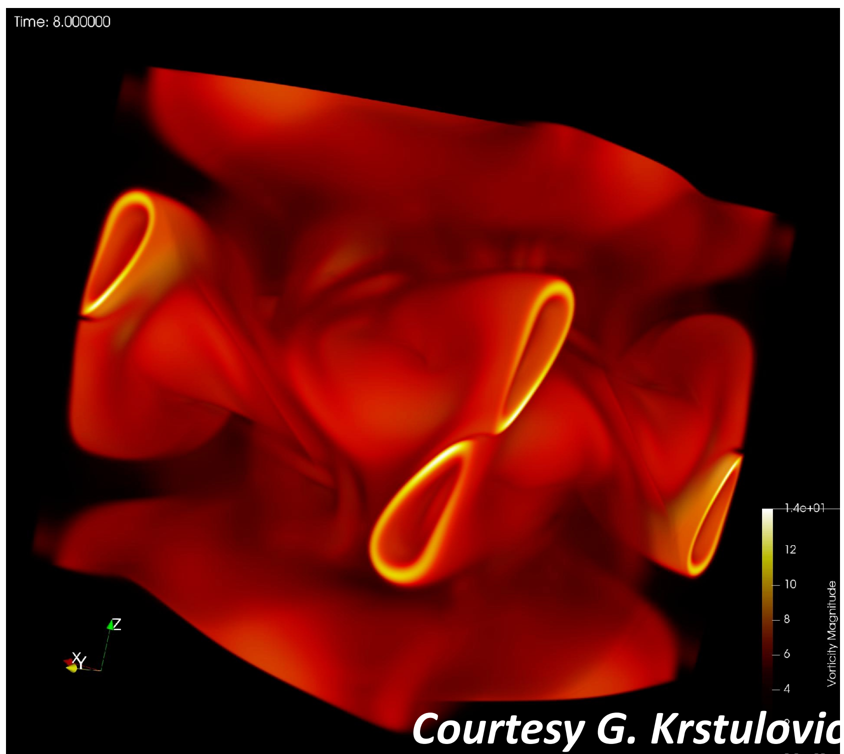
Reversible NS

Reversible Non-Equilibrium

t->--t

u->-u

E=cte



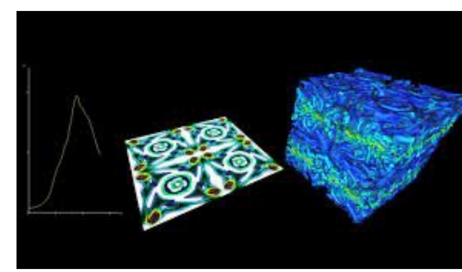
Euler flow

Reversible Equilibrium

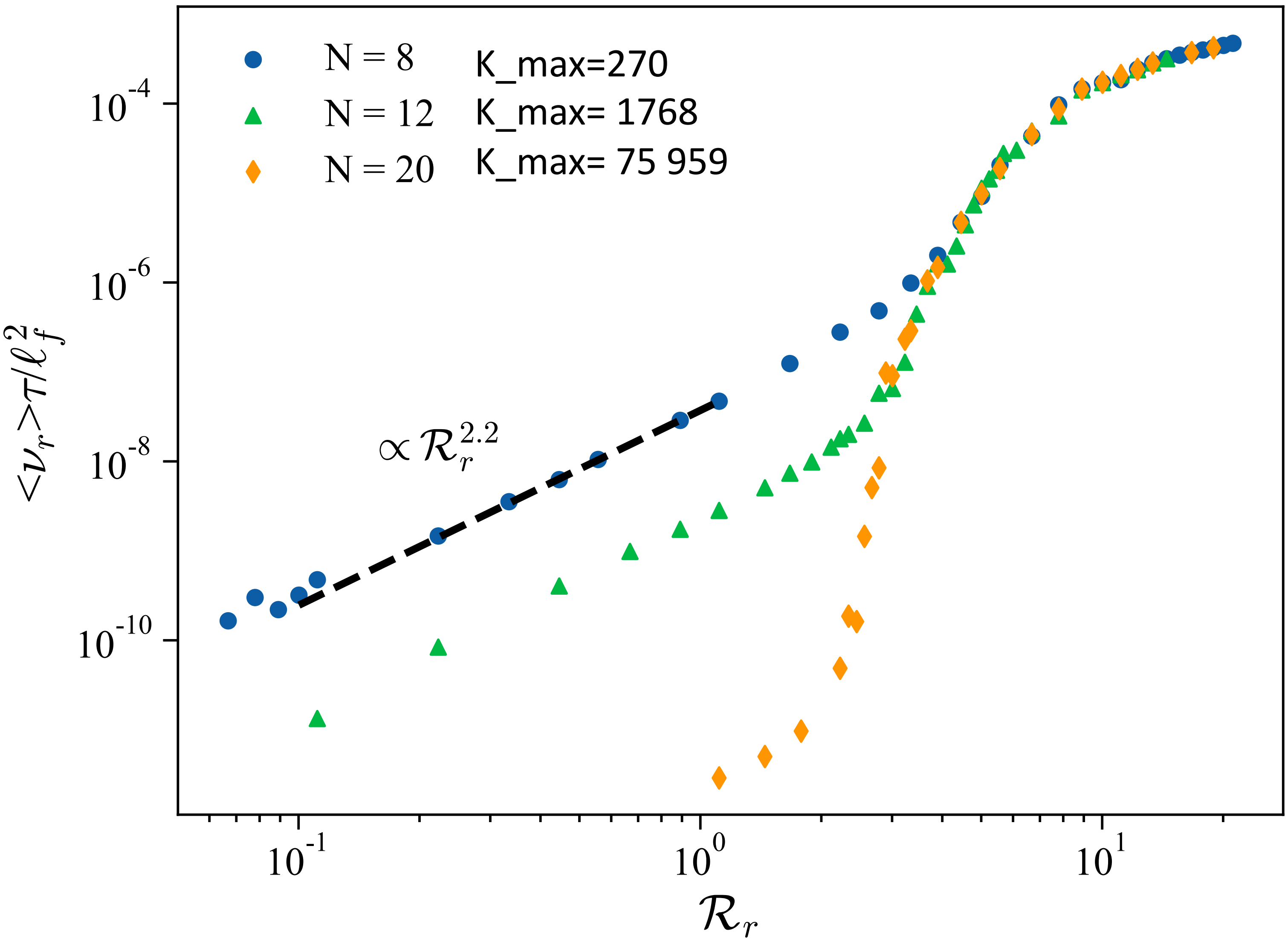
t->--t

u->-u

E=cte

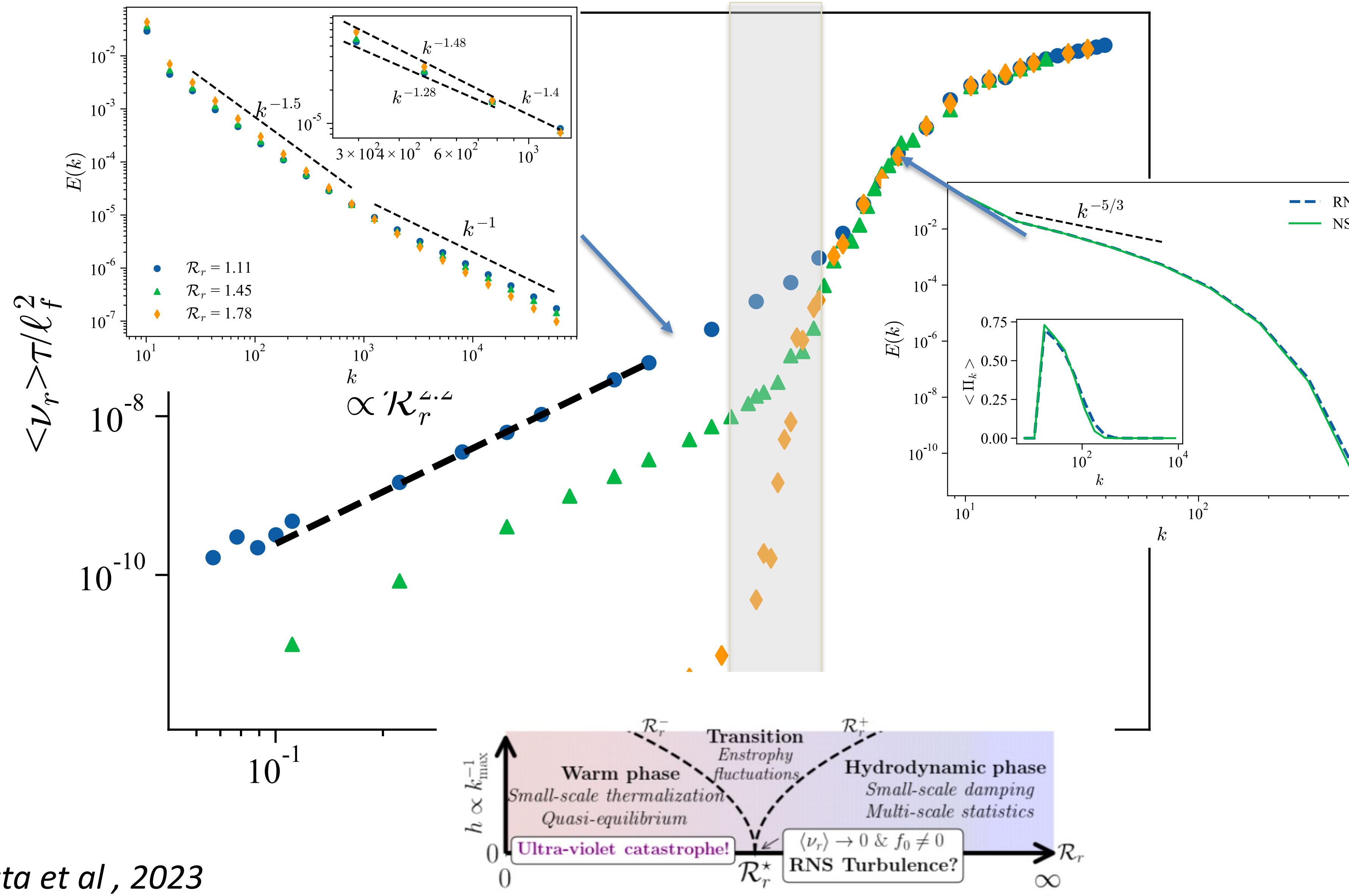


