Vortex Interactions in Dipolar Superfluids

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Introduction

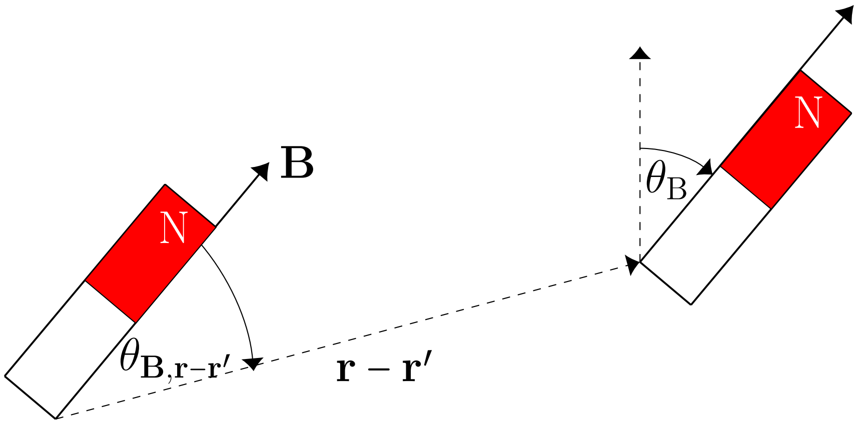
- Interactions in superfluids cannot always be simplified as local 'contact' interactions.
- Dipole-dipole interactions (DDIs) occur naturally in strongly magnetic Bose-Einstein condensates (BECs); must be treated separately to the short-ranged interactions
- Novel effects arise due to the long-ranged and anisotropic nature of the DDI – rotons, supersolids and stripes, modified vortex-vortex interactions and dynamics, exotic vortex lattices

Dipole-dipole interactions (DDIs)

- Cr, Er, Dy, Eu are *strongly dipolar*

$$V_d(\mathbf{r}) = \frac{3\varepsilon_{\rm dd}}{4\pi}\frac{1-3(\widehat{B}\cdot\hat{\mathbf{r}})^2}{r^3}$$

- Tuneable ratio (ε_{dd}) of DDI to 'contact' interaction strengths
- *Magnetostriction*: dipoles preferentially align parallel to **B**.



	ε_{dd}	Year
^{87}Rb	0.007	1995
^{52}Cr	0.15	2005
^{162}Dy	1.06	2015
^{164}Dy	1.42	2011
^{166}Er	1.06	2016
^{168}Er	0.48	2012
^{151}Eu	0.54	2022

Dipolar interactions in the mean-field

- Gross-Pitaevskii theory: we explicitly account for the DDI in addition to the short-ranged interactions

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\nabla^2\psi + V\psi + |\psi|^2\psi + \int \mathrm{d}^3r\, V_d(\mathbf{r}-\mathbf{r}')|\psi(\mathbf{r}')|^2\psi(\mathbf{r})$$

$$V_d(\mathbf{r}) = \frac{3\varepsilon_{\mathrm{dd}}}{4\pi}\frac{1-3(\widehat{B}\cdot\hat{\mathbf{r}})^2}{r^3}$$

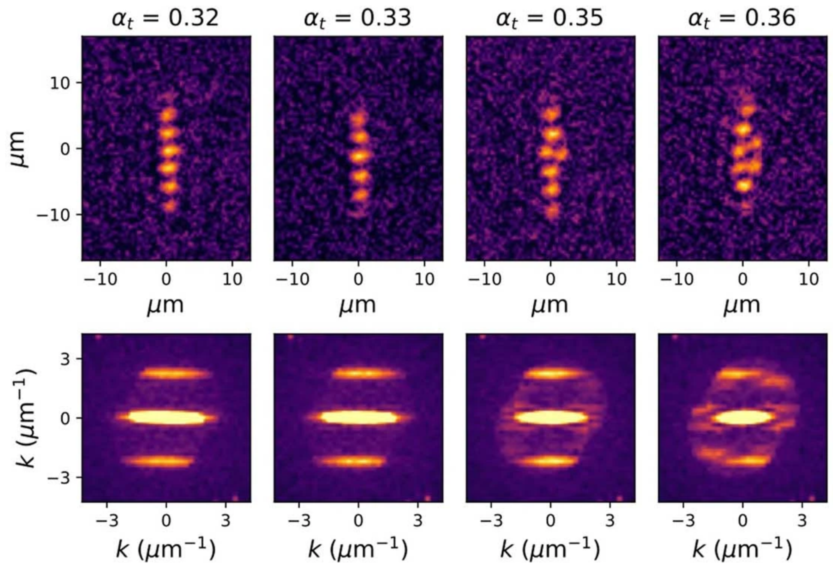
$$\widehat{V}_{\mathrm{d}}(\mathbf{k}) = \varepsilon_{\mathrm{dd}}[3(\widehat{B}\cdot\hat{\mathbf{k}})^2 - 1]$$

