Types of quantum turbulence

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Aim

- To introduce and review some of the main properties of **quantum turbulence**
- To distinguish various limiting types of quantum turbulence that have been identified and compare them to ordinary (classical) turbulence:
  - Thermal counterflow
  - Kolmogorov
  - Vinen
  - Weak / strong

Contexts

Superfluid ^{4}He, ^{3}He, atomic Bose-Einstein condensates (which provide most of the information about turbulence) and also polariton condensates, interior of neutron stars, bosonic dark matter, etc.

Two main properties

- **Superfluidity** (flow without viscous dissipation)
  Not as significant as one may think: there are other ways to dissipate kinetic energy, e.g. scattering of thermal excitations ("mutual friction"), density waves in ^{4}He, ^{3}He and atomic BECs, and excitation of Caroli-Matricon bound states in vortex cores in ^{3}He (Silaev, PRL 2012)
- **Quantised vorticity** (Onsager, Feynman)
  Significant: in classical fluids vorticity is **continuous** and **unconstrained**, in quantum fluids it is **discrete** and **constrained**

Quantised circulation

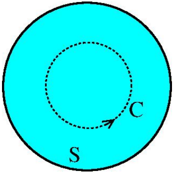
A quantum fluid is ruled by a complex wavefunction $\psi(\boldsymbol{r}, t)$

$$\psi(\boldsymbol{r}, t) = |\psi(\boldsymbol{r}, t)| e^{i\phi(\boldsymbol{r}, t)}$$

Quantum mechanics $\Rightarrow$ $\boldsymbol{v} = (\hbar/m)\nabla\phi$

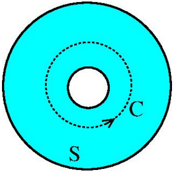
Consider the circulation of the velocity $\boldsymbol{v}$ along a closed path C in a region S which is simply-connected:

$$\Gamma = \oint_C \boldsymbol{v} \cdot d\boldsymbol{r}$$



Apply Stokes Theorem, use $\nabla \times \boldsymbol{v} = 0$, and conclude $\Gamma = 0$

If S is multiply-connected, Stokes Theorem does not apply, and



$$\Gamma = \frac{\hbar}{m} \oint_C \nabla\phi \cdot d\boldsymbol{r} = q\kappa$$

$\kappa = h/m$ (quantum of circulation)
$q = 1, 2, 3, \cdots$ (winding number)

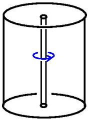
Hence the velocity around the hole at distance r is

$$v = \frac{q\kappa}{2\pi r}$$

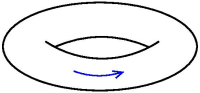
(cf. fluid dynamics textbooks)

Quantised circulation

A multiply-connected region can be created externally



(vibrating) wire
(Vinen 1961*,
Hough & Zieve, PRB 2001)

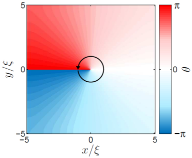


torus trap

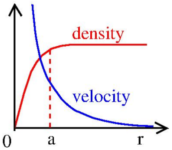
* the first evidence of **macroscopic quantum effect**
(before superconductivity)

Quantised circulation

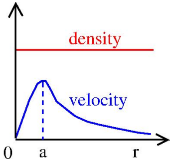
Vortex nucleation involves the spontaneous creation of a hole



Phase



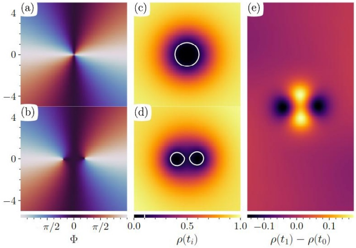
Velocity and density



cf. Rankine shape of classical vortex

In this case $q = 1$ is favoured:

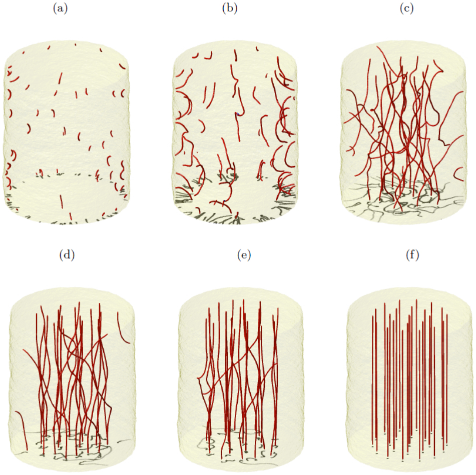
- Energy: for the same angular momentum, two $q = 1$ vorts have less energy than one $q = 2$ vort
- Dynamics: superradiant bound state inside vort core