Log-Lattice reversible Navier-Stokes



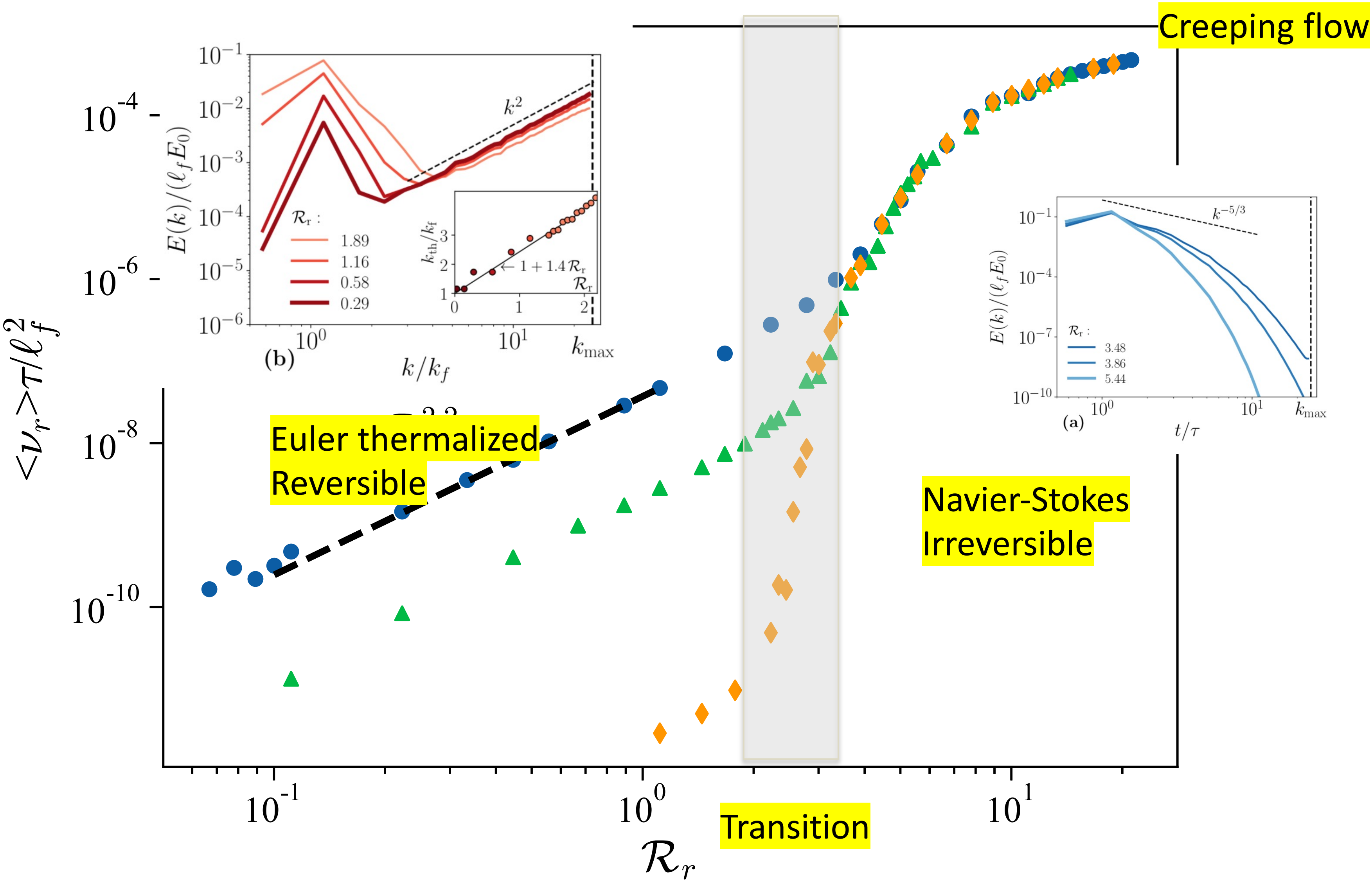
Costa et al , 2023

Log-Lattice reversible Navier-Stokes



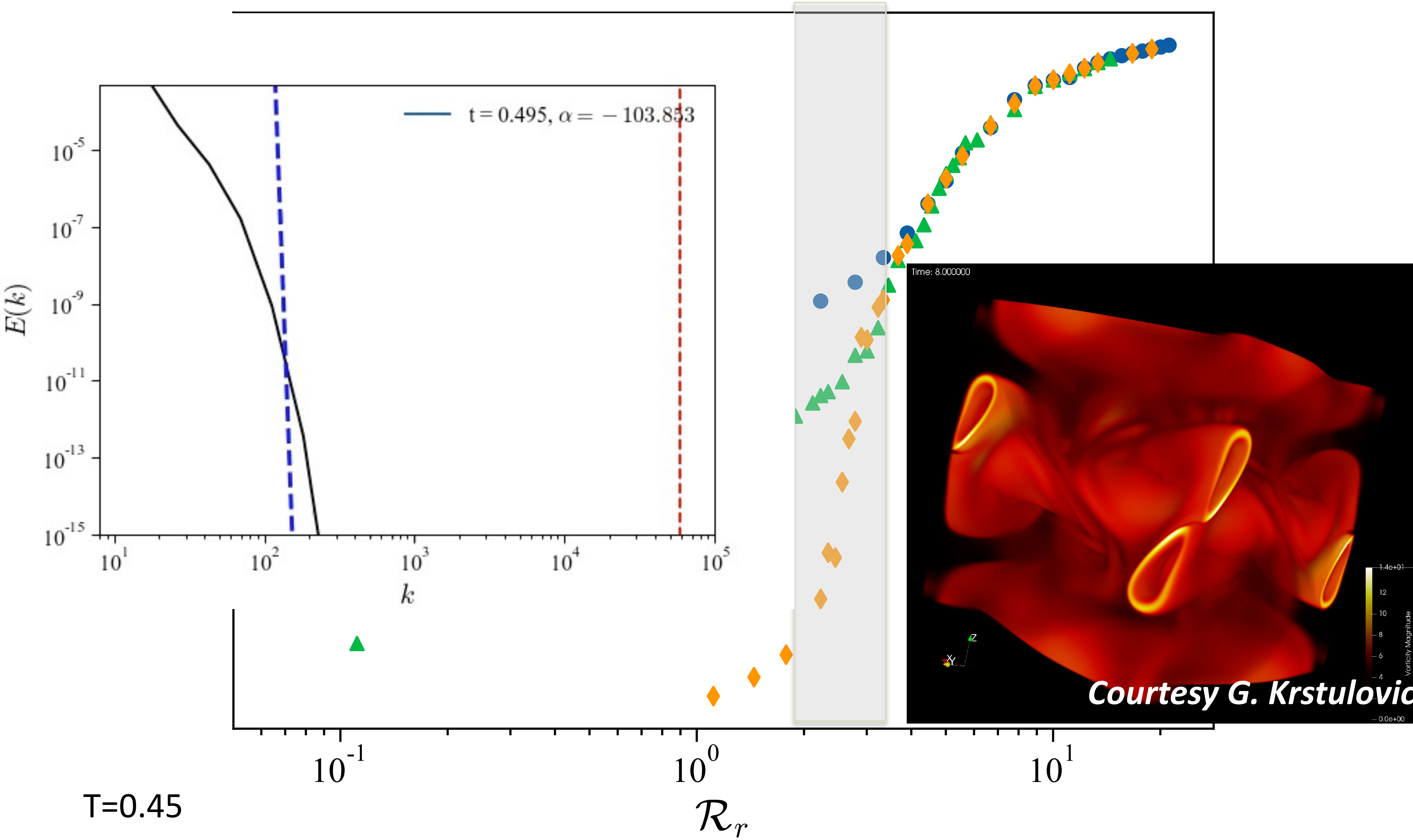
Costa et al , 2023

DNS reversible Navier-Stokes



Costa et al , 2023

What is happening in the transition zone?