- Mean-field theory stability occurs only for $-0.5 \leq \varepsilon_{dd} < 1$, cf. a modified Bogoliubov spectrum, speed of sound and Landau critical velocity

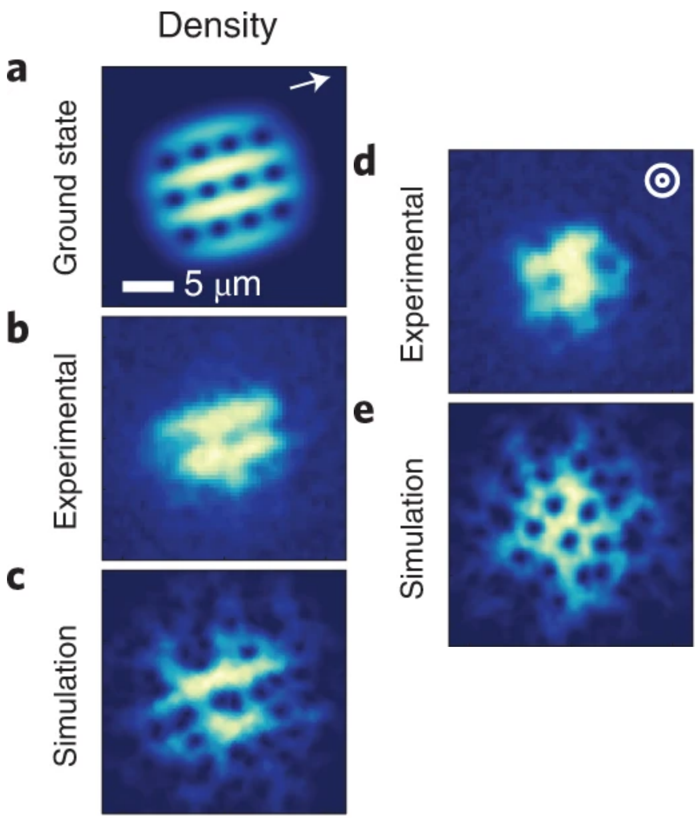
$$\omega^2(\mathbf{k}) = \frac{k^2}{2}\left\{\frac{k^2}{2} + 2n_0[1+\varepsilon_{\mathrm{dd}}(3\cos^2\theta_{\widehat{B},\hat{\mathbf{k}}} - 1)]\right\}$$

$$\frac{c^2(\hat{\mathbf{k}},\varepsilon_{\mathrm{dd}})}{c^2(\varepsilon_{\mathrm{dd}}=0)} = 1 + \varepsilon_{\mathrm{dd}}(3\cos^2\theta_{\widehat{B},\hat{\mathbf{k}}} - 1)$$

Dipolar BECs – miscellaneous phenomena



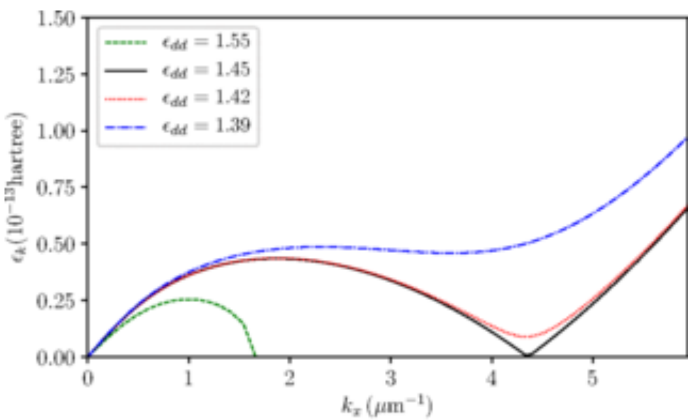
1D to 2D supersolid transition;
M. Norcia et al., Nature 596, 357 (2021)



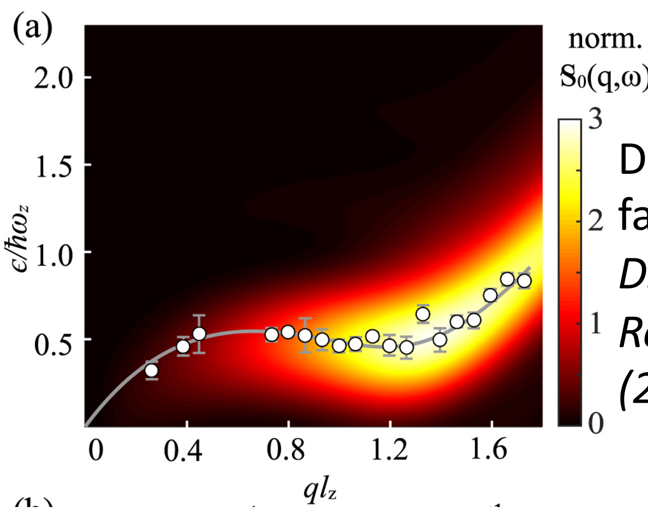
Experimental production of vortices;
L. Klaus et al., Nat. Phys. 18, 1453 (2022)

Dipolar rotons

- Localised depletion of superfluid (strong confinement, vortices…) results in DDI *screening* by *virtual* dipole moments polarised *antiparallel* to the real dipole moments.
- Dipole alignments with a nonzero projection orthogonal to the depletion boundary produce *density oscillations*, i.e. *rotons*!