(Patrick & al, PR Res 2022)

Vortex lattice

In a rotating system, vortex lines form a lattice:



Keepfer & al, PRB 2020

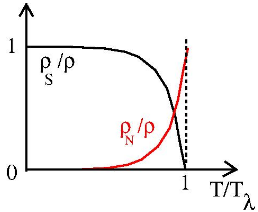
Landau's two-fluid model

Quantum turbulence was first studied by Vinen in the 1958 to understand heat transfer in ^{4}He. Recall Landau's two-fluid model:

- inviscid superfluid component ($\leftarrow$ the ground state)
- viscous normal fluid component ($\leftarrow$ thermal excitations)

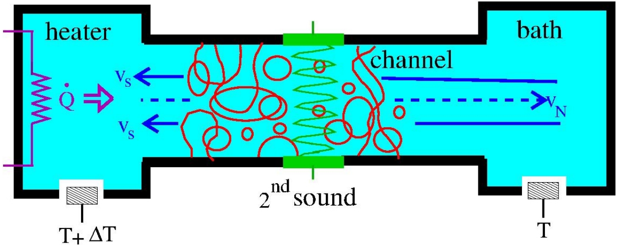
component	density	velocity	viscosity	entropy
normal fluid	ρ_n	v_n	η	S
superfluid	ρ_s	v_s	0	0

where the total density is $\rho = \rho_s + \rho_n$ is approximately constant and ρ_s and ρ_n are strong functions of the temperature T:



Quantum turbulence generated thermally

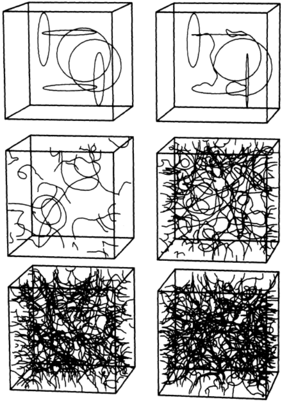
The heat flux $\dot{Q}$ is carried away by the normal fluid ($\dot{Q} = \rho S T v_n$), but, since the mass flux is zero ($\rho_s v_s + \rho_n v_n = 0$), a **counterflow velocity** $v_n - v_s = \dot{Q}/(\rho_s S T)$ is set up, proportional to $\dot{Q}$.



Vinen found that if $\dot{Q} > \dot{Q}_{crit}$, a second sound wave across the channel is attenuated. Inspired by Feynman 1955, he interpreted this effect as due to a **turbulent tangle of vortex lines**.

The vortex tangle

In the late 1980's, numerical simulations confirmed Vinen's scenario (direct visualization was achieved only 20 years later)

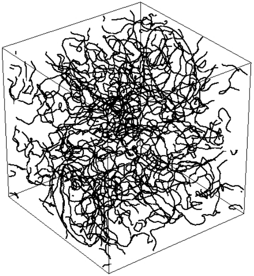


Quantum turbulence thus implies a new non-classical lengthscale ℓ (the **average intervortex distance**) estimated as $\ell \approx L^{-1/2}$, where the **vortex line density** L is the vortex length/volume

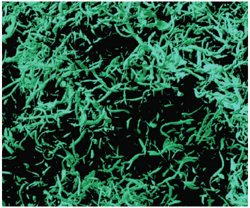
Schwarz, PRB 1988:
growth of vortex tangle from few seeding vortex rings

Quantum vs classical turbulence

In the 1990's numerical simulations of classical turbulence revealed coherent filamentary vortex structures.



Quantum turbulence (Baggaley)



Intense vorticity regions in classical turbulence (Okamoto & al, PoF 2007)

Question: Are these two pictures related ?
(hint: consider the orientation of the vortex lines)

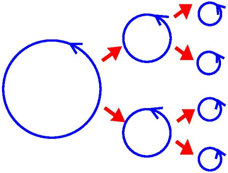
Recall the classical 5/3 law (Kolmogorov 1941)

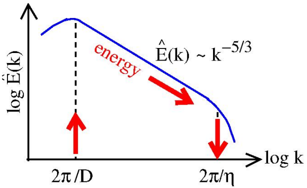
Energy cascade: Energy E, injected at large scale D, is transferred to larger and larger wavenumbers k (smaller and smaller lengthscales) until it is turned into heat at the dissipation scale η.