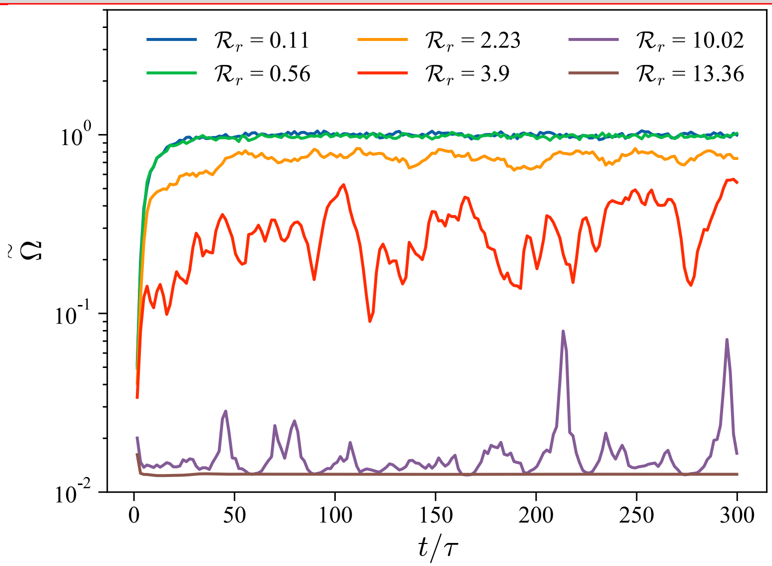


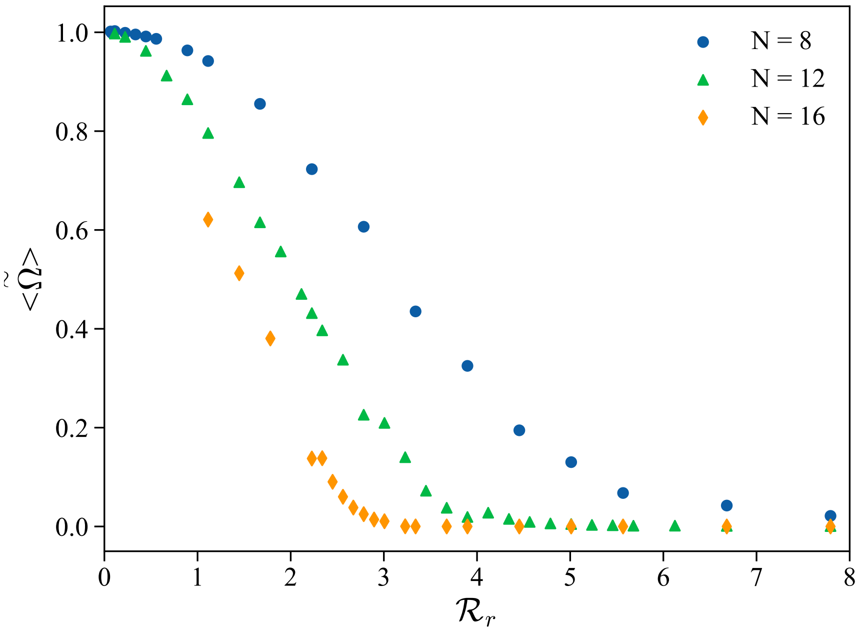
T=0.45

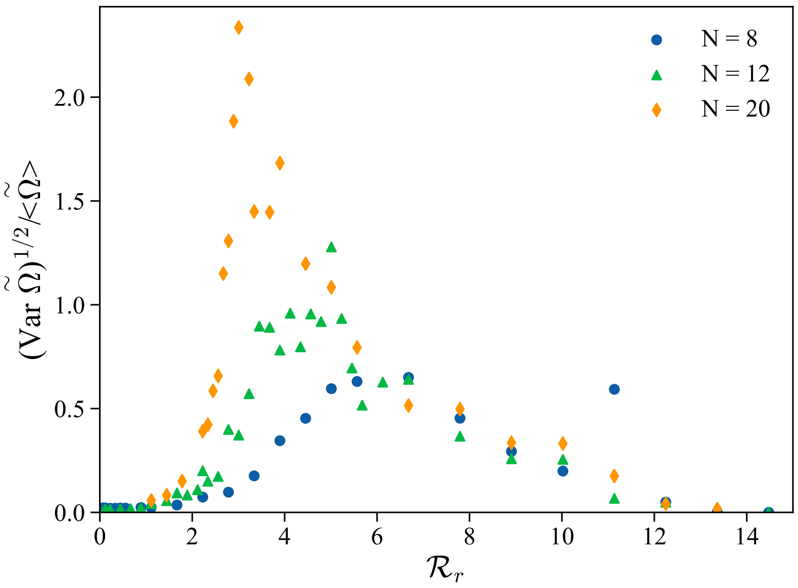
Costa et al , 2023

Phase transition in LL-RNS

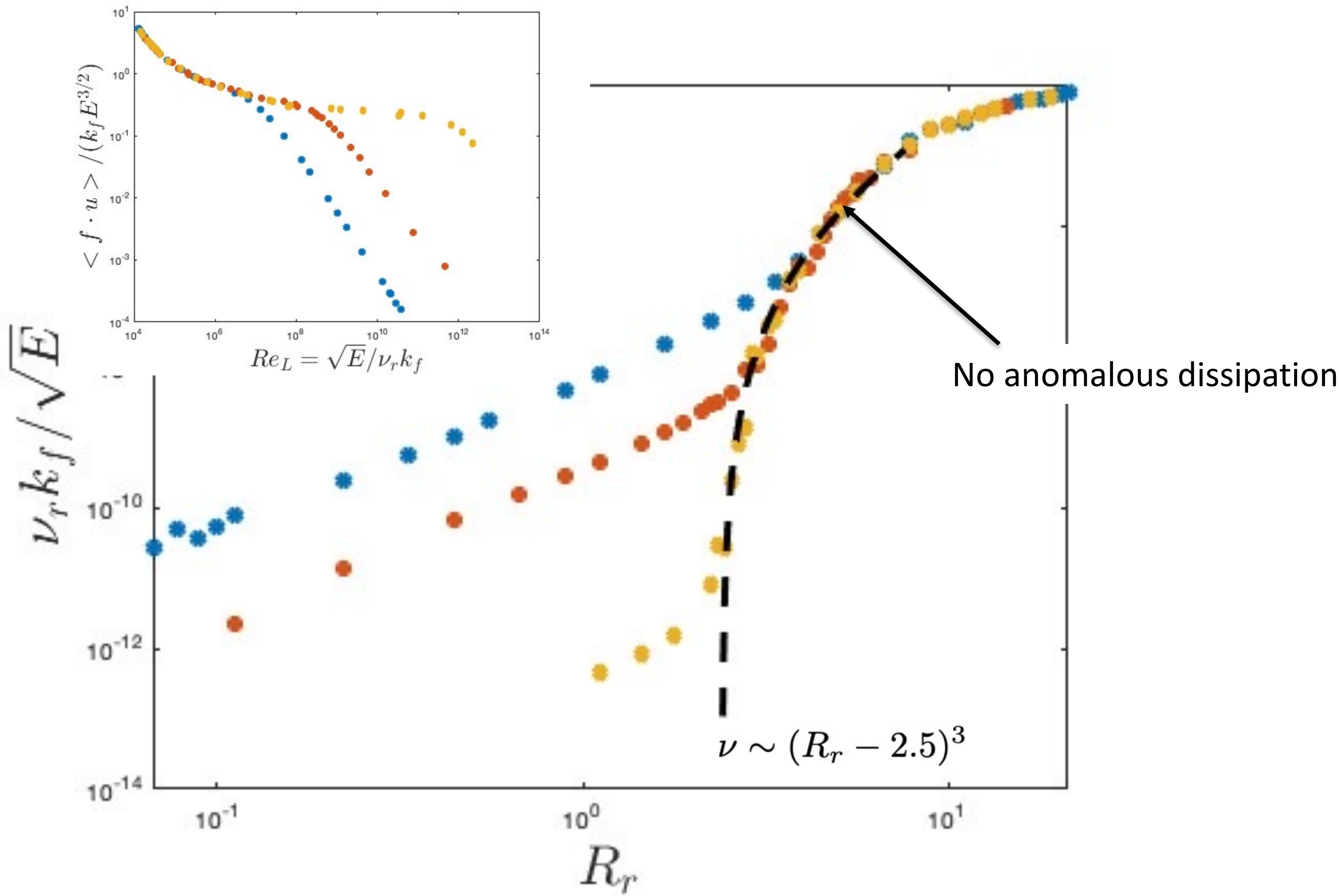