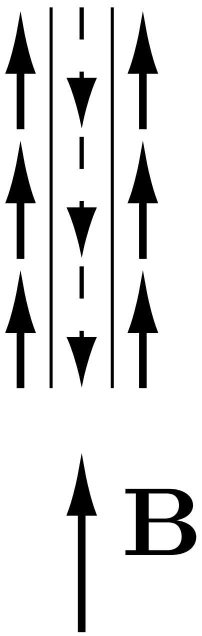


Roton instability spectrum
L. Santos et al., Phys. Rev. Lett. 90, 250403 (2003)

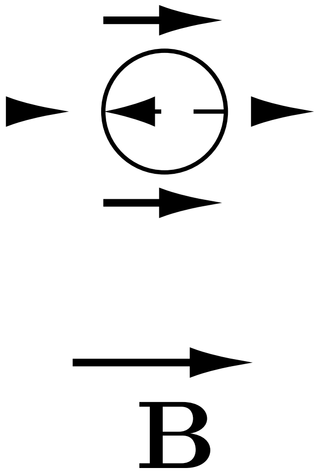


Dynamical structure factor measurement
D. Petter et al., Phys. Rev. Lett. 122, 183401 (2019)

Dipolar screening/virtual dipole moments

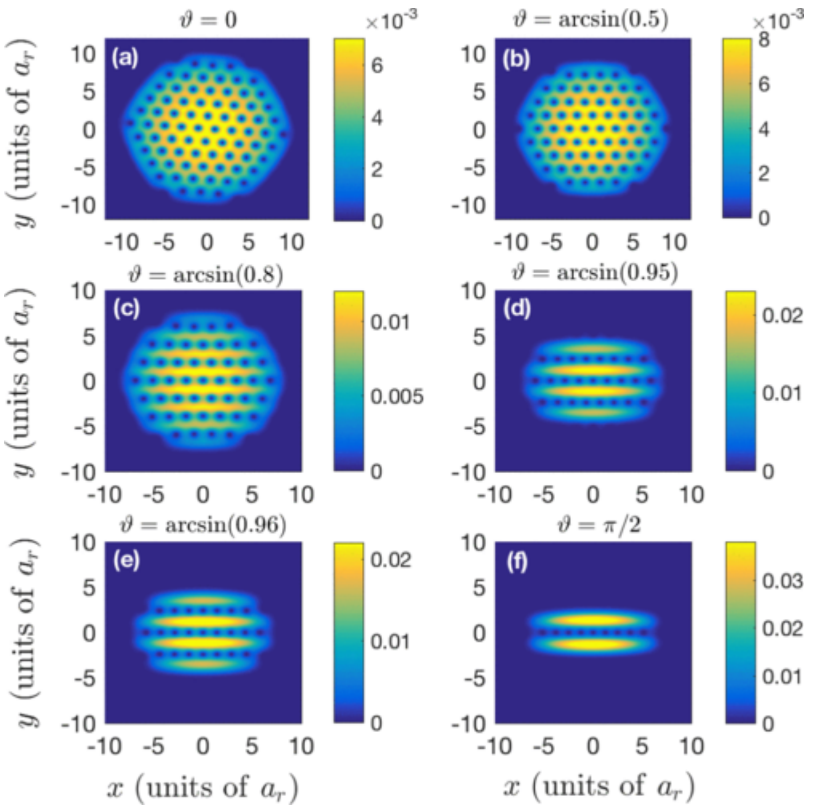


Case 1: magnetic field parallel or anti-parallel to the depletion zone. Real-virtual interactions cancel out
$\Rightarrow$ no screening