The energy distribution over the lengthscales is defined by

$$E = \int_0^\infty \hat{E}(k)dk \quad \text{where} \quad \hat{E}(k) = \text{ energy spectrum}$$

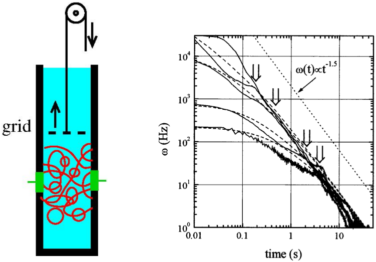
We have $\hat{E}(k) = C\epsilon^{2/3}k^{-5/3}$ (**Kolmogorov spectrum**) where $C \approx 1$ and $\epsilon = -dE/dt$ is the **energy dissipation rate**



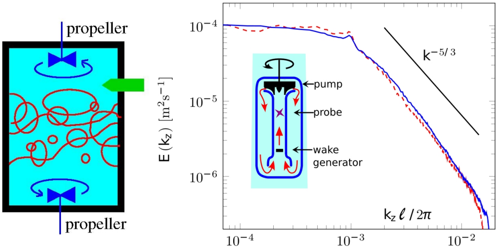


Kolmogorov type

Connections with classical turbulence was achieved in experiments in which the vortex tangle was excited **mechanically rather than thermally**: the some classical vorticity decay and the same $k^{-5/3}$ energy spectrum were observed

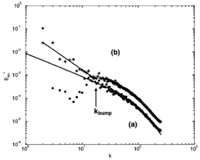


towed grid:
Smith & al, PRL 1993
Stalp & al, PRL 1999

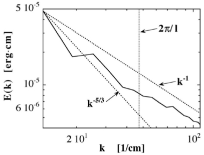


propellers/wind tunnel:
Maurer & Tabeling, EPL 1998
Salort & al, PoF 2012

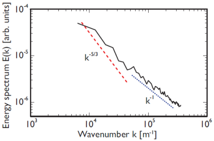
Numerical simulations confirmed the 5/3 scaling in $k_D < k < k_\ell$



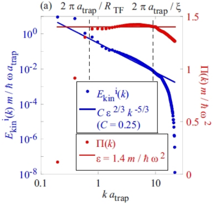
Nore & el, PRL 1997



Araki & al, JLTP 2002



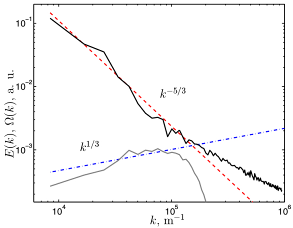
Tsepelin & el, PRL 2017



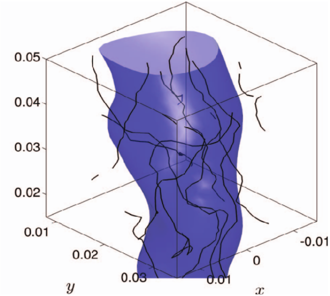
Kobayashi & Tsubota, PRA 2007

Kolmogorov type

Coarse-graining the vortex lines (i.e. accounting for their orientation) makes apparent their **partial polarization** (Baggaley & al, PRL 2012) and the classical $k^{1/3}$ scaling of the vorticity



Baggaley & el, EPL 2017



Baggaley, PoF 2014

Kolmogorov type

Other similarities between classical and quantum turbulence are

- **the 4/5 law**

$$\langle \delta v(r)^3 \rangle = -(4/5)\epsilon r$$

  where $r = 2\pi/k$, observed by Salort & al (EPL 2012)

- the **scaling of the circulation** around loops of area A, which Muller & al (PRX 2021) found is

$$\langle |\Gamma_A|^2 \rangle \sim A^{4/3} \quad \text{for large } A$$

  whereas $\langle |\Gamma_A|^2 \rangle \sim A^2$ for random loops,
  $\langle |\Gamma_A|^2 \rangle \sim A$ for fully polarised lines