There is a second order phase transition
Following mean field exponents





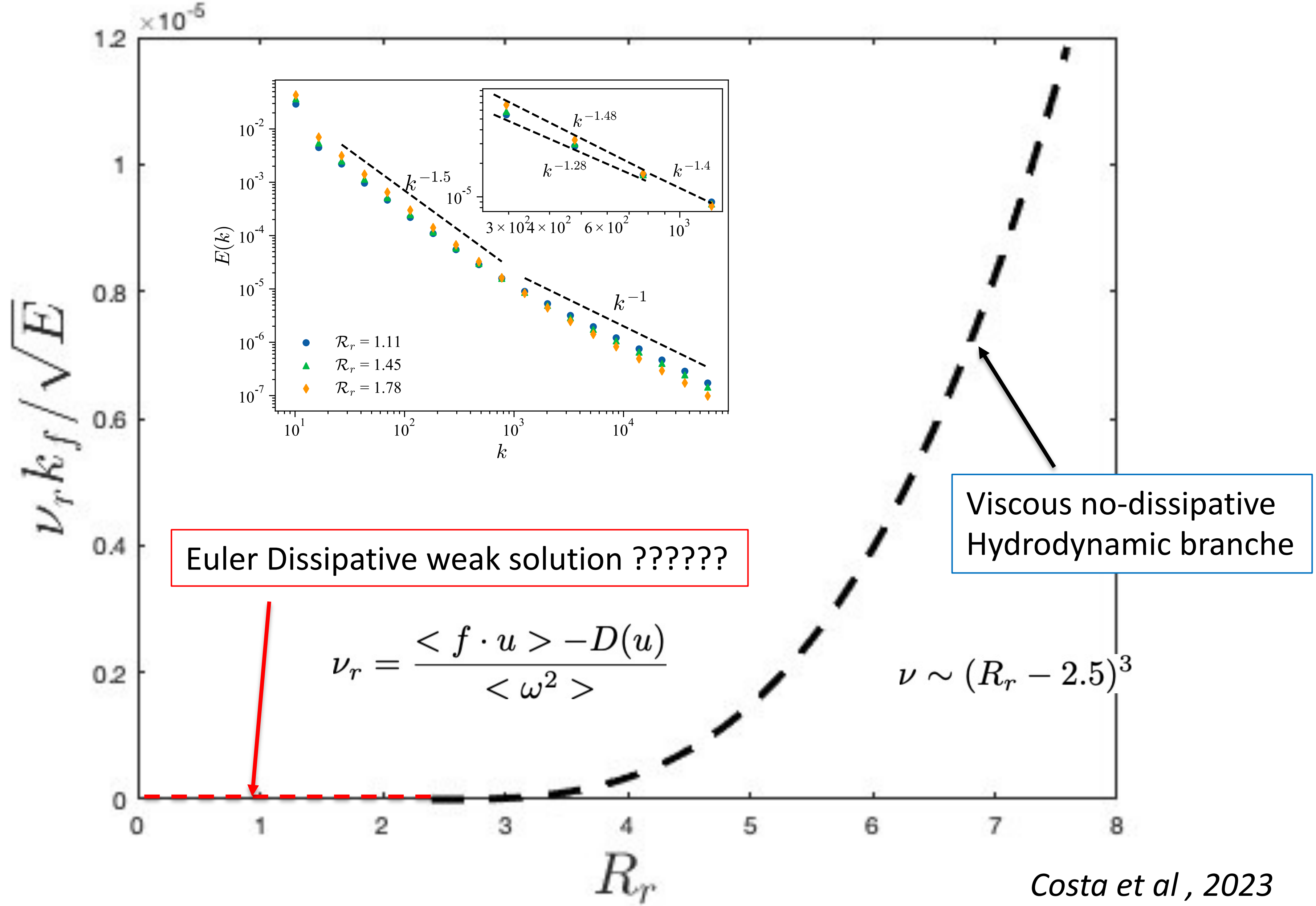


Is there anomalous dissipation?



Costa et al , 2023

What is the limit of infinite resolution?