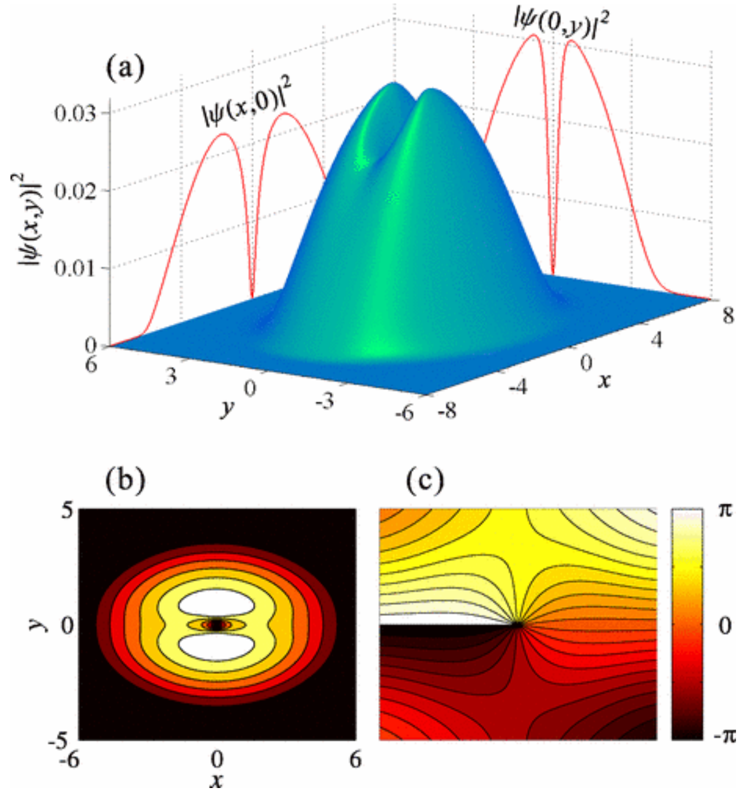


Case 2: magnetic field orthogonal to the depletion zone. Real-virtual repulsion along axis parallel to the field, and attraction along the binormal

Dipolar vortices



Vortex lattice ground states
Y. Cai … W. Bao, Phys. Rev. A 98, 023610 (2018)



Vortex in a harmonic trap
S. Yi and H. Pu, Phys. Rev. A 73, 061602(R) (2006)

Vortex ground states in a uniform background

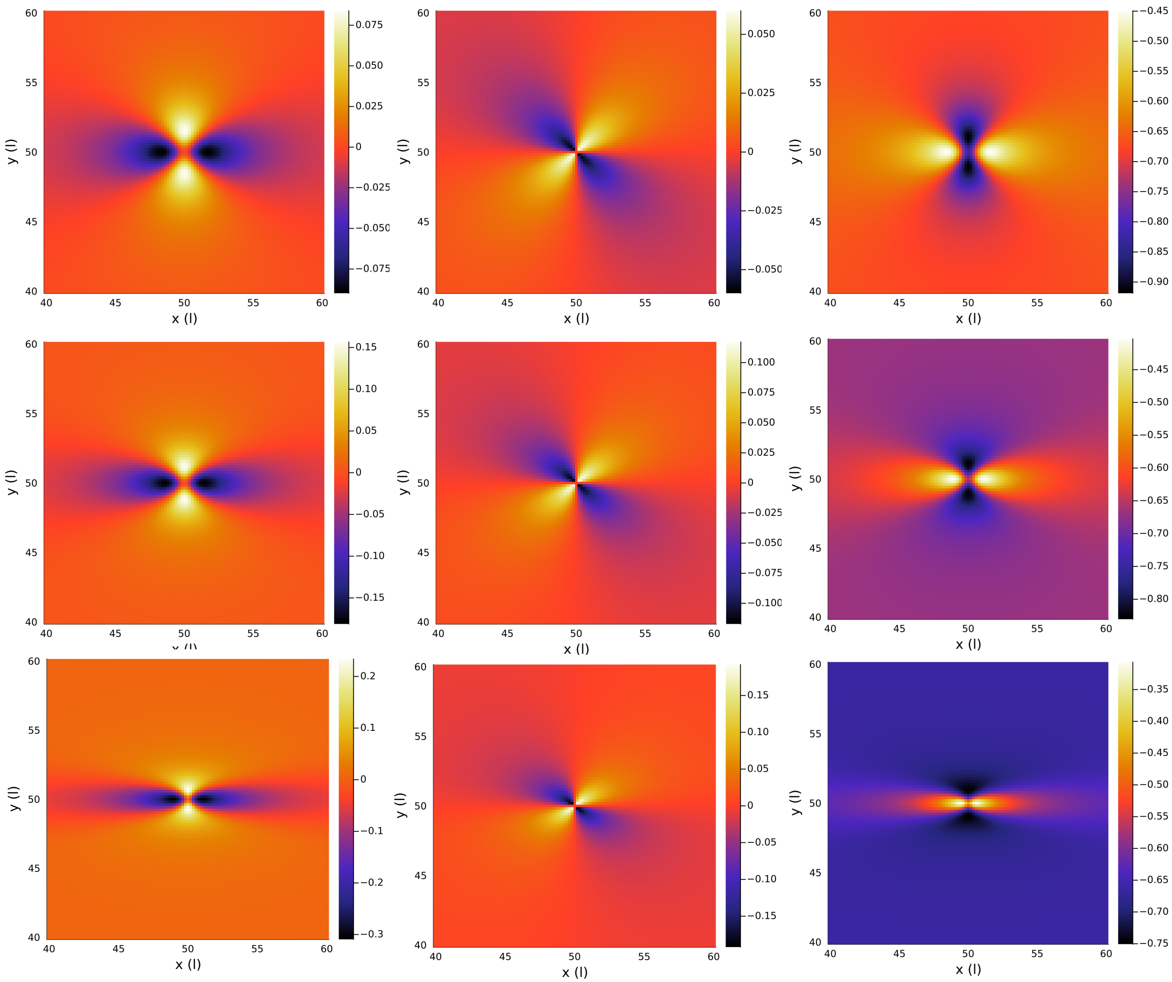
- Madelung transformation: $\psi = \sqrt{n}e^{iS}; \, \mathbf{v} = \nabla S$
- Consider vortices along the z-axis with dipoles aligned along the x-axis.