In the limit of vanishing viscosity, the energy dissipation rate ϵ tends to a constant (not to zero as in laminar flows). Hence, in the limit of $\mathrm{Re} \to \infty$, the solutions of the Navier-Stokes equation

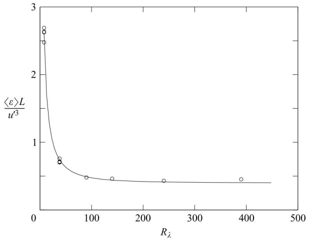
$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{\omega}) + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{\omega}, \qquad \mathrm{Re} = UL/\nu$$

(where $\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$) are not the solutions of the Euler equation

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{\omega})$$

Smaller and smaller viscosity is compensated by the creation of motions at smaller and smaller length scales

Question: in superfluids $\nu = 0$, so is $\mathrm{Re} = \infty$?



Donzis & al, JFM 2005

Solution: Interpret Reyolds as inertia/dissipation,

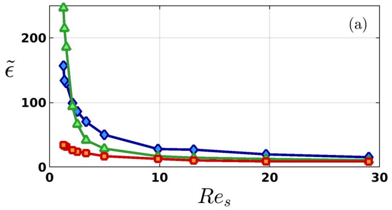
Consider the HVBK equations (which generalize Landau's model)

$$\frac{\partial \boldsymbol{\omega}_s}{\partial t} = (1-\alpha')\nabla \times (\boldsymbol{v}_s \times \boldsymbol{\omega}_s) + \alpha \nabla \times [\hat{\boldsymbol{\omega}}_s \times (\boldsymbol{\omega}_s \times \boldsymbol{v}_s)]$$

where α and α' are friction coeffs. Finne & at (Nature 2003) showed that the superfluid Reynolds is $\mathrm{Re}_s = (1-\alpha')/\alpha$.

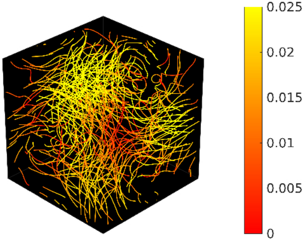
Galantucci & al (PR Fluids 2023) found that, upon increasing Re (for values corresponding to $1.3 < T < 2.16$), $\epsilon \to$ const as in classical turbulence.

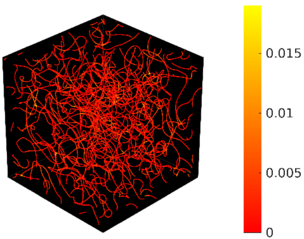
Experimental evidence has also appeared (Mäkinen & al, Nat Phys 2023)



Vinen type

Surprisingly, there are other types of quantum turbulence, as found experimentally (Walmsley & Golov, PRL 2008, Bradley & al, PRL 2006) and numerically (Baggaley & al, PRB 2012), depending on the forcing:



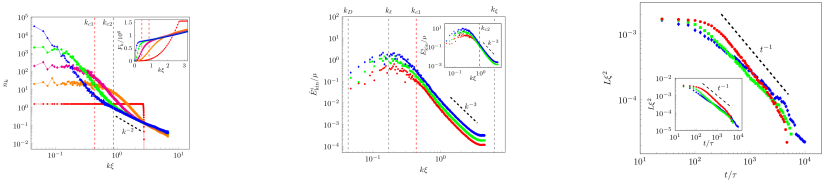


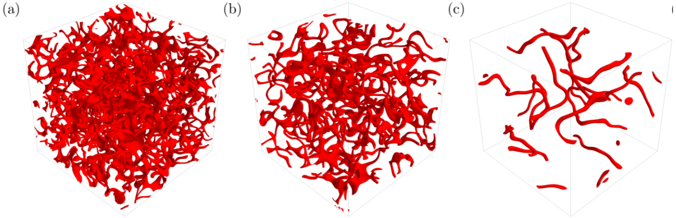
Kolmogorov type turbulence:
partially polarised bundles,
Kolmogorov spectrum,
energy cascade
decay as $L \sim t^{-3/2}$

Vinen type turbulence:
random flow,
no Kolmogorov spectrum,
no energy cascade
decay as $L \sim t^{-1}$