Costa et al , 2023

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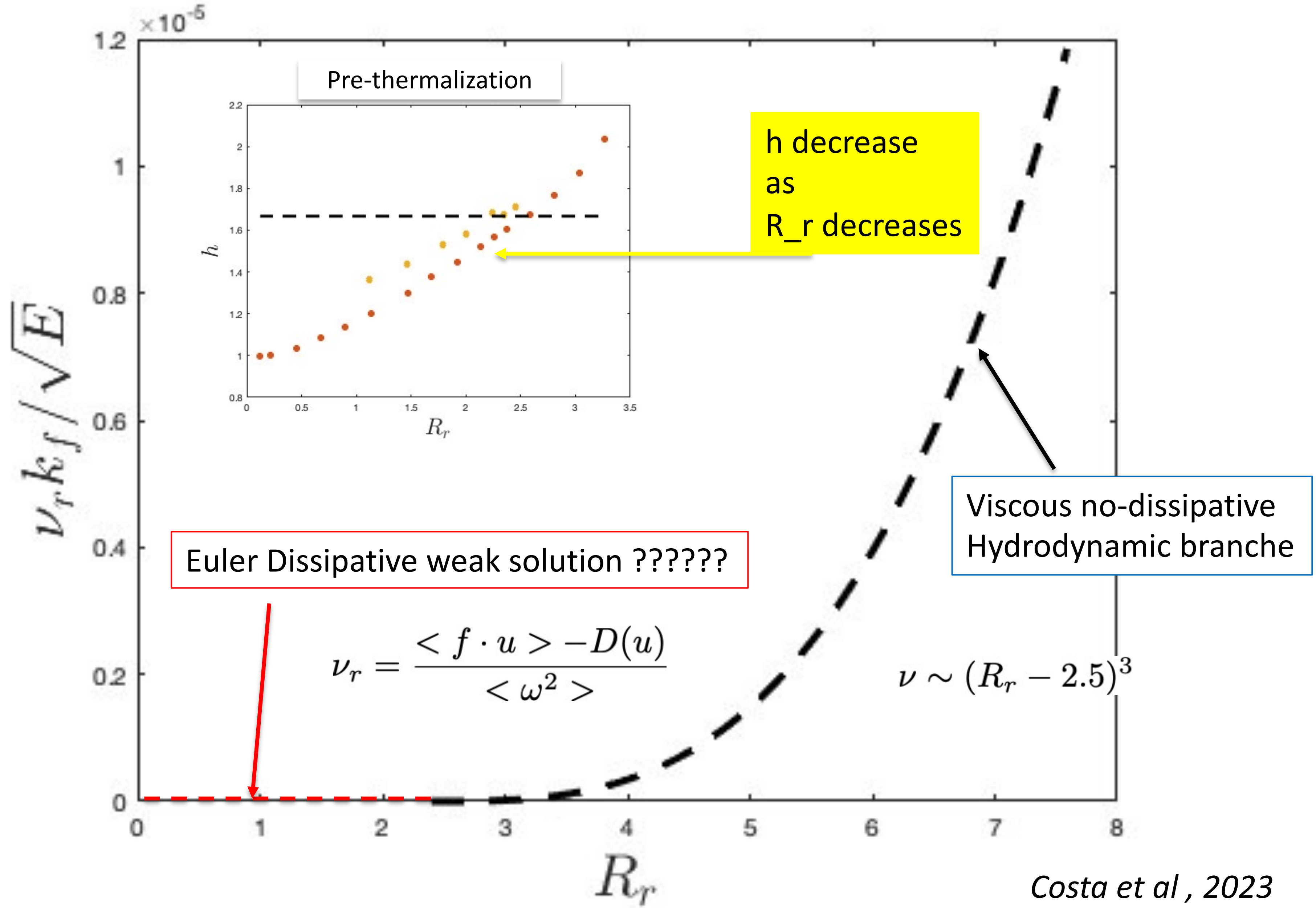
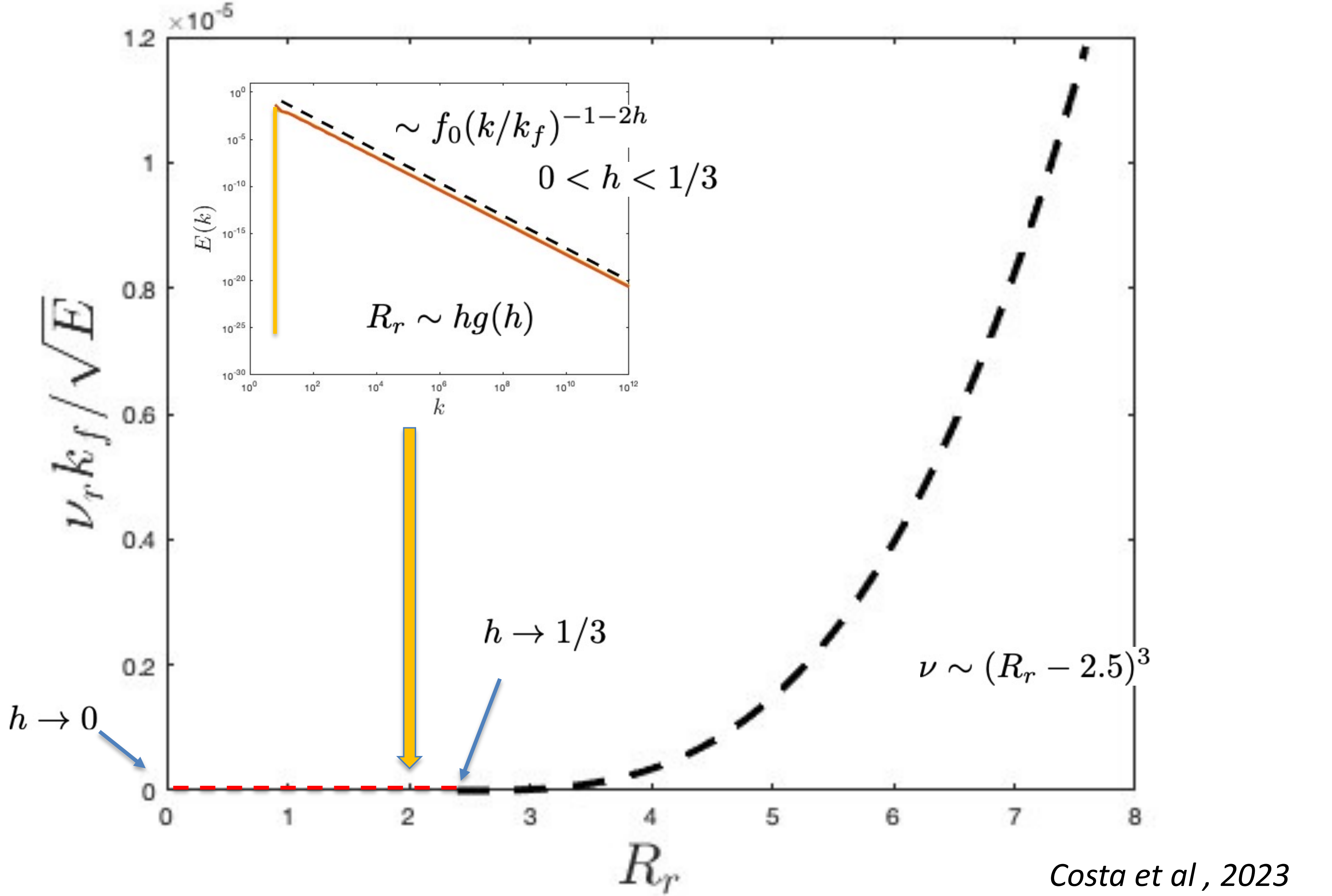