  Compressibility: anisotropic densities ⇄ anisotropic velocities
- Impose Neumann boundary conditions

$$\partial_x \psi(0,y) = \partial_x \psi(L_x, y) = \partial_y \psi(x, 0) = \partial_y \psi(x, L_y) = 0$$

  and find ground states of the GP equation with a vortex at the centre of the grid.
- Rotonic density peaks develop at minima of the local DDI potential and the vortex cores are elongated along the dipole alignment
- Lobes of compressibility surround the vortex cores
- In agreement with “quasi-2D” results of *Mulkerin et al., Phys. Rev. Lett. 111, 170402 (2013)*

Vortex ground states in a uniform background

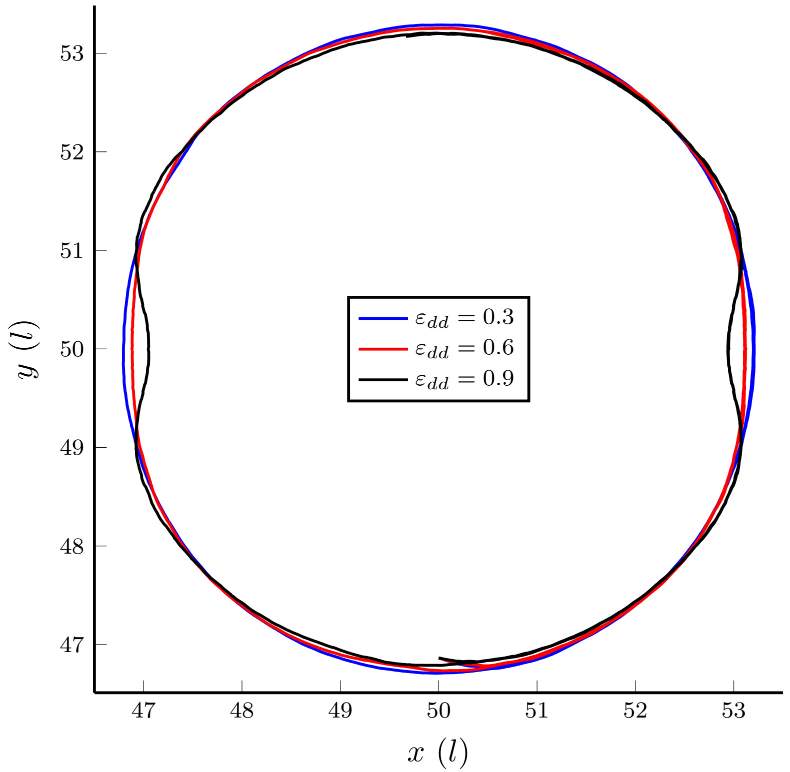


Rows: ε_{dd} = 0.3 (top row), 0.6 (middle row), 0.9 (bottom row).

Columns: n(x,y) - ⟪n(ρ)⟫ (left column), S(x,y) - ⟪S(ρ)⟫ (middle column), ⟨V_d(x,y)⟩ (right column)