Example of Vinen type: thermal quench of a Bose gas

- Uniform occupation number up to cutoff k, random phase
- Particle cascade to low k, energy cascade to high k
  (Berloff & Svistunov 2007)
- Out of equilibrum $\rightarrow$ turbulence $\rightarrow$ equilibrium



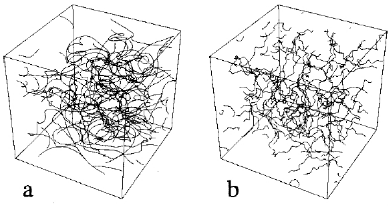


Stagg & al
PRA 2016

Also velocity correlation function decreases rapidly

Turbulence of Kelvin waves

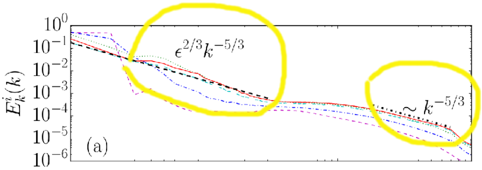
- Experiments by Davis & al (Physica B 2000): turbulence created at mK temperatures decays, despite the absence of normal fluid's friction.
- Svistunov (PRB 1995): for $T \to 0$, there exists a **cascade of Kelvin waves** (helical displacements of the vortex axis of frequency $\omega \sim k^2$) on individual vortex lines, which shifts the energy to such large k that acoustic radiation is possible.
- Tsubota & al (PRB 2000) noticed that for $T \to 0$ a vortex tangle looks different (short waves, cusps)



Vortex tangle with (a) and without (b) friction

Turbulence of Kelvin waves

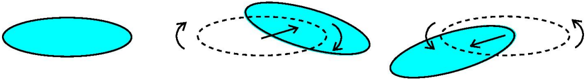
- L'vov & Nazarenko (JLTP 2010) developed a theory of the Kelvin cascade; their prediction that $n(k) \sim k^{-11/3}$ hence $\hat{E}(k) \sim k^{-5/3}$ was numerically verified by Krustulovic (PRE 2012)
- the $T \to 0$ scenario is thus of a **dual cascade**:
  a Kolmogorov cascade of eddies for $k \ll k_\ell$
  and a cascade of Kelvin waves for $k \gg k_\ell$



Leoni & al PRA 2017, Sasa & al PRB 2011

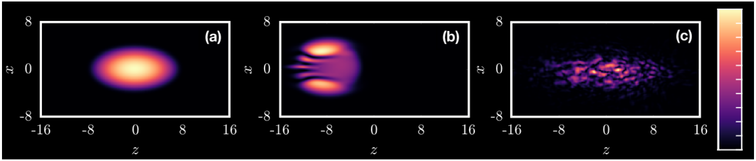
(3D) Quantum turbulence in atomic condensates

Small clouds of ultra-cold dilute gases ($T \approx 100$ nK) are made turbulent by shaking/oscillating them (Henn & al, PRL 2009; Nguyen & al PRX 2019; Navon & al, Nature 2016 and Science 2019)



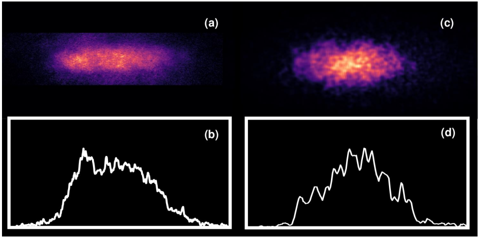
shaking the condensate

Ground state → solitons → waves/vortices



Middleton-Spencer & al arXiv 2022

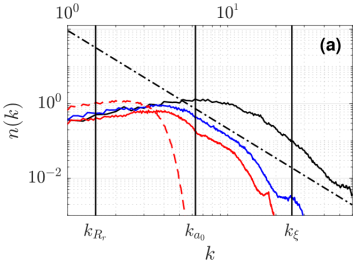
Note the large waves



Top: column-integrated density, bottom: transverse density profile
(a,b) experiments; (c,d) numerics

Middleton-Spencer & al arXiv 2022

Experimental/numerical evidence of inter-scale energy transfer:



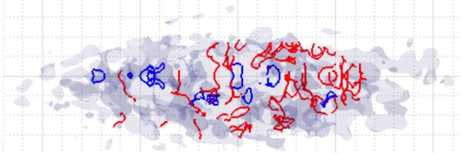
Momentum distribution $n(k)$ vs k at different expansion times assumes scaling behaviour $n(k) \sim k^{-2.6}$ in a small but consistent k-range in agreement with experiments.