Costa et al , 2023

Conjecture



Costa et al , 2023

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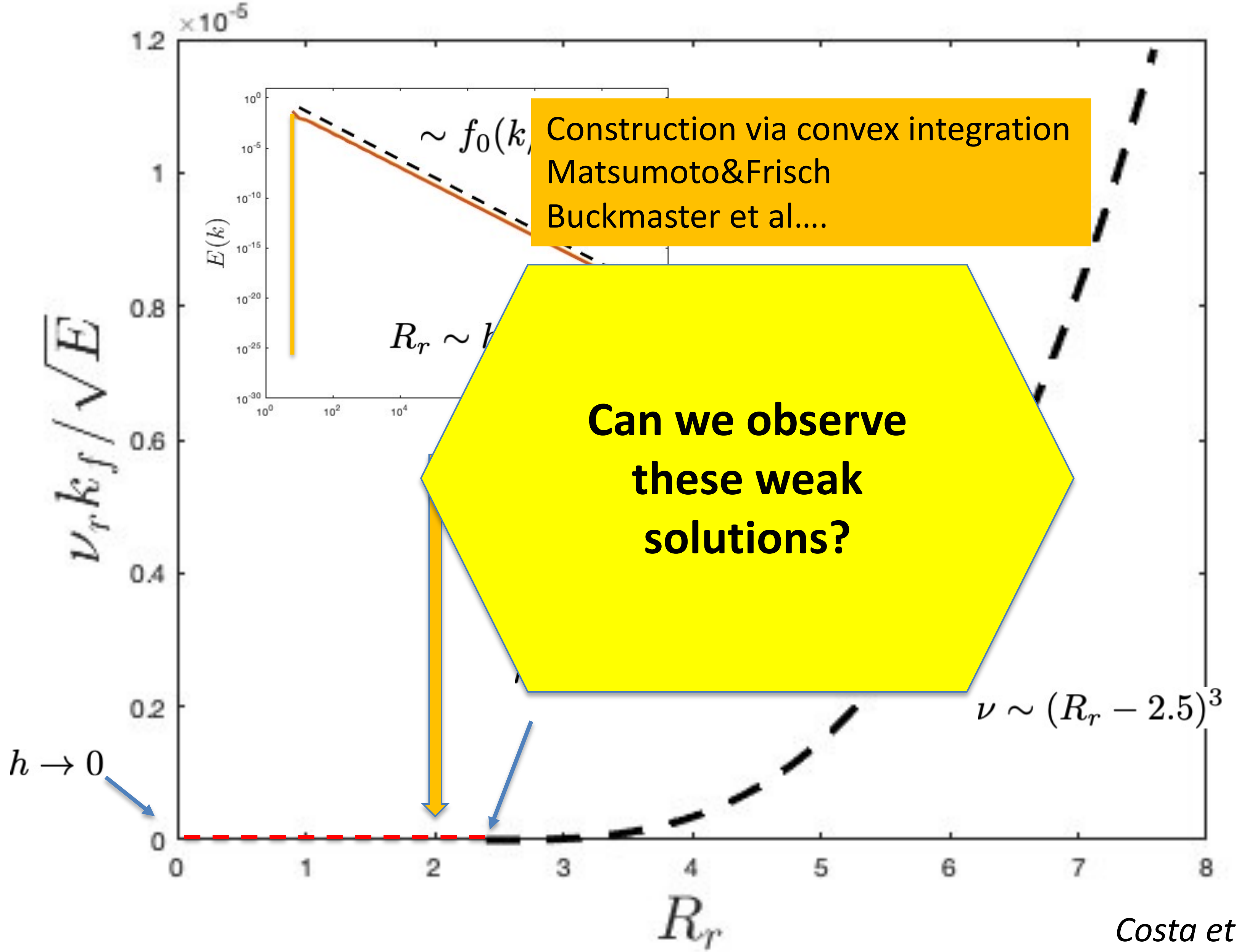
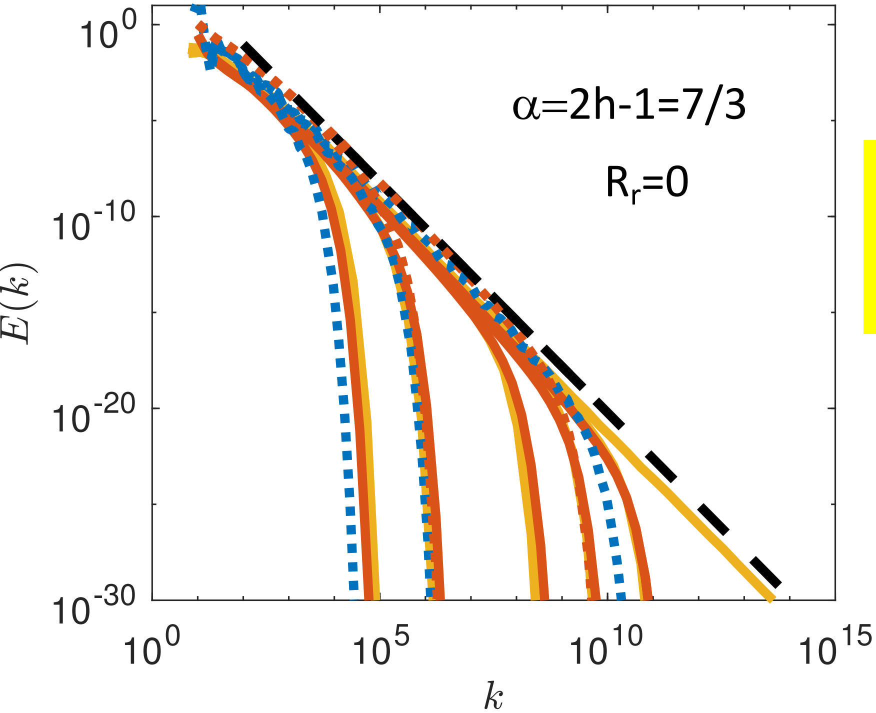
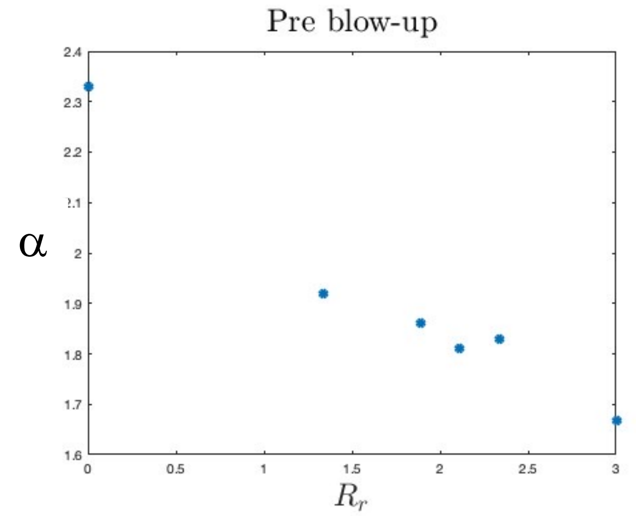


Costa et al , 2023

General case with adaptative grid

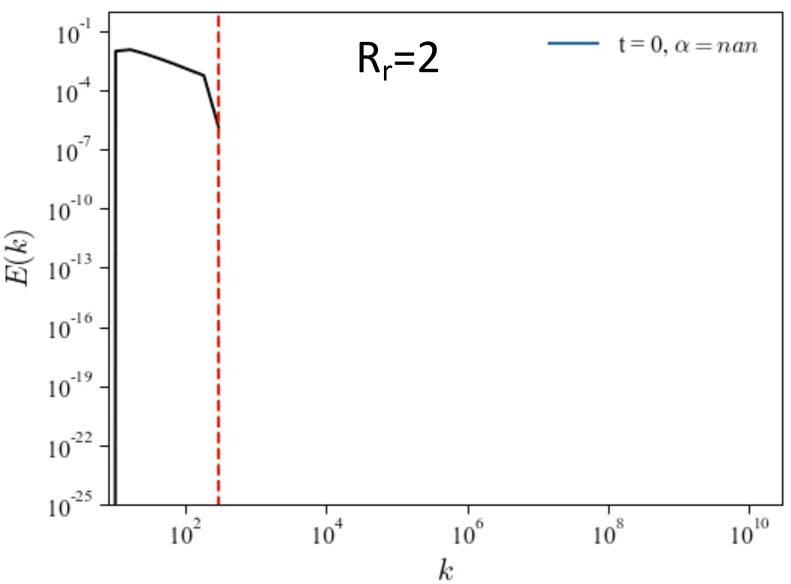


Blow-up with a different exponent



Non-dissipative Singularity !

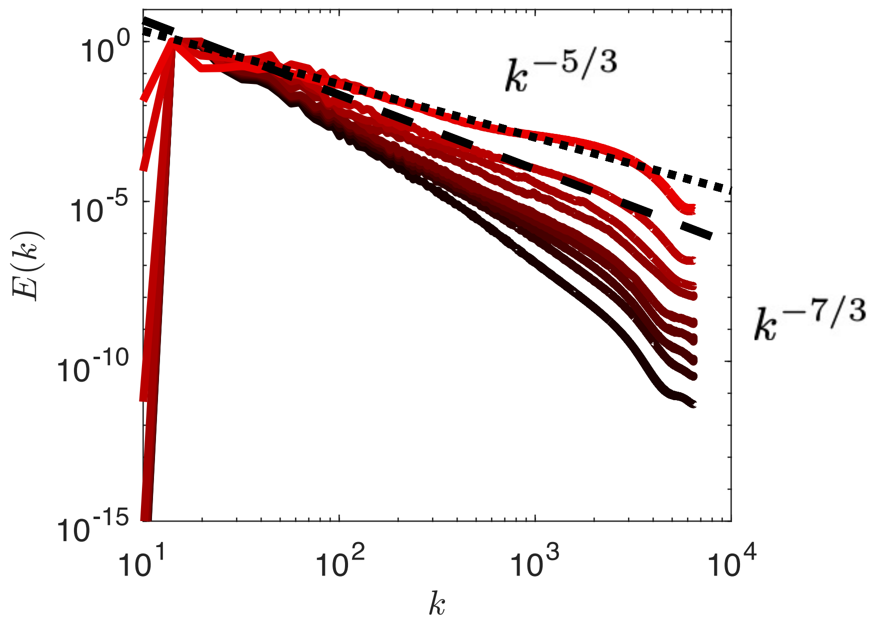
(h<1/3)

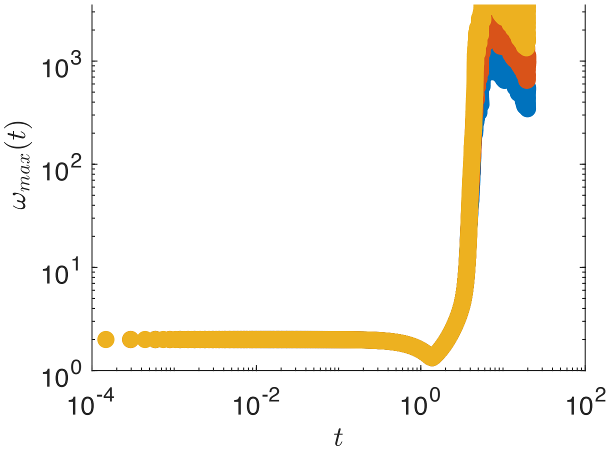


Costa et al, 2023

What is happening when k_max is reached?