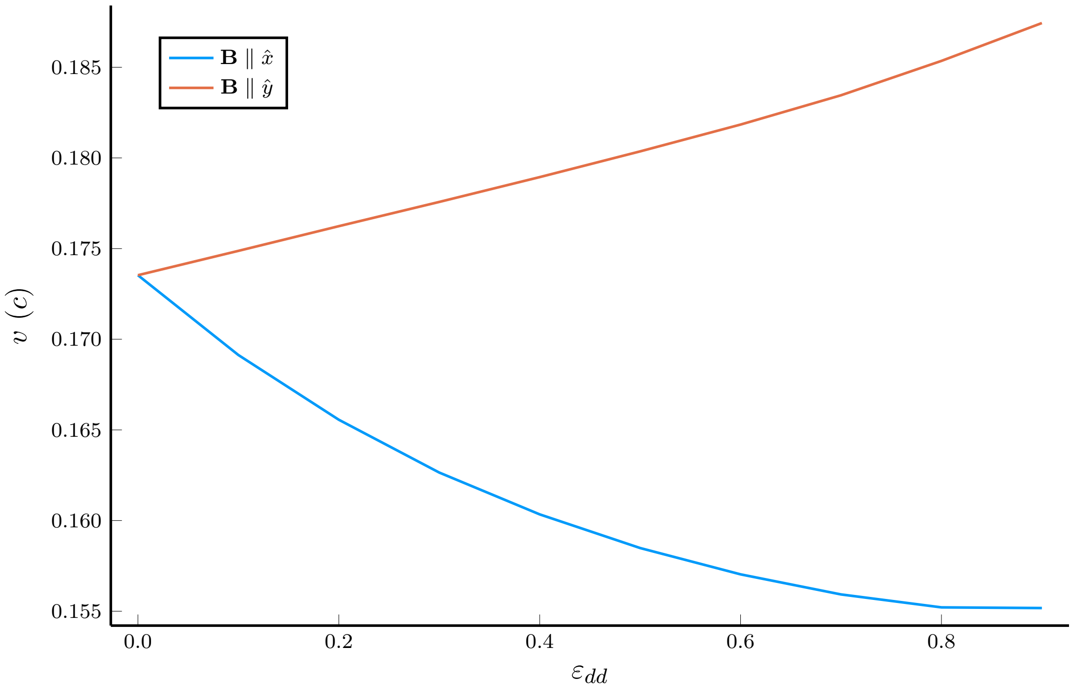
Same-signed vortex pair dynamics

- Again, consider dipole alignments along the x-axis but with a pair of vortices of the same circulation ($\Gamma = 1$)
- The trajectories deviate from the circular orbits predicted by the point-vortex/Biot-Savart model
- The trajectories are elongated along the y-axis and become dumbbell/Cassini oval-like for large ε_{dd}.
- Interplay of effective vortex-vortex dipolar interaction energy and vortex-vortex hydrodynamic energy determines the pair trajectories



Opposite-signed pair dynamics

- Now, consider the opposite-signed, with a vortex dipole moment along the y-axis
- We allow the magnetic dipole moments to be polarized along either the x- or y-axes
- The vortex dipoles translate at constant velocity, but the velocities diverge from the nondipolar Biot-Savart law
- The vortex-vortex DDI interaction energy decreases (increases) for higher ε_{dd} along the x-(y-)axes, causing higher (lower) point-vortex energies



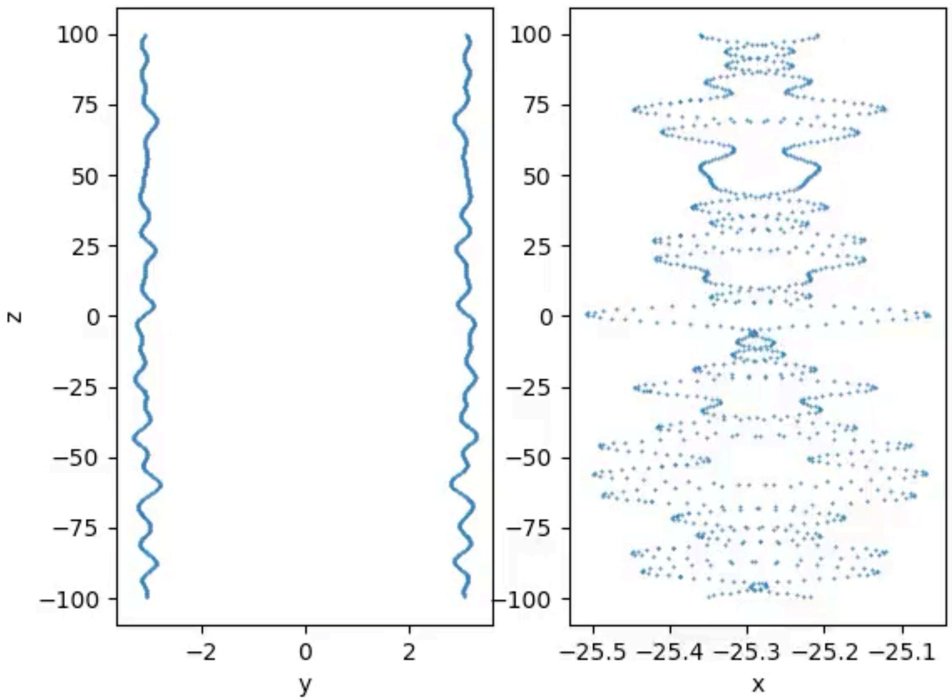
Vortex dipoles, Kelvin waves, reconnections

- Consider two vortex filaments with opposite circulations and with Kelvin waves (shifted in phase by π w.r.t. each other) on each filament
- Self-induction ⇒ time-dependent Kelvin mode population due to different segments of the filaments travelling at different speeds
- The *symmetric* modes are dynamically unstable (*Kuznetsov & Rasmussen 1995; Phys. Rev. E 51, 4479*), leading to reconnections into vortex loops that cascade into smaller loops
- Analogous to the classical turbulent phenomenon of aircraft and wingtip vortex interactions studied by Crow in 1970 (*AIAA Journal 8, 12, 2172*)