Compare with $n(k) \sim k^{-3.5}$ and $k^{-3.2}$ in oscillated boxtrap (Navon & al 2016; Dogra & al 2022)

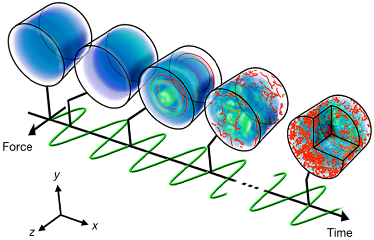
Middleton-Spencer & al arXiv 2022

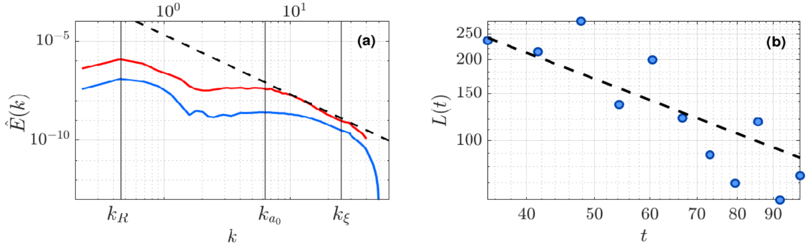
Quantum turbulence in atomic Bose-Einstein condensates

Large density waves and very small vortex loops distributed inhomogeneously; absence of long vortex filaments



Similar vortex configurations appear in numerical simulations of the experiment by Navon & al (Nature 2016) in an oscillated boxtrap





- The incompressible energy spectrum $\hat{E}(k)$ vs k (left) has no scaling features (besides k^{-3} due to vortex cores)
- The decay (right) is roughly Vinen like ($L \sim t^{-1}$)

Middleton-Spencer & al, arXiv 2022;

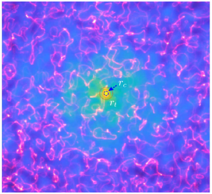
Quantum turbulence in atomic Bose-Einstein condensates

Shaking/oscillating the condensate creates a turbulence mainly consisting of large density waves. Waves and scaling of $n(k)$ recall the **classical weak turbulence of waves** (Zakharov 2012, Nazarenko 2011) for which $n(k) \sim k^{-3}$, but here waves have very large amplitudes (leading to fragmentation) and there are also small vortex loops. In analogy, we can call it the **Strong Turbulence** type.

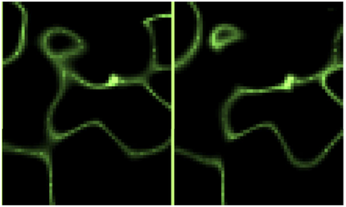
	Kolmogorov type	**Vinen type**	**Strong type**
energy peaks at	small k	intermediate k	-
$\hat{E}(k) \sim$ scaling range	$k^{-5/3}$ $k_D \ll k \ll k_\ell$	k^{-1} $k \approx k_\ell$	k^{-3} $k \lesssim k_\xi$
$L(t) \sim$	$t^{-3/2}$	t^{-1}	t^{-1}
vortex configuration	partially polarised	random	random

Quantum turbulence in astrophysics

Light bosons models of dark matter in galactic halos based on self-gravitating condensates

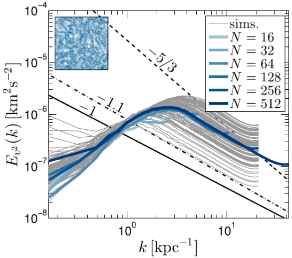


(I-K Liu & al, MNRAS 2023)



Vortex reconnections

Mocz & al, MNRAS 2017



Vinen type energy spectrum

- Identification of **Kolmogorov**, **Vinen** and **Strong types** to help thinking of quantum turbulence
- The original **thermal counterflow turbulence** of Vinen seems more complicated

Thank you!

Reference:

- CFB, Middleton-Spencer, Galantucci and Parker, AVS Quantum Sci. **5**, 025601 (2023)
- CFB, Skrbek, and Sreenivasan, *Quantum Turbulence*, Cambridge University Press, to appear (2023)