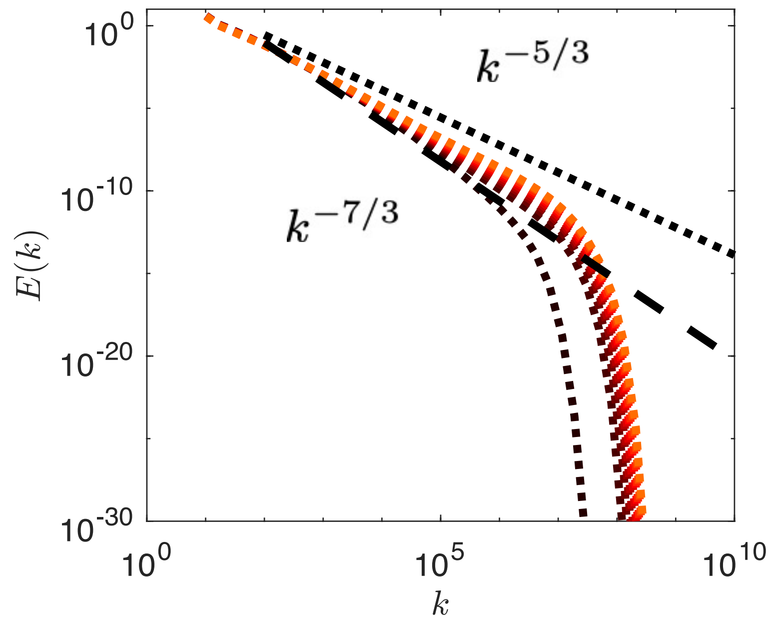
DNS

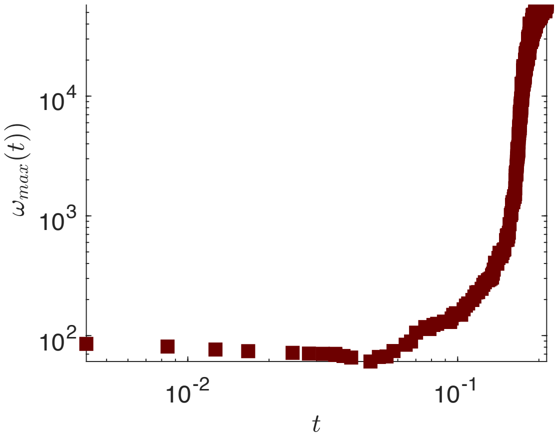




Fehn et al JFM, 2022

LogLattice





Barral et al, 2023

Conjecture (2): scenario to obtain weak solution

Anomalous spectral laws in differential models of turbulence

Thalabard et al, 2015

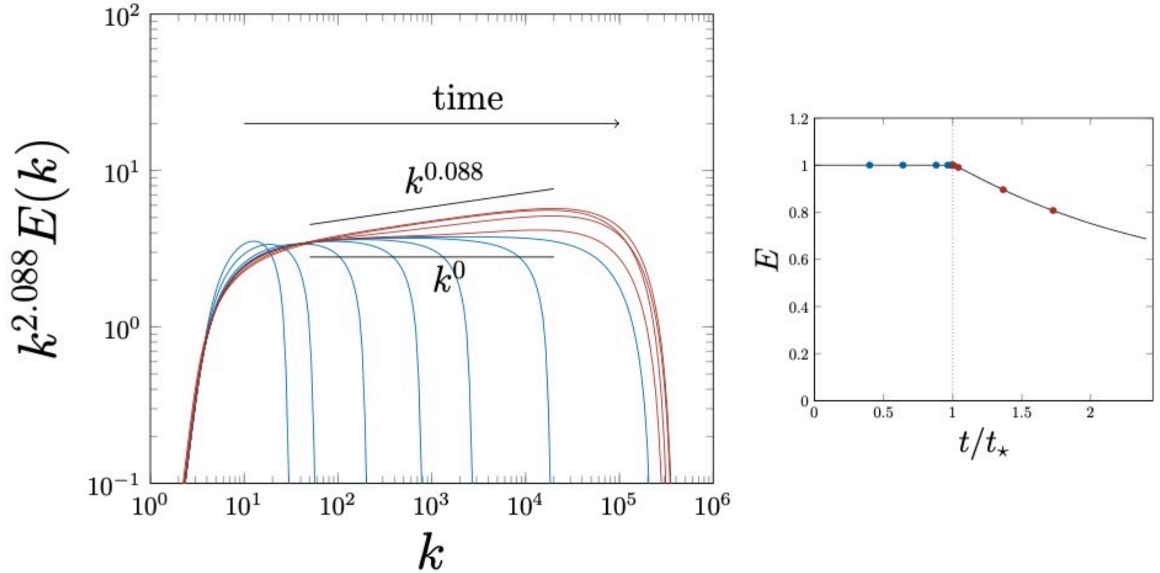


Figure 4. Left: Compensated MHD spectra obtained numerically from the second-order differential model at different times: earlier than $t_\star$ (in blue) and later than $t_\star$ (in red). The spectra are normalized by the total energy. The times are reported by the dots on the right inset. The blue dots correspond to $t/t_\star - 1 \simeq -6 \cdot 10^{-1}$, $-4 \cdot 10^{-1}$, -10^{-1}, $-4 \cdot 10^{-2}$, -10^{-2}, $-2 \cdot 10^{-3}$, 0; the red dots to $t/t_\star - 1 \simeq 4 \cdot 10^{-3}$, $4 \cdot 10^{-2}$, -10^{-1}, $4 \cdot 10^{-1}$, $7 \cdot 10^{-2}$. The right inset shows the time evolution of the energy. $t_\star \simeq 8.3 \cdot 10^{-3}$ marks the onset of energy dissipation. The figure is made using the data obtained by E.Buchlin and previously reported in [23] (in a different form).

Forward self-similar Blow-up in finite time → Backward self-similar front resulting in

$k^{-\alpha}$ (h>1/3, not dissipative) $k^{-\beta}$ (h<1/3, dissipative)

Bridging classical and quantum turbulence

Observations

GP Equations

$$i\varepsilon\psi_t = -\frac{\varepsilon^2}{2}\Delta\psi + h(|\psi|^2)\psi, \quad x \in \mathbb{R}^d,$$
$$\psi(\cdot,0) = \psi_I(\cdot)$$

Madelung

Quantum Hydrodynamic equations

$$\rho_t + \operatorname{div}(\rho u) = 0, \qquad t > 0,\ x \in \Omega \subseteq \mathbb{R}^d,$$
$$(\rho u)_t + \operatorname{div}(\rho u \otimes u) + \nabla P(\rho) = \frac{\varepsilon^2}{2}\rho\nabla\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right),$$

Blow-up in finite time with boundaries

ON THE BLOWING UP OF SOLUTIONS TO QUANTUM HYDRODYNAMIC MODELS ON BOUNDED DOMAINS *

IRENE M. GAMBA†, MARIA PIA GUALDANI‡, AND PING ZHANG§

Abstract. The blow-up in finite time for the solutions to the initial-boundary value problem associated to the multi-dimensional quantum hydrodynamic model in a bounded domain is proved. The model consists on conservation of mass equation and a momentum balance equation equivalent to a compressible Euler equations corrected by a dispersion term of the third order in the momentum balance. The proof is based on a-priori estimates for the energy functional for a new observable constructed with an auxiliary function, and it is shown that, under suitable boundary conditions and assumptions on the initial data, the solution blows up after a finite time.