Crow instability

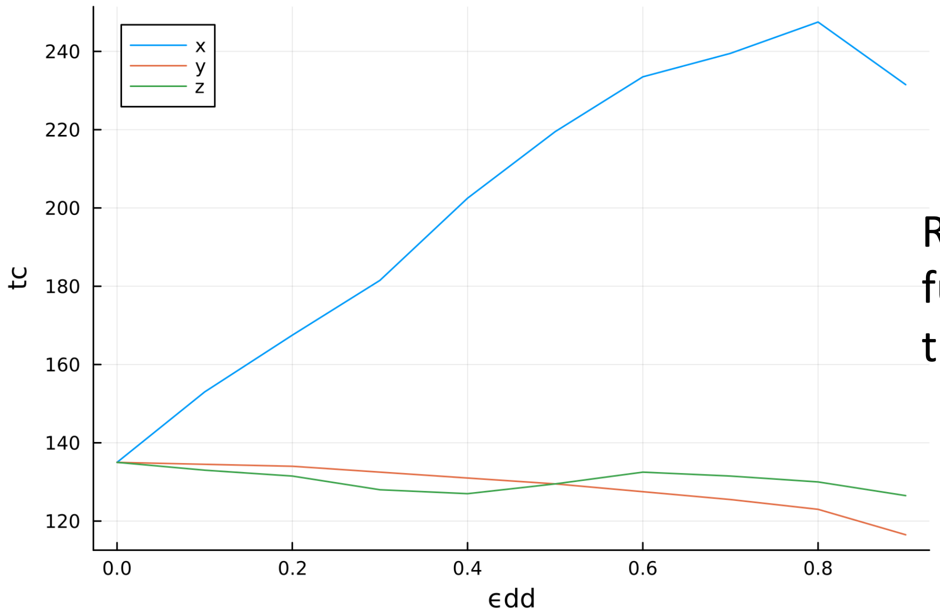
For $\varepsilon_{dd} = 0$, we initialise a vortex dipole with a separation of 6.25 healing lengths *along y*, populate the lowest 20 Kelvin modes on each vortex and propagate the GP equation to observe the evolution of the vortices.

(N.B. This ‘figure’ is really a video; please play the accompanying .m4v file!)



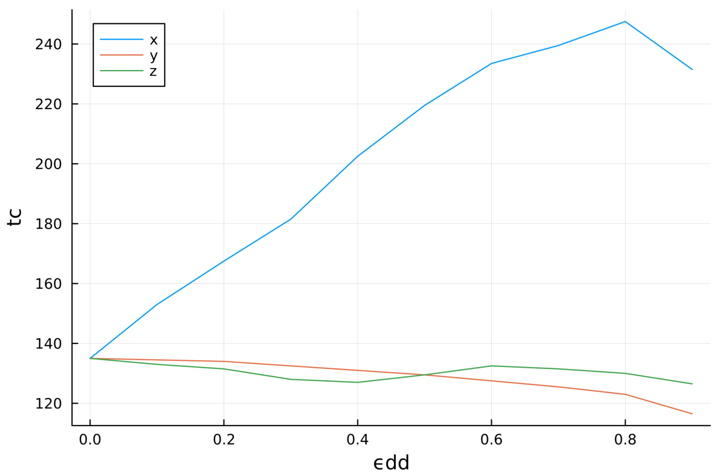
Dipolar vortex reconnection times

- How does the DDI influence the onset of the Crow instability?
- The instability is driven by curvature along the binormal axis ⇒ the Kelvin wave population is dependent on ε_{dd} and the direction of **B**.
- Use the same 'noise' profile and initial vortex-vortex displacement for **B** ∥ {**x**, **y**, **z**}, with an initial separation of 6.25 healing lengths along y.

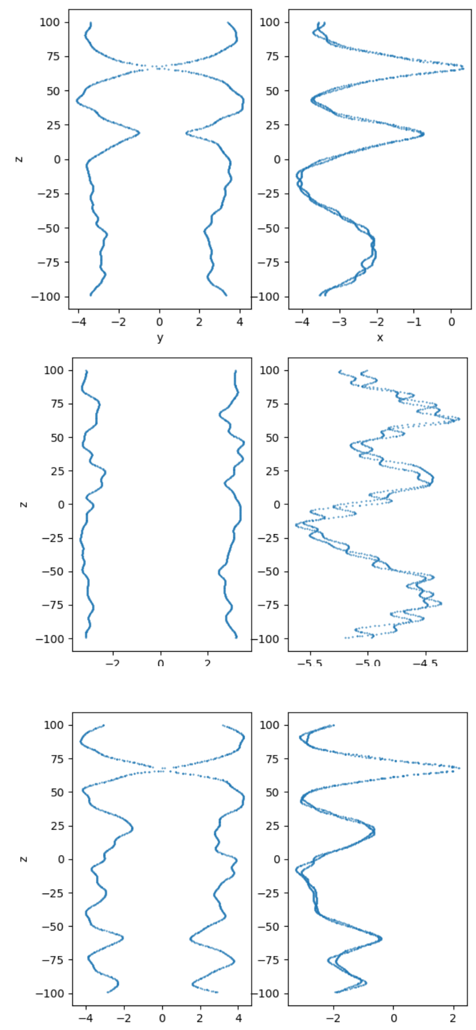


Reconnection times as functions of ε_{dd} and of the dipole polarisation

Crow instability: polarisation comparison



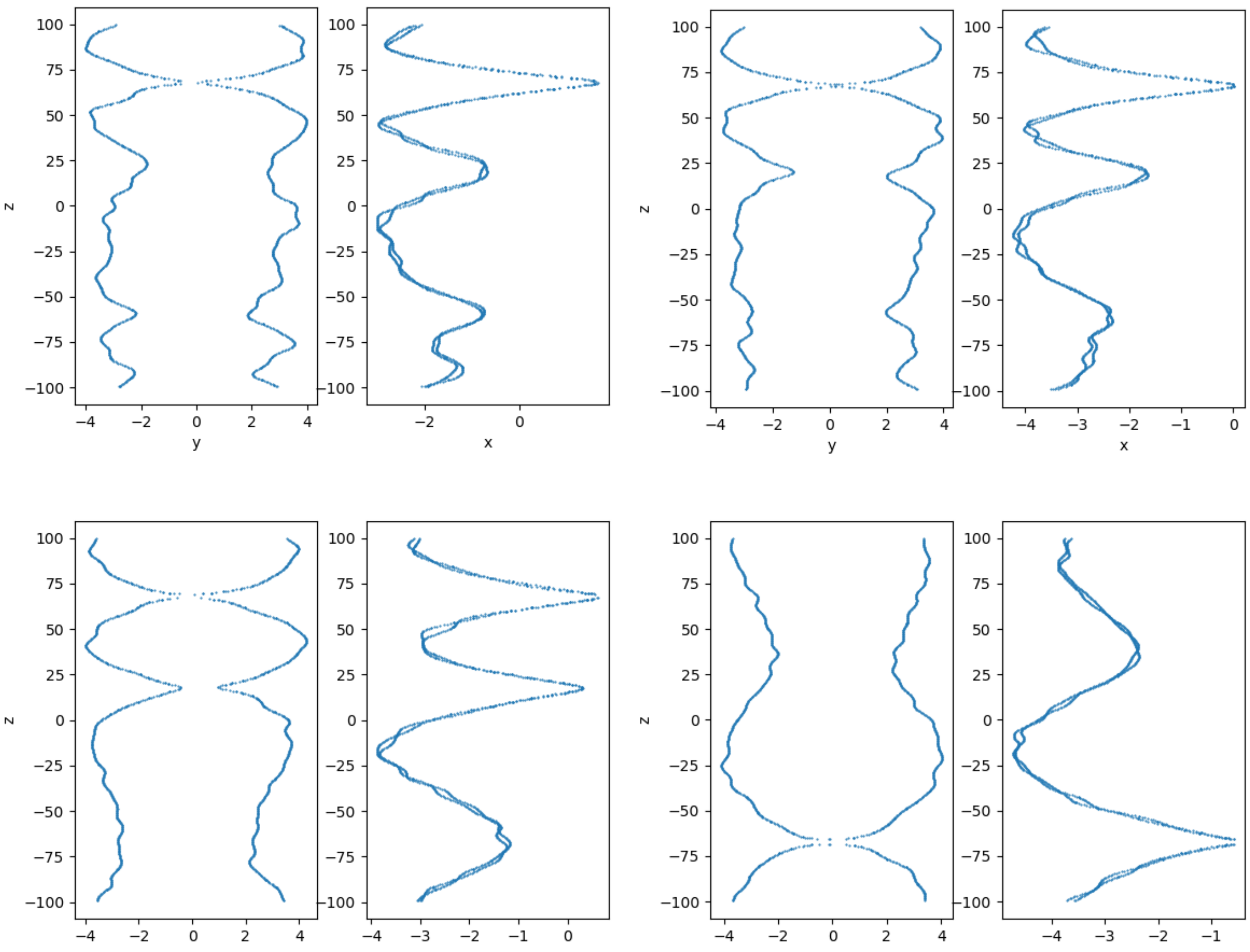
Reconnection times as functions of ε_{dd} and of the dipole polarisation



Vortex lines at time of the first reconnection along x and z with fixed ε_{dd} = 0.5. Dipole polarization along the x-axis (top), y-axis (middle) and z-axis (bottom)

The vortex filaments are fastest for polarisations along y and slowest along x, seemingly explaining the reconnection suppression when the DDI is parallel to the separation

Crow instability: ε_{dd} comparison (B ∥ z)



Vortex lines at time of first reconnection with a dipole polarization along the z-axis; $\varepsilon_{dd} = 0$ (top left), 0.3 (top right), 0.6 (bottom left), 0.9 (bottom right)

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Conclusions and Outlook

- Dipole-dipole interactions considerably alter the qualitative aspects of vortex ground state densities, phase profiles, vortex-(anti-)vortex trajectories *and* vortex-vortex reconnections
- Modifications to Biot-Savart in the presence of the DDI?
- Does the vortex-vortex distance still scale as $|t-tc|^{0.5}$?
- Curvature and torsion of vortex lines during reconnections?
- Time-dependent dipole polarisations can be used to generate vorticity; driven turbulence?