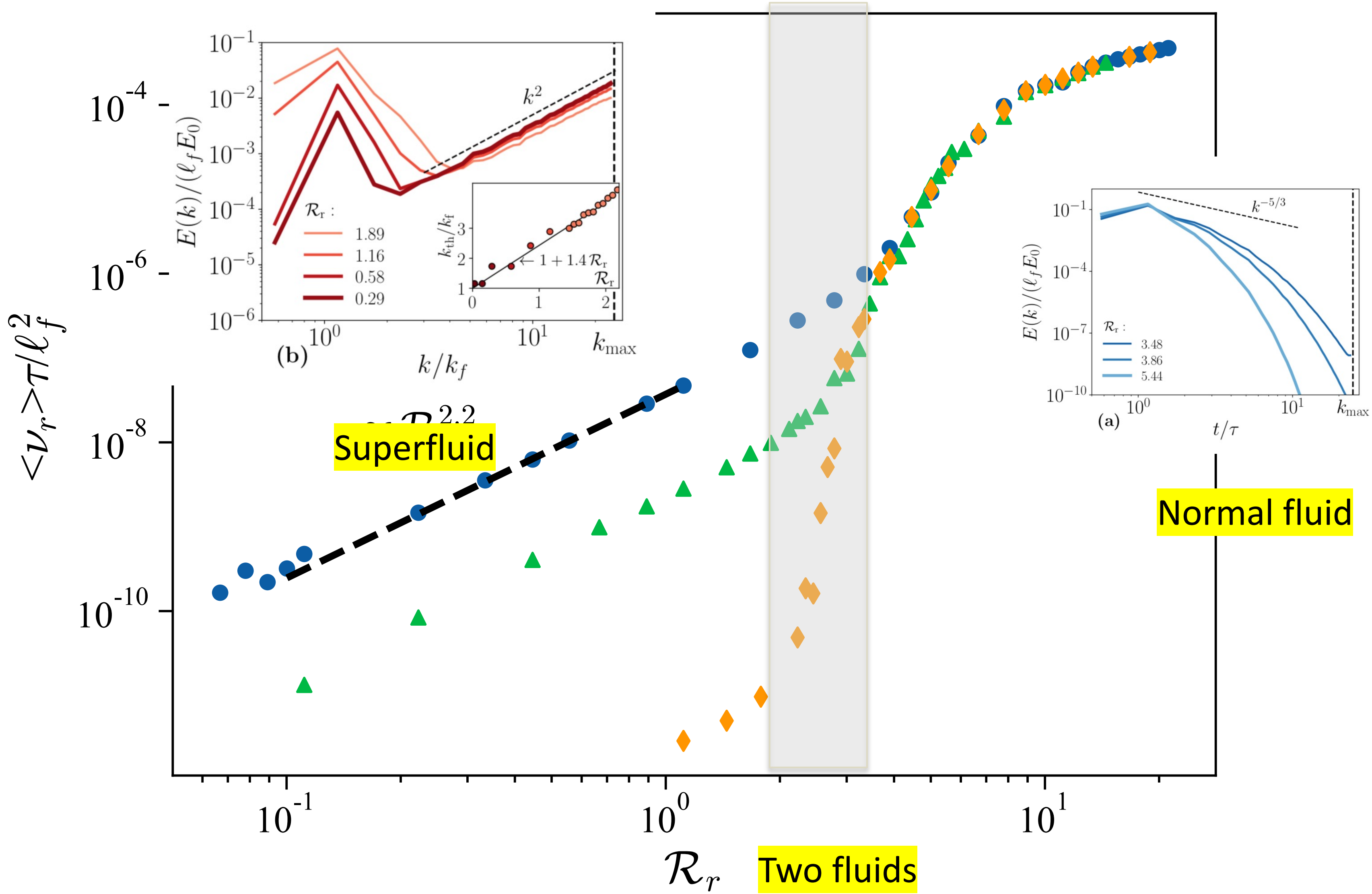
A R-quantum hydro model as a toy model for SF?

$$\rho_t + \operatorname{div}(\rho u) = 0, \qquad t > 0,\ x \in \Omega \subseteq \mathbb{R}^d,$$

$$(\rho u)_t + \operatorname{div}(\rho u \otimes u) + \nabla P(\rho) = \frac{\varepsilon^2}{2}\rho\nabla\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right) + \nu_r \Delta u + F$$

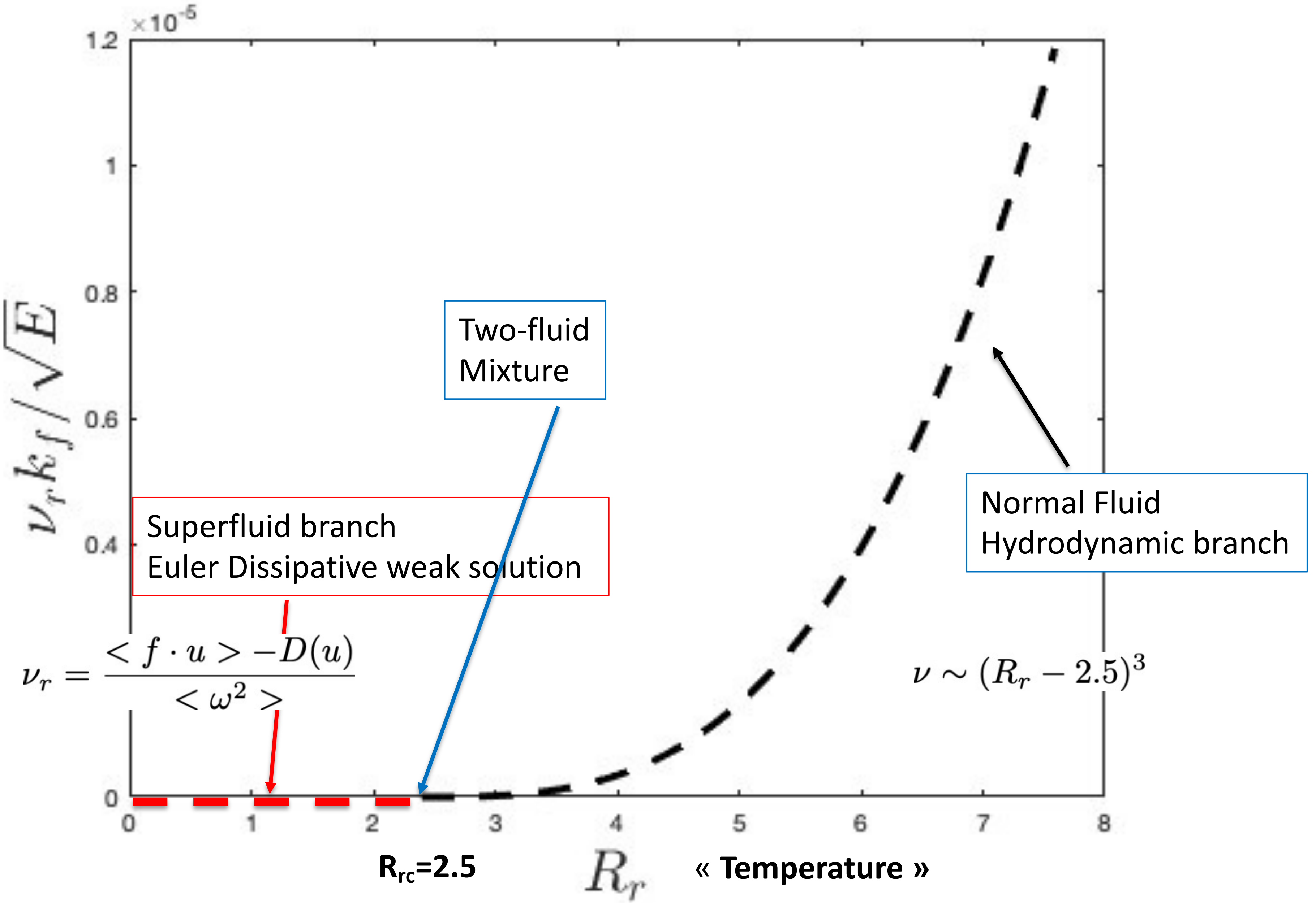
Advantage:
*We naturally have the phase transition
*Two fluid phase: « superfluid »= weak solution; « normal » = viscous solution
« sound »=tygers; coupling automatic between the two phases
*We can have finite dissipation by weak solutions like observed in superfluids
*Forcing is easy

Finite resolution



Costa et al , 2023

Infinite resolution



$$\nu_r = \frac{< f \cdot u > - D(u)}{< \omega^2 >}$$

Summary and Perspectives

Using log-lattices, we have identified in RNS a second order transition as a function of efficiency where the fluid changes :
from ***viscous Navier-Stokes*** flow to ***thermalized Euler Flow***

In the infinite resolution limit, the thermalized Euler Flow could tend to ***dissipative weak solutions of Euler equation*** of finite energy

- Can we get the possible weak dissipative solutions of LL-RNS?
- Are these things true for DNS-RNS?
- Is RQNS a good model of superfluid? What kind of structures are observed during the blow-up phase and normal fluid building?(role of quantum pressure)